

EFFECTS OF VARIABLE HEAT SOURCES AND ACTIVATION ENERGY ON CASSON NANOFLUID FLOW: A CATTANEO-CHRISTOV HEAT FLUX APPROACH

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Article Info	ABSTRACT
Article history: Received October 11th, 2025 Revised December 18th, 2025 Accepted January 8th, 2026 *Corresponding Email: chandrakala.maths@anurag.edu.in	This study investigates MHD convective heat transfer of Casson nanofluid past a stretched cylinder using the Cattaneo-Christov model, incorporating activation energy, Brownian motion, thermophoresis, and variable heat sources. The governing equations are reduced via similarity transformations and solved numerically using the Runge-Kutta shooting method and BVP4C in MATHEMATICA. Results indicate that velocity increases with the Casson parameter, Brownian motion, thermophoresis, Eckert number, activation energy, and heat sources. Temperature rises with Deborah numbers, while nano-concentration decreases with the first Deborah number and increases with the second; activation energy lowers nano-concentration. Heat transfer rates vary inversely with Deborah numbers. This novel integration of advanced heat flux modelling and activation energy provides valuable insights for optimizing heat and mass transfer in chemical, energy, electronic, and biomedical systems. Keyword: Cattaneo-Christov model, activation energy, Brownian motion, thermophoresis, stretching cylinder

Introduction

The study of Casson nanofluids has gained increasing attention due to their enhanced thermal and rheological characteristics, making them highly suitable for applications in chemical processing, biomedical engineering, and electronic cooling systems. Numerous researchers have investigated the dynamics of Casson nanofluids under various physical effects. The magnetohydrodynamic (MHD) radiative Casson nanofluid flow over an exponential stretching sheet by incorporating a heat source/sink and the Cattaneo–Christov heat flux model, reporting that both magnetic field intensity and radiation parameters significantly affect the velocity and temperature profiles (Younus M, Venkatalakshmi, 2024). Heat and mass transfer in Casson nanofluid flow over a three-dimensional Riga plate using the Cattaneo–Christov double flux model, emphasizing the influence of thermal and mass diffusivity in microfluidic system design (Loganathan *et al.*, 2023).

Activation energy has been recognized as a critical parameter influencing reactive nanofluid behavior. The effects of activation energy on MHD radiative Casson nanofluid flow with Darcy–Forchheimer drag and the Cattaneo–Christov heat flux model, concluding that activation energy significantly governs chemical reaction rates and temperature gradients (Younus *et al.*, 2023). Similarly, it was reported that unsteady radiative–convective Casson hybrid nanofluid flow over an inclined disc with the Cattaneo–Christov heat flux and entropy generation analysis, demonstrating that activation energy enhances the control of thermal management in reactive systems (Kar *et al.*, 2024).

Geometric configuration also plays a major role in determining heat and mass transfer characteristics. Sakkaravarthi and Reddy examined entropy generation in MHD Casson and Williamson hybrid nanofluid flows over curved stretching sheets using the Cattaneo–Christov heat flux model and revealed that curvature substantially influences entropy production and flow uniformity (Sakkaravarthi and Reddy, 2023a, Sakkaravarthi and Reddy, 2023b). Despite these contributions, most studies have focused on flat or simply curved surfaces, with limited consideration of more complex

geometries such as stretching cylinders, which are representative of many industrial and biomedical applications. From a computational perspective, recent advances in numerical techniques have enabled detailed simulations of Casson nanofluid dynamics. A finite element approach to solve modified Navier–Stokes and energy equations for hybrid nanofluids in complex domains, providing accurate predictions of flow and thermal fields (Dey and Rathish, 2023). A higher-order super-compact scheme for three-dimensional heat transfer issues in nanofluids involving conducting fins, achieving improved convergence and computational precision (Punia and Ray, 2024).

Incorporating multiple physical effects, Ahmed *et al.* analyzed Casson nanofluid flow with Brownian motion, thermophoresis, and chemical reactions in porous media using the Cattaneo–Christov heat flux model (Ahmed *et al.*, 2024). Their findings indicated significant impacts on temperature and concentration distributions. Similarly, Zuberi, *et al.* investigated blood-based Casson nanofluid flow over a nonlinear stretching sheet with Ohmic dissipation and partial slip, suggesting potential applications in biomedical cooling technologies (Zuberi *et al.*, 2024). However, these studies often overlook the combined influence of surface curvature and space- and temperature-dependent heat sources, thereby limiting their practical applicability.

Furthermore, Sakkaravarthi and Reddy demonstrated the role of nonlinear radiation in entropy generation and temperature variation for hybrid nanofluids over curved surfaces (Sakkaravarthi and Reddy, 2023c). It emphasized that nanoparticle shape and size significantly affect thermal conductivity and flow stability in Casson-type nanofluids (Zainal *et al.*, 2023, Mishra *et al.*, 2023). Despite these extensive contributions, there remains a significant research gap in integrating the combined effects of Casson fluid rheology, Cattaneo–Christov heat and mass fluxes, activation energy, Brownian motion, thermophoresis, and variable heat sources in the context of curved stretching cylinders. Most available studies are limited to simplified geometries or consider only partial physical effects. Additionally, only a few works employ advanced numerical algorithms, such as the Runge–Kutta shooting technique coupled with boundary value solvers in *Mathematica*, for solving the resulting nonlinear differential systems.

The present study addresses these gaps by investigating the MHD convective heat and mass transfer of Casson nanofluid flow past a stretching cylinder using the Cattaneo–Christov model. The formulation incorporates activation energy, Brownian motion, thermophoresis, and space- and temperature-dependent heat sources. This comprehensive approach provides a realistic representation of industrial and biomedical transport processes and contributes valuable insights towards optimizing thermal performance in energy systems, chemical processes, and micro-scale cooling applications.

Methods

Mathematical Formulation

The present work explores the boundary layer characteristics of Casson fluid driven by natural convection around a vertical circular cylinder that undergoes exponential stretching with radius a . The stretching velocity along the radial direction is denoted by U_∞ . The cylinder stretches exponentially in the radial direction with velocity as depicted in Figure 1, the surface of the cylinder is maintained at temperature T_w , whereas the uniform ambient temperature is T_∞ . A temperature difference drives buoyancy: the flow is assisted when $T_w > T_\infty$ and opposed when $T_w < T_\infty$. An externally imposed uniform magnetic field of strength B_0 is oriented along the radial direction, and its interaction with the electrically conducting Casson fluid gives rise to magnetohydrodynamic (MHD) effects.

