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Specialized Content Knowledge Lower Secondary School Teachers on Quadrilaterals

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ABSTRACT

Pengetahuan guru tentang konten tertentu memiliki hubungan positif dengan prestasi matematika siswa. Guru matematika harus memiliki pemahaman yang sesuai untuk memastikan pembelajaran matematika secara efektif. Segi empat adalah salah satu konten mendasar dalam geometri. Namun, banyak guru tidak berhasil menyampaikan dan membelajarkan konten ini dalam pengajaran di kelas. Oleh karena itu, penelitian kualitatif ini bertujuan untuk mengeksplorasi pengetahuan konten khusus guru sekolah menengah pertama dalam mendefinisikan dan mengklasifikasikan segi empat. Empat guru terdiri atas dua laki-laki dan dua guru perempuan menjadi partisipan dalam penelitian ini. Semua partisipan memiliki pengalaman mengajar yang sama dan tidak memiliki sertifikat pendidik. Tes dan wawancara semi-terstruktur ditugaskan untuk memeroleh pengetahuan konten khusus guru tentang segiempat. Hasil wawancara guru dianalisis dalam tiga tahap: kondensasi data, penyajian data, dan penarikan kesimpulan dan verifikasi. Hasil analisis data menunjukkan bahwa hanya satu guru yang memahami secara hierarkis dalam mendefinisikan dan mengklasifikasikan segi empat. Dua guru berada pada tingkat prototipe parsial karena mereka tidak dapat melihat hubungan hierarkis di antara segi empat. Selanjutnya, seorang guru lainnya berada pada tingkat pemahaman prototipe karena ia bergantung pada prototipe bentuk segi empat. Temuan ini menunjukkan bahwa pengetahuan konten khusus guru sekolah menengah pertama tentang segi empat perlu diperkuat melalui lokakarya dan pelatihan pengembangan profesional.

The teachers' knowledge of specific content has a positive relationship with the students' mathematics achievement. Mathematics teachers must have an appropriate level to ensure mathematics learning effectively. The quadrilateral is one of the essential contents in geometry. However, many teachers did not successfully deliver and teach this content in classroom instruction. Therefore, this qualitative study explores the specialized content knowledge of lower secondary teachers in defining and classifying quadrilaterals. Four teachers, two male and two female teachers, were recruited to become participants in this work. All participants have similar teaching experience and do not hold an educator certificate. A test and semi-structured interviews were assigned to obtain specialized content knowledge of the teachers on quadrilaterals. The interview data were analyzed in three stages: data condensation, data presentation, and conclusion drawing. The findings show that only one teacher understands hierarchically in defining and classifying quadrilaterals, two teachers are at the partial prototype level because they cannot see the hierarchical relationship between the quadrilaterals, and another teacher is at the prototype understanding level because it relies on the prototype of quadrilaterals' shape. These findings suggest that lower secondary teachers' special content knowledge of quadrilateral needs to be strengthened through workshops and training professional development.

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INTRODUCTION

The challenge for teachers in the learning process in the current era is not only how to have qualified content knowledge and pedagogy but also how to respond to the quick advances of the disruption era. Teachers play a crucial role in enhancing the quality of learning, including student achievement. Teachers must master competency standards that include pedagogical, personality, social, and professional competence. In mathematics instruction, the teacher plays a paramount role in providing opportunities for students to become mathematically proficient and, at the same time, fostering or creating a classroom atmosphere that supports the students' representations to build and understand the mathematical idea (Pagiling & Munfarikhatin, 2020). One of the frameworks widely used by scholars to explore a teacher's pedagogic and professional competencies is pedagogical content knowledge (PCK), which is a combination of content knowledge with pedagogical knowledge initiated by Sulman (Shulman, 1986).

PCK is the basic knowledge needed by teachers to guide them in making decisions or taking any action in teaching in class (Atay et al., 2010; Hidayati & Widodo, 2015; Pagiling & Taufik, 2022; Setyaningrum et al., 2018). This framework was further developed by Ball and her colleagues (Ball et al., 2008) into two major domains, namely (1) Subject Matter Knowledge (SMK) and (2) Pedagogical Content Knowledge (PCK). The knowledge content scientific field of mathematics known as MKT (mathematical knowledge for teaching) consists of Common Content Knowledge (CCK) which refers to knowledge and skills of general math, Specialized Content Knowledge (SCK) which refers to knowledge and math skills specific content, and Horizon Content Knowledge (HCK) which refers to what knowledge to connect different math topics (Hill et al., 2008). PCK consists of Knowledge of Content and Students (KCS), Knowledge of Content and Teaching, and Knowledge of Content and Curriculum. Thus, there are six domains of teacher knowledge. The MKT framework provides a valuable lens to view teacher knowledge required for effective teaching from mathematics's practical teaching (Hill et al., 2008; Marbán & Sintema, 2020).