The following provides the governing equations for steady-state conditions based on the aforementioned assumptions (Sarojini and Srinivas, 2023):

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r}\right) + \beta_T g(T - T_\infty)(1 - \phi_\infty) + \frac{1}{\rho} (\rho_\infty - \rho)(\phi - \phi_\infty) - \left(\frac{\sigma B_o^2}{\rho}\right) w - \left(\frac{\mu}{k_p}\right) w \quad (2)$$

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \left(\frac{k_f}{\rho C_p} + \frac{16\sigma^* T_\infty^3}{\rho C_p \beta_R}\right) \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r}\right) + \frac{(\rho C_p)^u}{(\rho C_p)} \left(D_T \frac{\partial T}{\partial r} \frac{\partial \phi}{\partial r} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial z}\right)^2\right) + \frac{q'''}{\rho C_p} + \gamma_1 \left[u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z}\right] \frac{\partial T}{\partial r} + u^2 \frac{\partial^2 T}{\partial r^2} + w^2 \frac{\partial^2 T}{\partial z^2} + 2uw \frac{\partial^2 T}{\partial z \partial r} + \left(u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z}\right) \frac{\partial T}{\partial z} + 2\mu \left(\frac{\partial w}{\partial r}\right)^2 + \sigma \mu_e^2 H_o^2 (w^2) = 0 \quad (3)$$

$$u \frac{\partial \phi}{\partial r} + w \frac{\partial \phi}{\partial z} = (D_B) \left(\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r}\right) + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r}\right) - k(\phi - \phi_\infty) \left(\frac{T}{T_\infty}\right)^n \text{Exp}\left(-\frac{E_a}{KT}\right) + \gamma_2 \left[u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z}\right] \frac{\partial \phi}{\partial r} + u^2 \frac{\partial^2 \phi}{\partial r^2} + w^2 \frac{\partial^2 \phi}{\partial z^2} + 2uw \frac{\partial^2 \phi}{\partial z \partial r} + \left(u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z}\right) \frac{\partial \phi}{\partial z} = 0 \quad (4)$$

The internal heat generation or absorption term q''' is modelled according in the following equation:

$$q''' = \frac{k_f U_w(z)}{a\nu} [A'_{11}(T_w - T_\infty)f' + B'_{11}(T - T_\infty)] \quad (5)$$

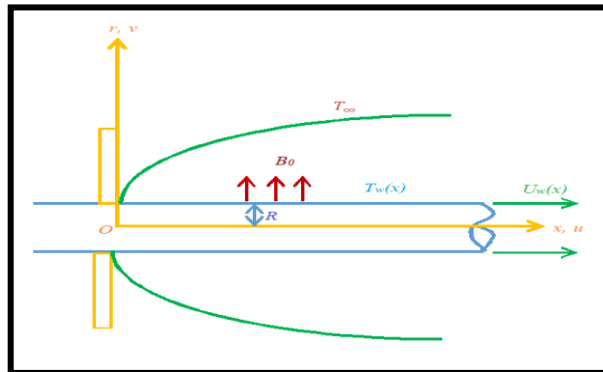


Figure 1. Sketch of the physical model

The thermal conductivity of the fluid is represented by k_f , while the space- and temperature-dependent heat source/sink parameters are represented by A'_{11} and B'_{11} , respectively. This formulation accounts for both spatially and temperature-dependent heat generation or absorption in the flow system.

Introducing the following similarity variables:

$$\eta = \frac{r^2}{a^2}, u = -\frac{1}{z} U_w \frac{f(\eta)}{\eta}, w = U_w f'(\eta), \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty} \quad (6)$$

where the following equation is used to compute the characteristic temperature and nano-concentration difference.

$$T_w = T_\infty + c \exp\left(\frac{z}{a}\right) \text{ and } C_w = C_\infty + \exp\left(\frac{z}{a}\right).$$

Equations (2)–(4) and equation (6), which employ transformations, reduce to

$$\left(1 + \frac{1}{\beta}\right) (\eta f''' + f'') + \text{Re} (ff'' + (f')^2) + \text{Re} G (1 - \phi_\infty) (\theta + Nr \cdot \phi) - (M^2 + K) f' = 0 \quad (7)$$

$$\eta\theta'' + \theta' + \text{Re} \text{Pr}(f\theta' - f'\theta) + \eta\theta'(Nb\theta' \cdot \varphi' + Nt(\theta')^2) + \text{Re}(A_{11}f' + B_{11}\theta) + Ec \cdot \text{Pr} \cdot (f'')^2 + Ec \cdot \text{Pr} \cdot M^2(f')^2 + \text{Pr} \cdot \text{De}_1(ff'\theta' + f^2\theta'') = 0 \quad (8)$$

$$\left. \begin{aligned} \eta\varphi'' + \varphi' + \left(\frac{Nt}{Nb}\right)(\eta\theta'' + \theta') + \text{Re} \cdot (f\varphi' - f'\varphi) - \text{Le} \gamma \cdot \varphi \cdot (1 + n\delta\theta) \\ \text{Exp}\left(-\frac{E_1}{1 + \delta\theta}\right) + \text{Le} \cdot \text{De}_2 \cdot (ff'\varphi' + f^2\varphi'') = 0 \end{aligned} \right| \quad (9)$$

where

$$Nb = \frac{(\rho C_p)_{nf} D_B (C_w - C_\infty)}{\rho C_p \alpha}, Nt = \frac{(\rho C_p)_{nf} D_T (T_w - T_\infty)}{\rho C_p \alpha T_\infty}, \text{Re} = \frac{aU}{4\nu}, \alpha = \frac{k_f}{\rho C_p}, \gamma = \frac{kc}{a}$$

$$K = \frac{a^2}{k_p}, \text{De}_1 = \lambda_1 a, \text{De}_2 = \lambda_2 a, Ec = \frac{U_w}{C_p (T_w - T_\infty)}, \varepsilon_1 = \varepsilon_1' \sigma_0, \varepsilon_2 = \varepsilon_2' \sigma_0$$