SCK acknowledges the specialized aspect of a teacher's mathematical knowledge in contrast to the mathematical knowledge required of other professionals who utilize mathematics (Carrillo-Yañez et al., 2018). Beyond knowing how specialized content knowledge also includes knowing why. Teachers need specialized mathematical expertise when recognizing student faults or determining if a nonstandard technique will be effective (Copur-Gencturk & Lubienski, 2013). From the abovementioned explanation, it can be concluded that SCK is one of the knowledge components teachers should possess under subject matter knowledge. Thus, SCK can be defined as unique mathematics knowledge and the ability to teach.

Several studies have established the relationship between teachers' PCK and student achievement (Barut et al., 2021; Hill et al., 2005; Saleh et al., 2021). Barut et al. (2021) pointed out that most of the teachers had PCK in the low category, and there is a significant positive correlation between mathematics teacher PCK and student achievement with teacher PCK to student achievement of 16.1%. Saleh and colleagues (2021), who explored 69 in-service teachers in Zanzibar, demonstrated that the level of PCK for mathematics teachers was moderate, and the implementation of PCK in classroom practices was low. These results were consonant with the findings of the previous study which emphasized teachers' mathematical knowledge was significantly related to student achievement gains in both first and third grades (Hill et al., 2005).

Although some attempts have been made to address this issue (Aslan-tutak & Adams, 2015; Budiarto et al., 2021; Türnüklü et al., 2013; Zeybek, 2018), it is still not clear whether teacher SCK in defining and classifying quadrilaterals. Aslan-tutak & Adams (2015) reported that pre-service teachers have limited geometry content knowledge. On the other hand, Budiarto et al. (2021) explored 82 teachers' specialized content knowledge about squares and pointed out that specialized content knowledge for first teachers is better than young and intermediate teachers since it is able to reconstruct concepts of a square. Meanwhile, young Teachers and intermediate teachers are still influenced by concept images and figural concepts. Türnüklü et al. (2013) suggested that individuals make two types of quadrilateral classifications. The first is a hierarchical classification by connecting the rectangles under the subset according to their properties. The second is partition classification, which means quadrilaterals in separate sets according to their properties. Zeybek (2018) showed that although student teacher candidates have a formal definition, prototypical images influence a person's conception, especially the description of the shape of a concept. Thus, inclusive relations between quadrilaterals are one of the difficulties for prospective teacher students. For example, prospective teacher students have difficulty seeing the relationship between a trapezoid and other quadrilaterals.

Villiers (1994) scrutinized that the quadrilateral family diagram should be used as a medium to show the hierarchical relationship between quadrilateral shapes to make it easier for students to understand the concept of quadrilateral accurately. Many teachers depend on definitions written in textbooks not to facilitate students' construct an understanding of the definition of quadrilaterals shapes (Disnawati et al., 2012; Fujita et al., 2019; Miller, 2018). According to Fujita (2012), teachers must understand a geometric image's conceptual and figural theory in quadrilateral learning. Furthermore, the teacher needs to understand students' thinking process in studying quadrilaterals as reflective material in designing learning based on students' geometric thinking levels.

The teacher's ability to possess quadrilaterals content significantly impacts students' teaching of the material. The previous study showed that the content knowledge of prospective teachers and teachers in the quadrilateral was inadequate (Butuner & Filiz, 2017; Çontay & Duatepe-Paksu, 2012). Twenty mathematics teachers could define a kite hierarchically; only one sample could hierarchically define a trapezoid. In addition, participants struggled with identifying the kite and trapezoid family and displaying the relations of kite-rhombus, trapezoid-parallelogram, and trapezoid-rectangle quadrilateral (Butuner & Filiz, 2017). Moreover, Çontay & Paksu (2012) uncovered that only one prospective teacher understood inclusive relations and quadrilateral classification. Additionally, some research results investigated more students who often had difficulty with formal definitions of quadrilaterals shapes. In essence, defining a concept is one of the crucial mathematical activities because it can reveal a precise and clear understanding of a mathematical concept. Fujita's study scrutinized that students faced difficulty in hierarchical classification due to their inability to perceive hierarchical relationships among quadrilateral and have prototypical images in their minds (Fujita, 2012; Fujita & Jones, 2007).

This present study focuses on the teachers' specialized content knowledge (SCK), which is believed to be a tool to determine how teachers deeply understand the quadrilateral they will teach. SCK includes representing mathematical ideas accurately, providing mathematical explanations for standard rules and procedures, and examining and understanding the unusual solution to a problem (Hill et al., 2008; Ball et al., 2008). One of the materials that have challenges for teaching teachers is quadrilaterals, which are the geometry domain. Many geometric shapes contain family relationships (familial relations). Therefore, understanding the family relationship is essential in the mathematics curriculum for teaching geometry concepts. A robust understanding of quadrilaterals definition and classifications is crucial for teachers since it influences their pedagogical approach to ensure students understand and classify quadrilaterals (Avcu, 2022). For instance, teachers must be aware that the same quadrilateral can have several equivalents and non-equivalent definitions, which refers to specialized content knowledge.

Quadrilaterals classifications seem crucial to creating a relationship between the quadrilaterals, ultimately providing solutions related to geometric problems that need verification. This issue happens because if the quadrilateral is in the same family as other squares, it will apply to the other quadrilateral besides the explanation, evidence, and related properties. Teachers' understanding of the definition of quadrilaterals may be reflected in their knowledge of hierarchical or partitional classification of shape types, which may be influenced by both their knowledge of shape types and their conceptions of definition (Miller, 2018; Nur'aini & Pagiling, 2020). Partitional definitions separate shapes into distinct groups, whereas hierarchical definitions produce an ordered order among shape types in which some shapes can be considered as subtypes of another. In this study, we proposed definitions of every quadrilateral. A parallelogram is defined as a quadrilateral that has two pairs of parallel lines. The rectangle is a parallelogram that has a right angle, a rhombus is a parallelogram with two congruent adjacent sides, and a square is a rectangle that has two congruent adjacent sides.

Although numerous studies have been conducted on quadrilaterals, very few of them investigated specialized content knowledge mathematics teachers on quadrilaterals (Avcu, 2022; Cansiz Aktaş, 2016; Duatepe-Paksu et al., 2012 ; Haj-Yahya et al., 2022; Miller, 2018; Türnüklü et al., 2013). The majority of studies examined students' definitions (Cansiz Aktaş, 2016; Haj-Yahya et al., 2022), pre-service elementary teacher (Duatepe-Paksu et al., 2012; Miller, 2018), and pre-service secondary teacher (Avcu, 2022). Moreover, studies on mathematics teachers' knowledge or understanding of the quadrilateral in Indonesia have not been widely documented. At the same time, student achievement is mainly determined by teachers' knowledge and teaching practices in the classroom. Therefore, we argue that the SCK of lower secondary mathematics teachers on quadrilaterals needs to be further comprehensively examined.

METHODS

This study was designed to explore specialized mathematics content knowledge for lower secondary school mathematics teachers on a small scale. This study aims to explore SCK teachers in defining and classifying quadrilaterals. Therefore, an exploratory study using a qualitative approach was implemented in this work (Miles et al., 2014).

Four lower secondary mathematics teachers, two male teachers, and two female teachers in Merauke Regency were recruited to become participants. We purposively chose four participants; FT1 (the first female teacher), FT2 (the second female teacher), MT1 (the first male teacher), and MT 2 (the second male teacher). MT2 has a background education undergraduate in mathematics education and post-graduate in mathematics; the others only graduated in undergraduate mathematics education. They have equivalent teaching experience (more than 5-years of teaching) and do not get an educator certificate. We choose the uncertified teacher as participants to gain a deeper grasp of their pedagogical and professional competencies as inputs for professional development in the Teacher Professional Education Program.

We assigned a task to examine teachers' knowledge of defining and classifying quadrilaterals. Subsequently, the researchers conducted in-depth interviews through semi-structured face-to-face interviews with four lower secondary mathematics teachers to get the SCK teacher's data. The results of the interviews were verbatim transcribed. The transcripts were also coded to relate to the teachers' reduced answers in Transcripts 1-4. To better see the data, we displayed it as a variable-by-variable matrix as one of the explaining methods in qualitative data analysis (Miles et al., 2014). We analyze teachers' interview transcripts line by line, sentence, or paragraph with the most appropriate word for participants to interpret teachers' understanding of the quadrilateral concepts. In order to validate the conclusions, investigator triangulation was employed (Rothbaeur, 2008). Each author contributed actively to data collection and verification to reach a consensus.