Now, the transferred boundary conditions are:

$$\begin{aligned} f(1) = 0, f_\eta(1) = 1, f_\eta(\infty) = \infty \\ \theta(1) = 1, \theta \rightarrow 0, \text{as } \eta \rightarrow \infty \\ \varphi(1) = 1, \varphi \rightarrow 0, \text{as } \eta \rightarrow \infty \end{aligned} \quad (10)$$

Skin friction coefficient C_{fx} , Nusselt number Nux , Sherwood number Shx , shear stress at the surface τ_w , heat flux q_w and mass flux m_w are defined as:

$$Cf = \frac{\tau_w}{\rho U_w^2}, Nux = \frac{ae^{z/a} q_w}{k_f (T_w - T_\infty)}, Shx = \frac{e^{z/a} m_w}{D_B (C_w - C_\infty)}, \tau_w = (w_{rz})_{r=a}, q_w = -(k_f \left(\frac{\partial T}{\partial z}\right)_{r=a}) \text{ and } m_w = D_B \left(\frac{\partial C}{\partial z}\right)_{r=a} \quad (11)$$

Code Validation

Table 1. Comparison of $f''(1)$ with distinct values of different parameters

$f''(1)$	Malik et al. [22]	Kathyayani et al. [23]	Present Study	Malik et al. [22]	Kathyayani et al. [23]	Present Study
λ/β						
0.2	0.8494	0.849330	0.849332	1.1283	1.128345	1.128346
0.4	0.7873	0.783831	0.783832	0.9860	0.986947	0.986948
0.6	0.7263	0.726418	0.726429	0.8490	0.849088	0.849089
0.8	0.6662	0.666158	0.666159	0.7162	0.716402	0.716403
1.0	0.6070	0.607089	0.607090	0.5869	0.586524	0.586526

Table 2. Comparison of $-\theta'(1)$ with distinct values of different parameters

$-\theta'(1)$	Malik et al. [22]	Kathyayani et al. [23]	Present Study	Malik et al. [22]	Kathyayani et al. [23]	Present Study
Pr/Re						
1	0.2442	0.244209	0.244210	0.3094	0.309405	0.309406
3	0.3826	0.382592	0.382598	0.5627	0.562759	0.562761
5	0.5118	0.511784	0.511786	0.7844	0.784406	0.784407
7	0.6327	0.632701	0.632702	0.9814	0.871392	0.871393
15	1.0499	1.049942	1.049944	0.6121	1.612129	1.612131

Method of Solution

The equations (10)-(11) governing the flow, heat, and mass transfer have been executed by the fourth-order Runge-Kutta shooting technique.

$$f = f_1, f' = f_2, f'' = f_3, \theta = f_4, \theta' = f_5, \phi = f_6, \phi' = f_7 \quad (13)$$

$$f''' = [Kf_2 - (A_1 A_2 (f_3 - f_1 f_2) - A_1 A_6 M^2 f_2 + f_3 f_2^2 - G A_1 A_3 (f_4 + \beta_1 f_4^2))] \quad (14)$$

$$\theta'' = [A_4 f_1 f_6 - (A_{11} f_2 + B_{11} f_4) - Ec Pr M^2 f_2^2 - Nb(f_5 f_7) - Nt(f_5^2) - Ec Pr(f_3^2)] / (A_5 + \frac{4Rd}{3}) \quad (15)$$

$$\phi'' = [f_1 f_7 + (\frac{Nt}{Nb}) [A_4 f_1 f_6 - (A_{11} f_2 + B_{11} f_4) - Ec Pr M^2 f_2^2 - Nb(f_5 f_7) - Nt(f_5^2) - Ec Pr(f_3^2)] / (A_5 + \frac{4Rd}{3})] - \gamma(1 + n \delta f_6) Exp(-\frac{E1}{1 + \delta f_4}) \quad (16)$$

$$f_1(1) = 0, f_2(1) = 1, f_3(1) = 1, f_4(1) = 1, f_2(\infty) = 0, f_4(\infty) = 0, f_6(\infty) = 0 \quad (17)$$

Results and Discussion

This study investigates the influence of Cattaneo-Christov heat and mass fluxes, activation energy, and viscous dissipation on non-Darcy hydromagnetic convective transport of a Casson nanofluid over a stretching cylinder with a non-uniform heat source. The governing nonlinear equations for momentum, energy, and species concentration are solved numerically using the fourth-order Runge-Kutta method combined with the shooting technique. Graphical results are provided to highlight the significant effects of the considered parameters on velocity, temperature, and nanoparticle concentration profiles. For fixed parameters, $\beta = 0.25$, $Nr = 0.5$, $Nb = 0.1$, $Nt = 0.2$, $A_{11} = 0.1$, $B_{11} = 0.1$, $Re = 0.3$, $\gamma = 0.5$, $Le = 1$, and $Pr = 0.71$, the velocity, temperature, and concentration distributions exhibit noticeable variations, and their interdependence is illustrated through the plotted graphs. The results highlight the significant role of these governing parameters in shaping the fluid motion, thermal field, and nanoparticle concentration.

Figure 2 illustrated the effects of the Grashof number (G) and magnetic parameter (M) on velocity, temperature, and nanoparticle concentration. Increasing G enhances velocity, temperature, and concentration, while larger M values suppress velocity and concentration but raise temperature. The resistive Lorentz force thins the momentum and solutal boundary layers while thickening the thermal boundary layer. Figure 3 showed how the inverse Darcy parameter (K) and Forchheimer parameter (fs) affect flow variables. Higher flow area temperature, nanoconcentration, and velocity decrease with lower porous permeability. Inertia and boundary effects in the momentum equation decrease the thickness of the momentum boundary layer. Figure 4 demonstrates that the temperature (θ) falls when both Nr and β increase, indicating that the boundary layer cools more when buoyancy is higher and Casson fluid properties are more noticeable. The Casson parameter reduces the temperature.