Fujita's (2012) work has been modified to build a more robust method to explore teachers' specialized content knowledge in defining and classifying quadrilaterals (see Table 1). Quadrilateral definitions are not uncomplicated in mathematics, and one of their tasks is to allow us to classify quadrilaterals 'hierarchically' (Villiers, 1994). A parallelogram, for example, is defined as a quadrilateral with two pairs of parallel lines.'. Though not explicitly stated, this definition suggests that the rhombus, rectangle, and square are also parallelograms (special sorts of) because they contain two pairs of parallel sides. On the other hand, the tendency for someone to perceive shape relying on figural aspects is known as the 'prototype phenomenon. Hershkowitz, as cited in (Okazaki & Fujita, 2007), distinguishes two types of prototypical assessments: 'type 1 prototype example serves as a frame of reference, and visual assessment is applied to other examples' and 'type 2 prototype example serves as a frame of reference, but the subject bases his judgment on the prototype's self-attributes and attempts to impose them on another concept example.

Table 1**.** Teachers' knowledge in defining and classifying quadrilaterals

Level	Description
Hierarchical	The teacher is able to understand a square as a particular case of rectangles and a rhombus, a teacher can accept a square, a rectangle, and a rhombus as a particular case of a parallelogram, and a teacher as a parallelogram is a particular case of a trapezoid.
Partial Prototypical	The teacher has started learning to develop figural concepts. For example, the teacher has accepted that a rhombus is a parallelogram but not for squares and rectangles.
Prototypical	Teachers have their own limited personal figural concepts.

RESULTS

This section presents each teacher's work and excerpt of interview transcripts, indicating their definition and concept of quadrilaterals.

1. The first male teacher's knowledge of concepts and definitions of quadrilateral

    The first is the first male teacher's (MT1) solution and excerpts of interviews to define and classify quadrilaterals.

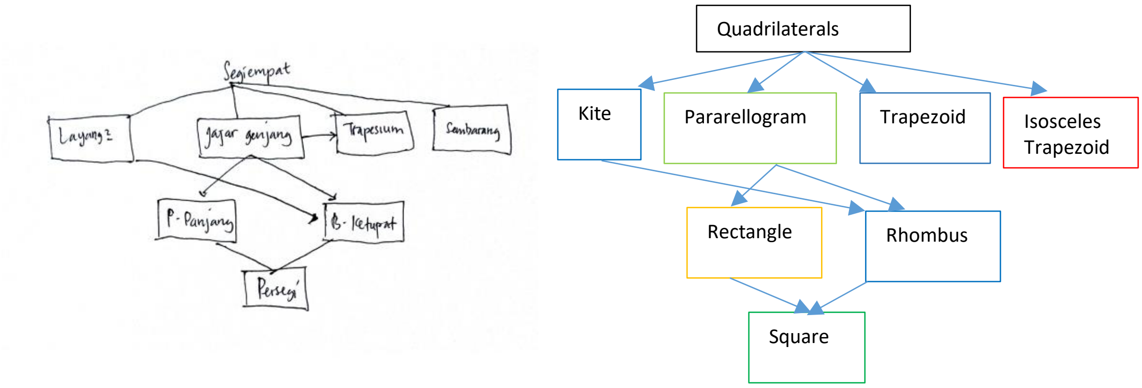

Figure 1. The first male teacher's work

Based on Figure 1, it can be seen that MT1 draws the shapes of quadrilaterals comprehensively. He constructs a relationship between different kinds of quadrilaterals in which some shape types are considered subsets of other shape types (e.g. a square as a subset of a rectangle). He builds a classification of quadrilaterals by portraying them in a family relationship chart. The MT1 is able to identify squares as a subset of the rhombus, given that the critical attribute of the rhombus is also applied to squares. Similarly, the MT1 is able to portray that a rhombus and rectangle were also a parallelogram. However, he failed to recognize that parallelogram then forms a subset of the trapezoid.

Transcript 1.

R	:	*According to you, is every square a rectangle?*
MT1	:	*Yes, Sir*
R	:	*Why?*
MT1	:	*Because if it is a rectangle, it is not sure that it is different in terms of edge size, but if it is a square, it can be said to be a rectangle. So in terms of edge size, we can see it there.*
R	:	*So, from the point of view, yes, Sir?*
MTI	:	*Yes, the length of the sides*
R	:	*So we can define a square as a rectangle whose four sides are equal. Then the second one, Sir, is each square is a rhombus?*
MT1	:	*A square is not a rhombus because every angle opposite it is the same.*
R	:	*For example, the rhombus is a rectangle with the same side length, while the square's four sides are the same, Sir. If I may say, this square is a rhombus with one angle of 90 degrees. Do you think you can, Sir? A square is a rhombus whose side is 90 degrees.*
MT1	:	*Yes, you can*
R	:	*So we can say that the square is part of a rhombus, correct?*
MTI	:	*Yes, Sir*
R	:	*So we can say that this square is a rhombus with one angle of 90 degrees, whereas we know the property of the shape of the quadrilateral is 180. If one angle is 90 degrees, automatically, the other angle is 90 degrees. Then the last one, Sir, is each rectangle a parallelogram?*
MT1	:	*Yes, each rectangle is a parallelogram because a parallelogram is not a rectangle in terms of the angle. If it is a parallelogram, it does not have to be 90 degrees at one angle. If on a rectangle, all the angles are 90 degrees.*
R	:	*Yes, that means we agree that a rectangle is a parallelogram with a right angle or 90 degrees. Yes Sir, then we go on. Is every rhombus a parallelogram?*
MT1	:	*Yes, it is a parallelogram. Because if it is a parallelogram, it is not necessarily a rhombus. If we look at the sides, the rhombus must have the same length on all sides. However, if it is on a parallelogram, it is not the same.*
R	:	*Yes, because there are parallel sides, yes, Sir. Then go on, is each parallelogram a trapezoid?*
MTI	:	*The parallelogram is not part of the trapezoid.*
R	:	*So because of the side characteristics, Sir, if you have a parallelogram, you can have more than one parallel side if it is a trapezoid?*

MTI : *Only one side is parallel.*

Figure 1 and Transcript 1 show that the male teacher MT1 has an understanding that a square is a particular occurrence of a rectangle based on its side size, a square is also a particular case of a rhombus based on its angle, and a rectangle and a rhombus are particular occurrences of a parallelogram based on its sides and the size of the angles. However, in understanding the relationship between a parallelogram and a trapezoid, he does not accept that a parallelogram is a particular trapezoid case. MT1 refers to a trapezoid in Indonesian mathematics textbooks that mostly contain an exclusive definition, emphasizing that the trapezoid is a closed curve with precisely one pair of parallel sides. It can be seen that the MT1 teacher's understanding of the concepts and definitions of square, rectangle, rhombus, parallelogram, and trapezoid is still at the partial prototype level.

2. The second male teacher's knowledge of concepts and definitions of quadrilateral
   The following are the second male teacher's (MT2) answers and excerpts of interviews defining and classifying quadrilaterals.

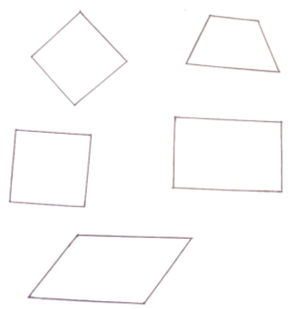

Figure 2. The second male teacher's work

Figure 2 shows evidence that the MT2 cannot depict quadrilaterals family relationships. He has difficulties determining the connections between the kind shapes of quadrilaterals as he sketched them separately. Thus, the MT1 struggle with identifying inclusion relations of the quadrilateral. The following shows the interview transcript between the researcher and the second male teacher (MT2).
Transcript 2

R : *Do you think that every square is a rectangle?*
MT2 : *No, Sir*
R : *Why?*
MT2 : *A square has four equal sides, whereas a rectangle has only two equal sides.*
R : *What square is a rhombus?*
MT2 : *No, Sir. A square is not a rhombus because the square of each angle is 90 degrees and the opposite angle of the rhombus is the same.*
R : *Is each rectangle a parallelogram?*
MT2 : *No, Sir. A square has an angle of 90 degrees. In contrast, the parallelogram is not necessarily an angle of 90 degrees.*
R : *What parallelogram is a trapezoid?*
MT2 : *The parallelogram is not part of the trapezoid.*
R : *Why, Sir?*
MT2 : *The trapezoid has only a pair of parallel sides.*

Figure 2 and Transcript 2 uncover that the male teacher (MT2) has an understanding that the square is not a particular occurrence of rectangles based on the size of the sides, squares are not a particular case of rhombuses based on their angles, and rectangles and rhombus are not a particular occurrence of a parallelogram based on the sides and the size of the angles. He does not understand that a parallelogram is a special case of a trapezoid since MT2 refers to the trapezoid's definition in Indonesian mathematics textbooks that mostly contain an exclusive definition, namely that the trapezoid is a closed curve that has exactly one pair of parallel sides. It can be seen that MT2's understanding of the concept and definition of the square, rectangle, rhombus, parallelogram, and trapezoid are at the prototype level, which indicates that the MT2 teacher has limited figural concepts. The second male teacher struggles to comprehend the hierarchy of quadrilaterals because he conceptualizes shapes in terms of prototype examples rather than their definitions.