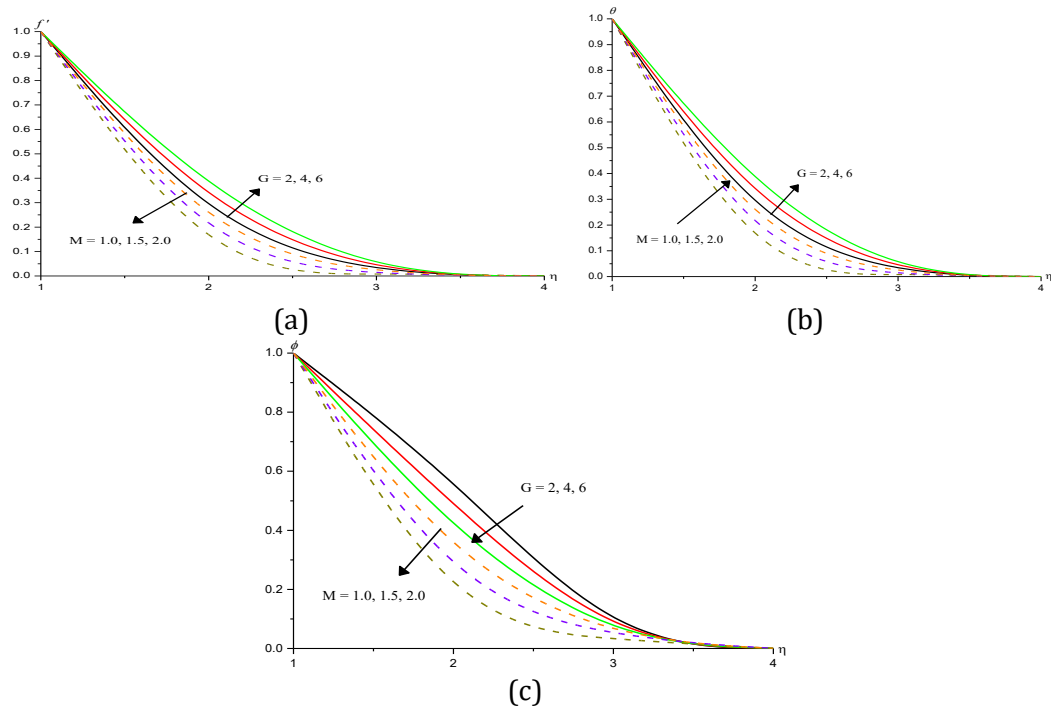


Figure 2. Effect of G and M on (a). velocity (f'); (b). temperature (θ) and (c). nano-concentration (ϕ)

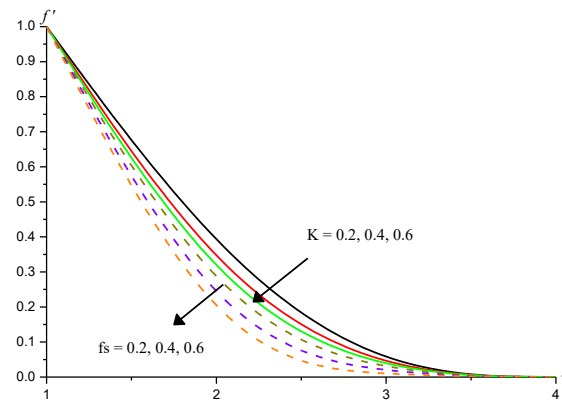


Figure 3. Effect of K and f_s on f'

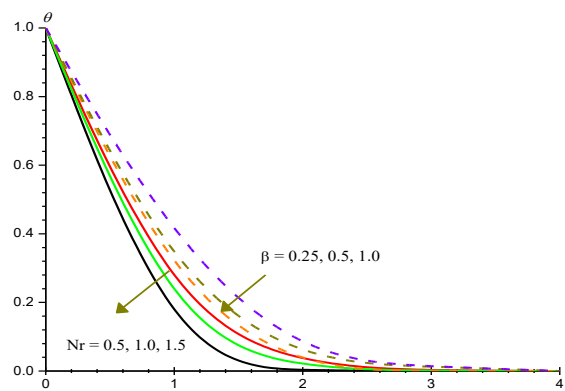


Figure 4. Effect of Nr and β on θ

In Figures 5, the Brownian motion parameter (Nb) and thermophoresis parameter (Nt) influence the flow variables. Brownian motion (Nb) improves micro mixing and increases nanofluid thermal conductivity. Hence, it can be predicted that Brownian motion increases temperature. It is apparent that increasing Nb increases boundary layer temperature. Nanoparticle concentration decreases in flow. Nanoparticle concentration decreases while temperature increases with greater thermophoresis parameter (Nt) values.

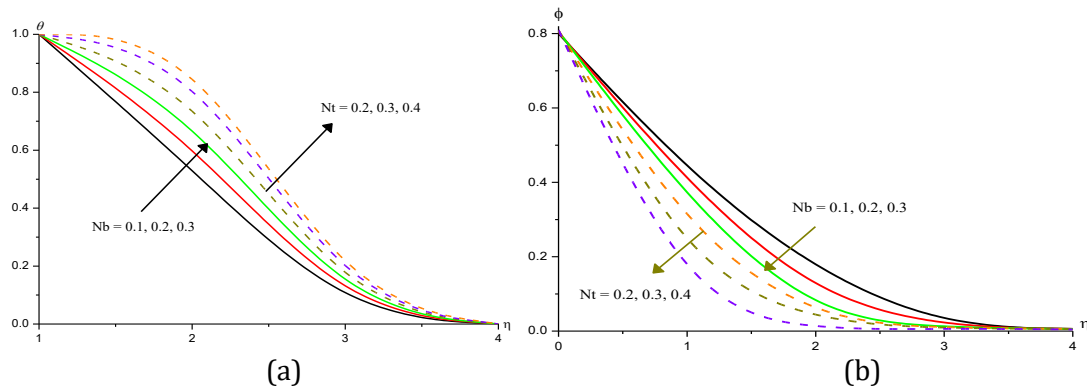


Figure 5. Effect of (a). Nb and Nt θ and (b). Nb and Nt ϕ

From Figure 6, raising A11 increases fluid temperature and nanoparticle concentration. Heat sources (A11>0) provide energy and raise temperature, whereas A11<0 (heat sink) has the opposite effect, cooling the fluid.