3. The first female teachers' knowledge about the concept and definition of quadrilaterals

The first female teacher's (FT1) solution and interview excerpts in defining and classifying quadrilaterals are as follows.

Figure 3. The first female teacher's work

Based on Figure 3, the FT1 portrays the shapes of quadrilaterals comprehensively. She builds a connection between various kinds of quadrilaterals in which some shape types are considered subsets of other shape types (e.g. a square as a subset of a rectangle). She generates a classification of quadrilaterals by portraying them in a family relationship chart. The FT1 is able to identify squares as a subset of rhombus and a rectangle. Similarly, the FT1 is able to illuminate that a rhombus and rectangle were also a parallelogram. Moreover, she is able to recognize that parallelogram as a subset of the trapezoid.

Transcript 3

R	:	*According to you, whether each square includes a rectangle?*
FT1	:	*Yes Sir, whereas if a rectangle is not necessarily a square because we see it from the sides, right, if all the sides are square, then two pairs of sides are parallel to the same length. Suppose the rectangle is two pairs of sides that are parallel and the same length. For example, a square can be said to be a rectangle too.*
R	:	*Yeah, that is right, so the views from the properties of sides and angles. Then go on, is each square a rhombus?*
FT1	:	*If it is a rhombus, the angles opposite each other are the same. Since the angles on the square opposite each other are the same, we can conclude that the square is also a rhombus.*
R	:	*Then, the third question is whether each rectangle is a parallelogram?*
FT1	:	*Same as the second point, yes Sir, if this is the same, the sides facing each other are parallel to the same length, and the angles facing each other are the same. So it can also pack a rectangle said to be a parallelogram.*
R	:	*Is every rhombus a parallelogram?*
FT1	:	*Maybe not, Sir, because if we look at its characteristics, it has a base and a height, then it has two pairs of parallel sides that are the same length, while this rhombus has the same length and only has a diagonal and has no base.*

R : *Hmm. However, if you look at the sides, what do you have in common, ma'am?*
FT1 : *From the sides, it means that the opposite side is the same length*
R : *If we consider parallels' properties, what is your point?*
FT1 : *Same Sir, if it is parallel, along the drawn lines, it will not meet it means that you can, Sir.*
R : *Could you explain clearly?*
FT1 : *Because the angles opposite each other are the same, in other words, a rhombus is a parallelogram, but a parallelogram is not necessarily a rhombus.*
R : *Could we say a rhombus is a parallelogram whose sides are equal in length?*
FT1 : *Yes, you can, Sir*
R : *Continue to the following questions. Whether a trapezoid is a parallelogram?*
FT1 : *If seen from the line, if two pairs of sides are parallel and of the same length, for example, if there is just any trapezoid, there is only one pair of parallel sides, although not necessarily the same length. So if for a parallelogram, its properties are two parallel sides that are equal in length. It means that it can represent one of the properties of the trapezoid.*
R : *However, if the trapezoid has one side that is parallel, yes, ma'am?*
FT1 : *Yes, the trapezoid cannot necessarily be a parallelogram, but the parallelogram is included in the trapezoid.*

Figure 3 and transcript 3 indicate that the female teacher FT1 understands that a square is a special type of rectangle based on the side and alignment attributes; a square is also a special case of a rhombus based on its angle. A rectangle and a rhombus are special occurrences of a parallelogram based on the sides, the parallels' properties, and the angles' size. In understanding the relationship between a parallelogram and a trapezoid, she explained that a parallelogram is a particular trapezoid case. FT1 explained this since she referred to an inclusive trapezoid, a trapezoid is a closed curve with one pair of parallel sides to classify a parallelogram as part of a trapezoid. Thus, the FT1 teacher's understanding of square, rectangle, rhombus, parallelogram, and trapezoid concepts is already hierarchical.

4. The second female teacher's knowledge of the concept and definition of quadrilaterals

The second female teacher's (FT2) solution and interview excerpts in defining and classifying quadrilaterals are as follows.

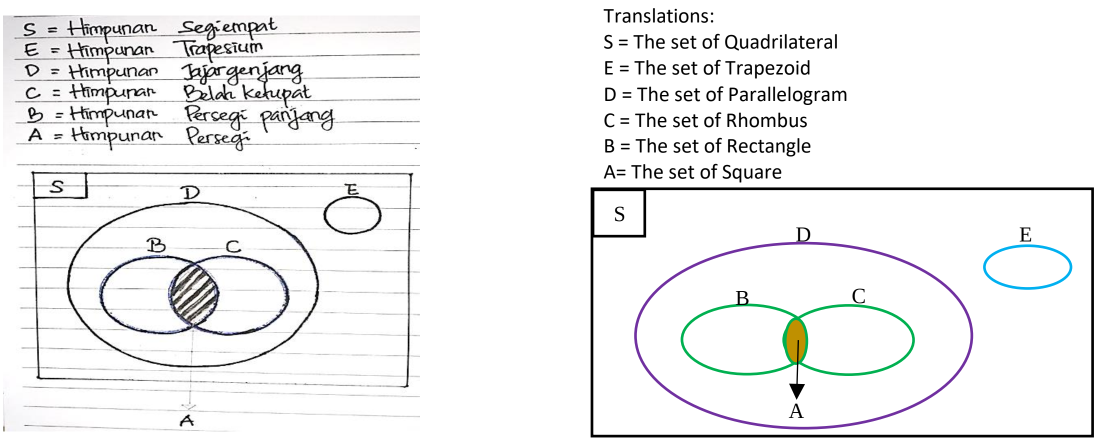

Figure 4. The second female teacher's work

Based on Figure 4, it can be seen that FT2 constructs a relationship between various kinds of quadrilaterals in the Venn diagram. She is able to generate representations of shapes in the form of a set. She illuminates that the shape types are considered subsets of other shape types (e.g. a square as a subset of a rectangle). She constructs a classification of quadrilaterals by portraying them in a family relationship in the Venn diagram. The FT2 is able to see the square as a subset of the rhombus and rectangle. Similarly, the FT2 is able to portray that a square, a rhombus, and a rectangle were also a parallelogram. However, she failed to recognize that parallelogram then forms a subset of the trapezoid.

Transcript 4

R : *Is each square a rectangle?*
FT2 : *A square and a rectangle is a rectangular shapes, and then when viewed from the shape, the sides have something in common. Every parallel side is always the same length, then every angle opposite it is the same, so the conclusion is that a square is also a rectangle.*
R : *So, do you agree that a square is a rectangle?*
FT2 : *Yes, I do.*
R : *Is each square a rhombus?*
FT2 : *Yes, because if you see a square and a rhombus, they have the same side, the length of each side is the same and the opposite side is the same, and the opposite angle is the same.*
R : *Is each rectangle a parallelogram?*
FT2 : *Yes, it is. A rectangle is also a parallelogram because it has two parallel sides; each other's angles are also equal.*
R : *Is the rhombus is also a parallelogram too?*
FT2 : *Yes, it is. Because judging from its properties opposite side is the same, has the same diagonal perpendicular, and every angle facing each other is equal.*
R : *Whether parallelogram is a trapezoid?*
FT2 : *As we know, there are an isosceles trapezoid and a right-angled trapezoid. If I equated parallelogram including trapezoid, I guess not since we see the shape and similarities in character. Suppose that a parallelogram has two sides parallel is the same length while the trapezoid is not necessarily parallel to the same length. Thus, it can be said that it is not the same length.*

Figure 4 and Transcript 4 demonstrate that female teacher FT2 understands that a square is a particular occurrence of a rectangle based on the attributes of sides, angles, and parallels; a square is also a special case of a rhombus based on side and angle attributes. Rectangles and rhombuses are occurrences specifically of a parallelogram based on sides, the property of the parallelism, and angle size. However, in understanding the relationship between a parallelogram and a trapezoid, she does not accept that a parallelogram is a special trapezoid case. FT2 refers to the definition of the trapezoid in Indonesian mathematics textbooks that mostly contain an exclusive definition, namely that the trapezoid is a closed curve with precisely one pair of parallel sides. Thus, the FT2 teachers' understanding of square, rectangle, rhombus, parallelogram, and trapezoid concepts is still at the partial prototype level.