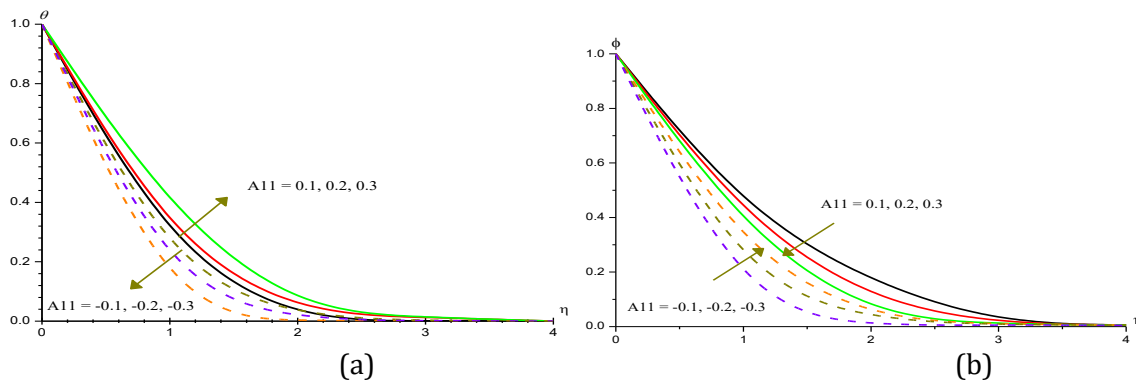


Figure 6. Impact of (a). A11 on θ and (b) A11 on ϕ

With a heat-generating source (B11>0), temperature increases, while with a heat-absorbing source (B11<0), temperature decreases, as in Figure 7.

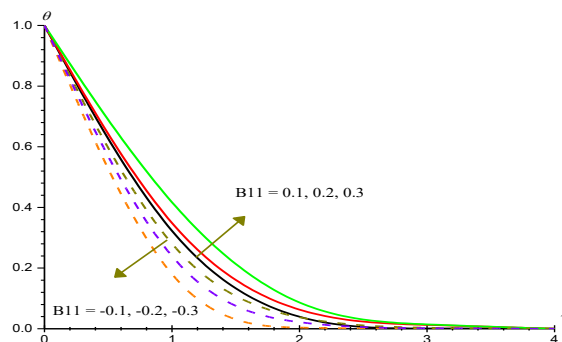


Figure 7. Impact of B11 on θ

Figures 8 illustrate how the Reynolds number (Re) and the chemical reaction parameter (γ) affect the velocity, temperature, and nanoparticle concentration distributions. As Re increases, all three quantities exhibit a noticeable decline within the boundary layer region.

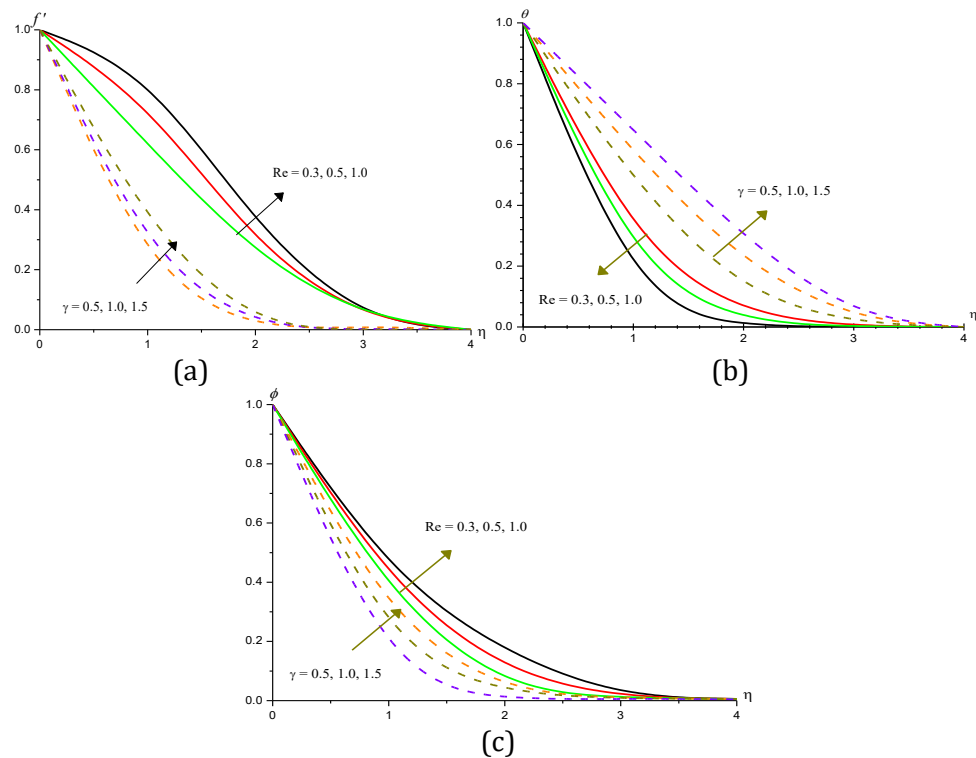


Figure 8. Power of (a) Re and γ on f' ; (b). Re and γ on θ and (c). Re and γ on ϕ

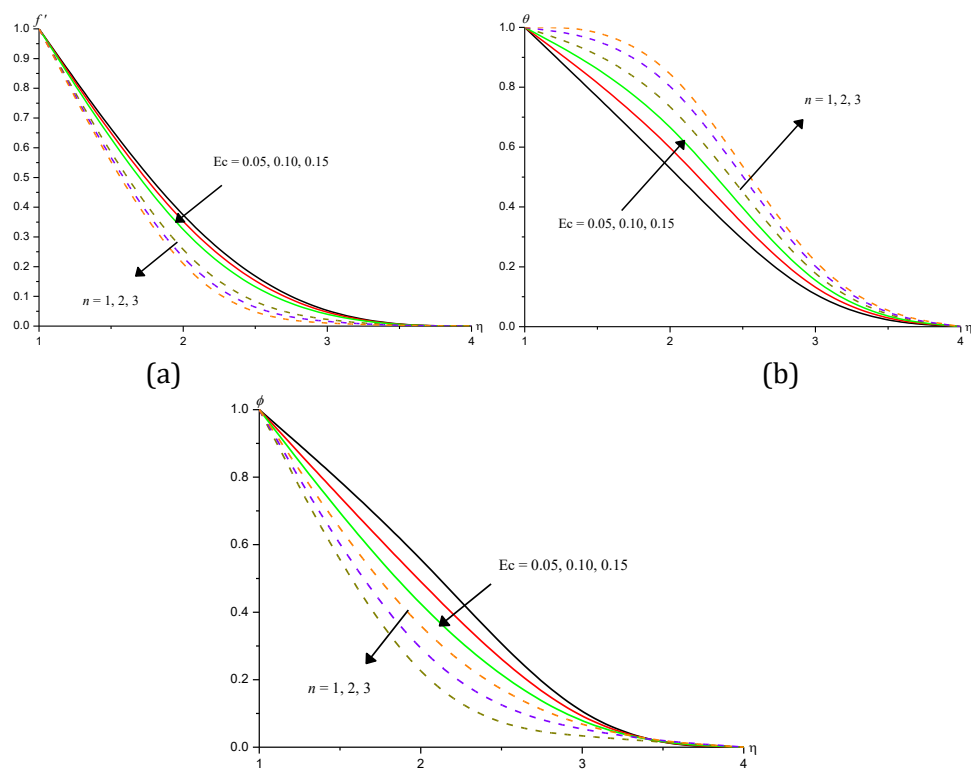


Figure 9. Control of Ec and n on (a). f , (b). θ and (c) ϕ

Figure 9 illustrate the impact of dissipative energy (Ec) and index number (n) on f' , θ and C . The flow area has lower velocity, nano=concentration, and greater temperature as dissipative energy increases. This may be because the Eckert number increases thermal boundary layer thickness. A higher index number (n) increases boundary layer f' , θ and C . Figure 10 show that increasing Deborah numbers $De1$ (heat flux relaxation) and $De2$ (mass flux relaxation) reduce velocity, temperature, and nanoparticle concentration. Higher $De1$ values correspond to longer heat relaxation times, leading to lower temperatures and nano-concentrations, while higher $De2$ values shorten mass flux relaxation times, further decreasing nano-concentration profiles.

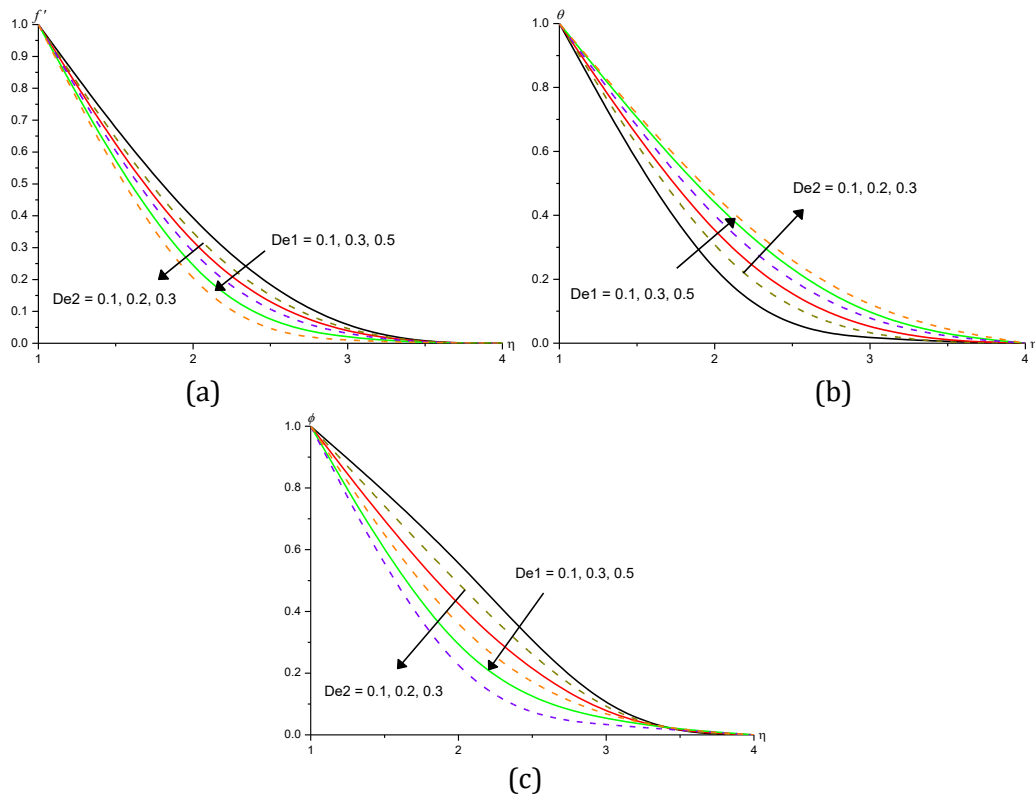


Figure 10. Effect of $De1$ and $De2$ on (a). f', θ and ϕ , (b). θ and (c). ϕ

The effects of Pr and Le on-flow variables are depicted in Figure 11. Temperature and nanoparticle concentration all drop as Pr rises. Increased Pr reduces thermal diffusivity, which lowers energy ability and causes the thermal boundary layer to shrink since Pr is the ratio of thermal diffusion to viscous diffusion. The thickness of the solutal boundary layer grows as the Lewis number (Le) rises, whereas the thermal boundary layer decreases.

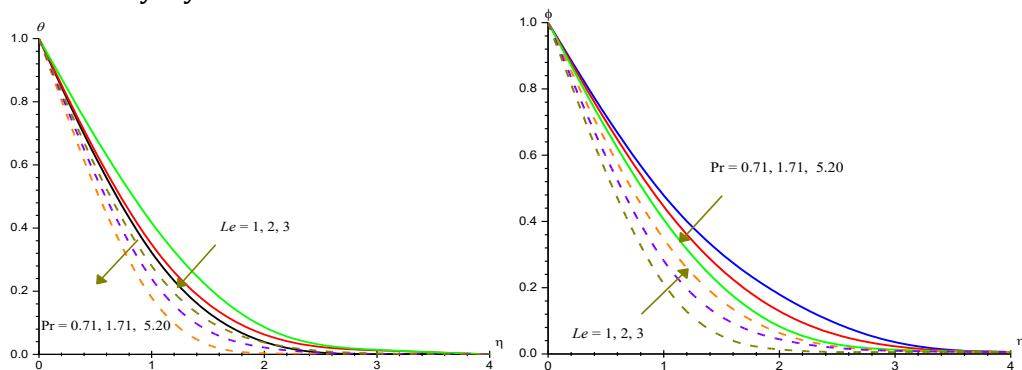


Figure 11. Bearing of Pr and Le on (a). θ and (b) ϕ

Figure 12 shows how activation energy (E_1) and temperature difference ratio (δ) affect f' , θ , and C . With increasing values of temperature difference ratio (δ) in the flow zone, an elevation in activation energy parameter (E_1) increases velocity (f'), decays temperature (T), and increases nanoparticle concentration (C).