DISCUSSION

We have displayed teachers' interviews and interpretations of teacher knowledge in defining and classifying quadrilaterals. Table 2 summarizes the findings of the present study.

Table 2**.** Summarized findings of teachers' SCK on quadrilaterals

Level	Description	Teachers' SCK on Quadrilaterals			
		MT1	MT2	FT1	FT2
Hierarchical	The teacher is able to understand a square as a particular case of rectangles and a rhombus, a teacher can accept a square, a rectangle, and a rhombus as a particular case of a parallelogram, and a teacher as a parallelogram is a particular case of a trapezoid.	_	_	√	_
Partial Prototypical	The teacher has started learning to develop figural concepts. For example, the teacher has accepted that a rhombus is a parallelogram but not for squares and rectangles.	√	_	_	√
Prototypical	Teachers have their own limited personal figural concepts.	_	√	_	_

Table 2 pointed out that three teachers (MT1, FT1, and FT2) had sufficient specialized content knowledge in understanding the relationship between squares and rectangles, rhombuses, and parallelograms. In contrast, one male teacher (MT2) had insufficient knowledge of quadrilaterals classification since he is very dependent on the shape prototype. Moreover, in constructing and understanding the trapezoid definition and inclusive relation, one male teacher and a female teacher are still at the partial prototype level, while other female teachers' knowledge is already hierarchical. The inclusive definition states that "a trapezoid is a closed curve having four sides with a pair of parallel sides," whereas the exclusive definition states that "a trapezoid is a closed curve having four sides with precisely one pair of parallel sides.

Based on the data analysis, in general, specialized content knowledge of quadrilateral material, two female teachers and a male teacher constructed the definition of each type of quadrilateral based on the side, angle, and alignment attributes. It can be concluded that the three teachers understand the concept and definition of quadrilaterals by paying attention to the properties of the shape. In contrast, the other male teacher understands dependently on sightings or prototype quadrilateral. For example, he can not conclude that the square is a particular shape of the rectangle, the square is a special case of the rhombus, and the rhombus and the rectangle are the special shapes of a parallelogram. We highlighted that teachers habitually use familiar quadrilaterals figures rather than displaying basic properties of geometric shapes (Ersen & Karakus, 2013; Fujita & Jones, 2007; Okazaki & Fujita, 2007; Žilková, 2015).

Furthermore, the findings of this study corroborate (Zazkis & Leikin, 2008) that, in general, the definition of a concept taught to students is based on the teacher's choice or from textbooks. The teachers must investigate the relationship between the quadrilateral concept, prerequisite concepts, and concept illustrations (Budiarto et al., 2021). Additionally, as evidenced by the numerous studies examining creating definitions, involving students in defining can help students appreciate the arbitrariness of definitions and make explicit characteristics of definitions (Disnawati et al., 2012; Yavuzsoy-Köse et al., 2019; Zandieh & Rasmussen, 2010; Zaslavsky & Shir, 2005). This study implies that policymakers sustain the professional development of mathematics teachers to ensure the quality of mathematics instruction, especially in quadrilaterals content.

This study focused only on four mathematics teachers who do not have an educator certificate. Different findings might be achieved if a similar study involves teachers who are certificated in professionalism. However, we argue that the findings of this study provide a valuable lens to understand teachers' knowledge in defining and classifying quadrilaterals. In mathematics teaching and learning, we might find that some teachers could not reach the hierarchy level in defining and classifying quadrilaterals. In this case, further study is necessary to understand teachers in understanding quadrilaterals fully.

CONCLUSION

Specialized content knowledge on the quadrilateral of a teacher is hierarchical in understanding square, rectangle, rhombus, and parallelogram definitions. Two teachers are in the prototypical partial level. However, the other teacher's knowledge about the relationship between the definitions of the rectangle and the hierarchical relationship between the quadrilaterals shapes is insufficient because of the dependence on the phenomenon of shape prototypes. He could not conclude that a square is a particular shape of a rectangle and a rectangle is a particular case of a parallelogram. Future studies can examine challenging geometry assignments to explore teachers' content and pedagogical knowledge in teaching quadrilaterals with representations. This study's findings may provide teacher-training institutions feedback on the specialized topic knowledge that secondary mathematics teachers should possess regarding the classification and definitions of quadrilaterals. In the meanwhile, such input may encourage mathematics teacher educators in the Teacher Professional Education Program to consider more fruitful strategies for fostering the growth of secondary mathematics teachers' grasp of the definitions and relationships of quadrilaterals.

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