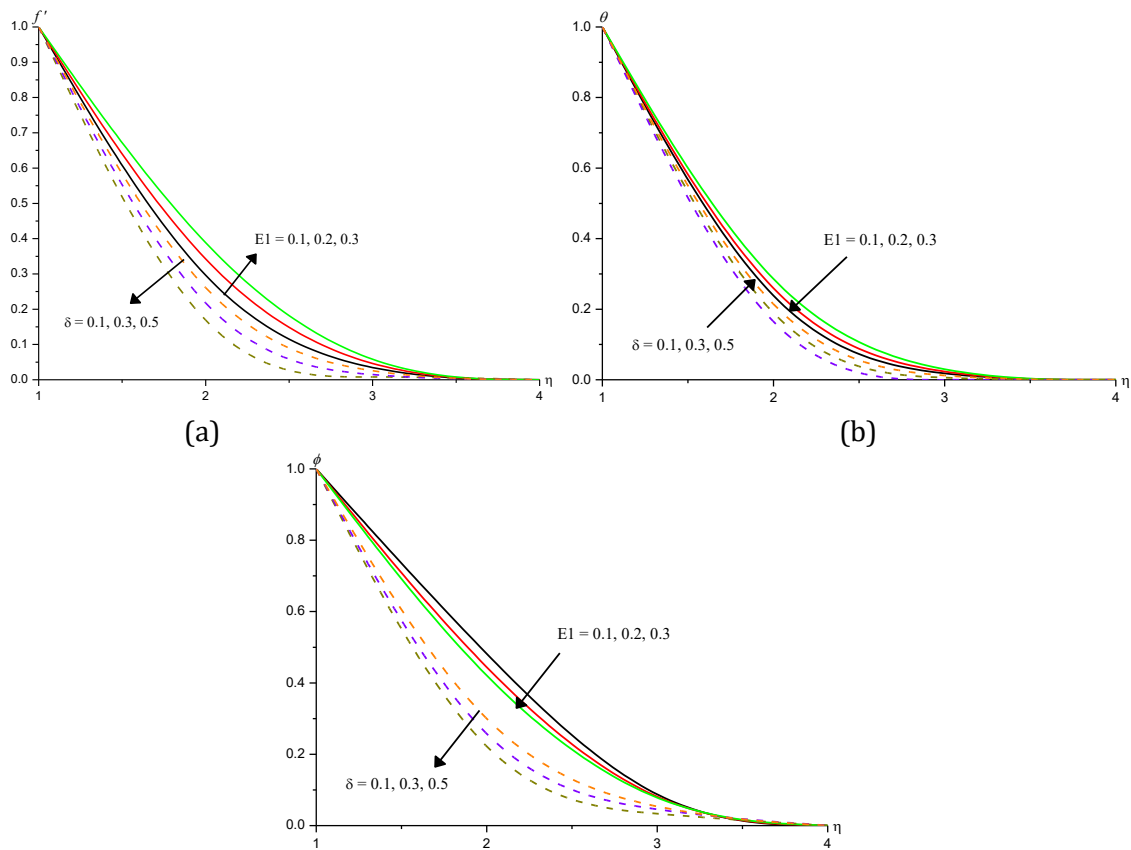


Figure 12. Impact of E_1 and δ on (a) f' , θ and ϕ ; (b). θ and (c). ϕ

Figures 13-21 display the fluctuation of skin friction (τ), Eckert number (Ec), Reynolds number (Re), and temperature-dependent heat source (B_{11}). $\beta=1$ (figs. 12-16) and enhances on $\eta=1$ with magnetic parameter (M), inverse Darcy parameter (K), Forchheimer parameter (fs), and Casson parameter (β). Increased G /Boyancy ratio (Nr)/Space-dependent heat source (A_{11})/Debora number (De_1 , De_2)/Activation energy (E_1) reduces skin friction on values. $\delta/n/Le$, and decays with increasing $Nb/Nt/A_{11}/B_{11}/Ec/Pr/De_1/\delta/Pr/Nt/E_1/\gamma/De_2/E_1$, declines with $M/K/Nb/fs/De_1/\beta$.

Heat transfer rate (Nu) rises with $\delta/\gamma/Le$ and decreases with $Ec/Nt/E_1$. Mass transfer rate (Sh) increases with $Nb/De_1/De_2$ ($\eta=1$), Nusselt number (Nu), and Sherwood number (Sh) on the wall, while De_2 increases Nu and Sh on the stretching cylinder. The Catano-Christov heat flux model (De_1) decreases skin friction (τ) reverses these effects. $\delta=1$, but increasing temperature difference ratio (η), Sh , and Nu at τ Increasing activation energy (E_1) decreases skin friction (τ) Inertia and boundary effects (fs) increase cylinder skin friction in the momentum equation.

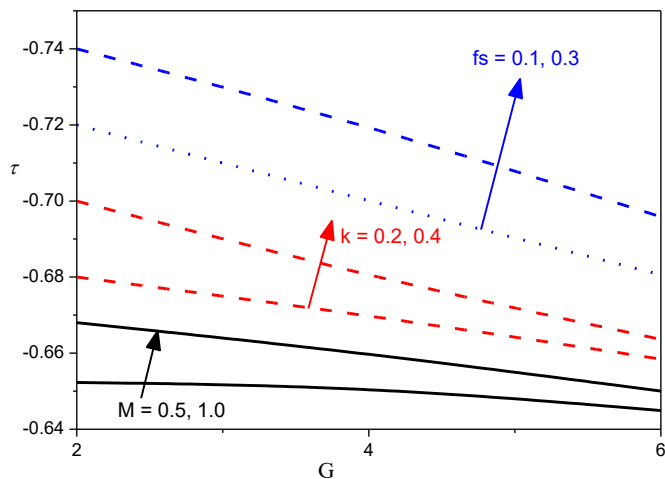


Figure 13. Control of G , M , k , and fs on τ at $\eta=1$

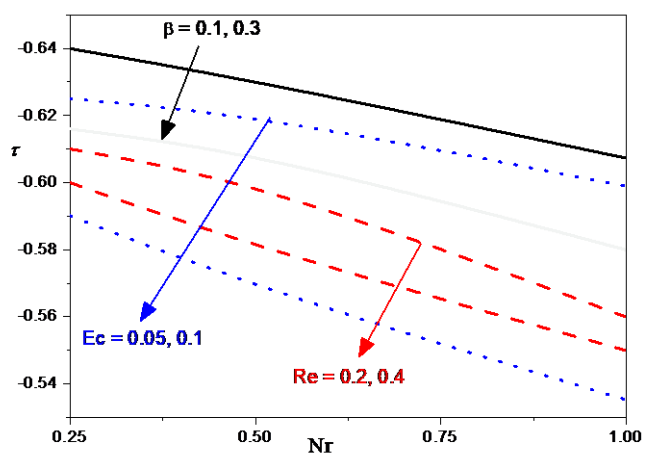


Figure 14. Impact of Nr , β , Re , and Ec on τ at $\eta=1$

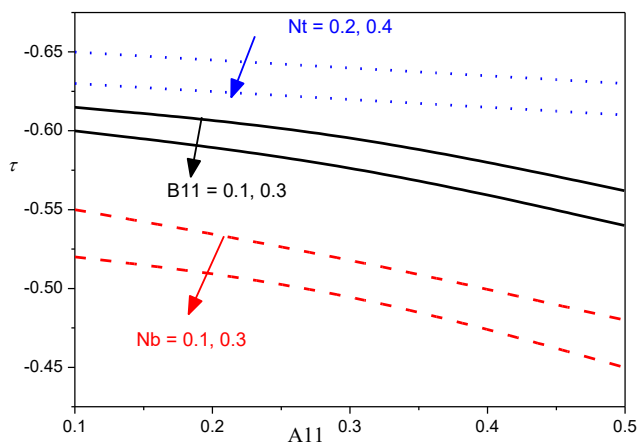


Figure 15. Impression of A_{11} , B_{11} , Nb , and Nt on τ

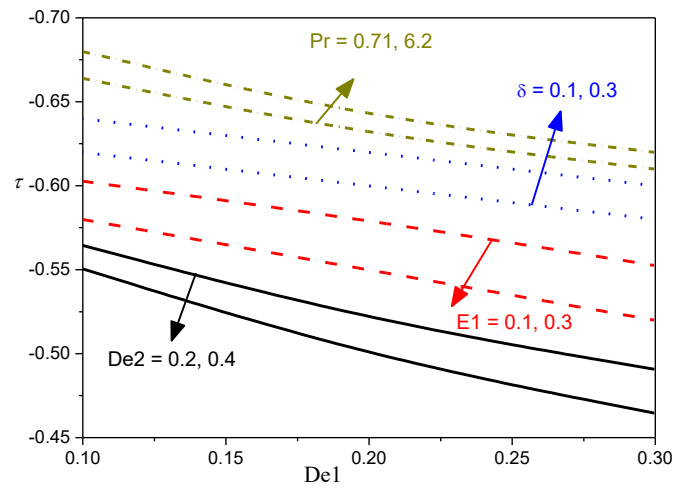


Figure 16. Stimulus of De1, De2, E1, δ , Pr on τ at $\eta=1$

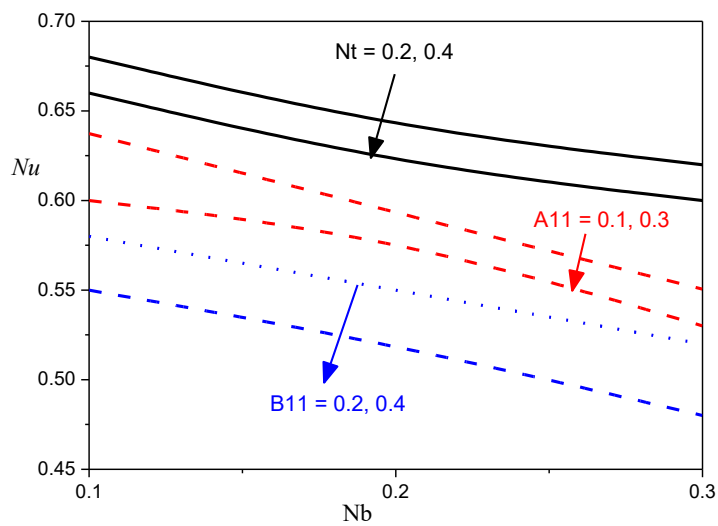


Figure 17. Effect of Nb, Nt, A11, and B11 on Nu at $\eta=1$

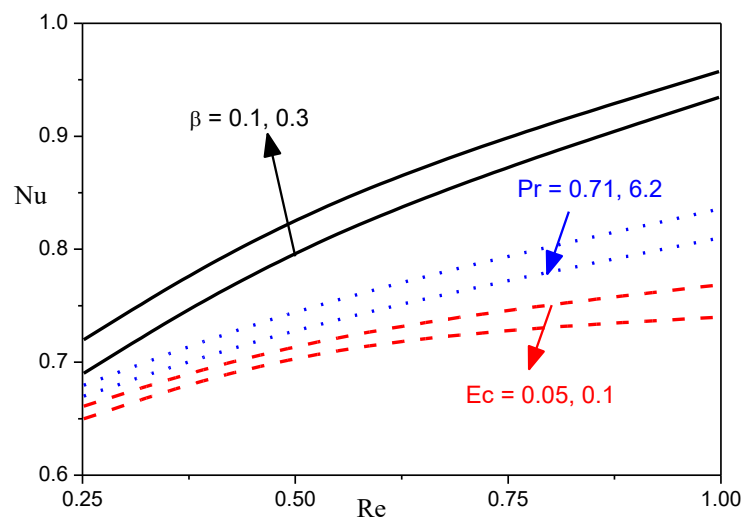


Figure 18. Effect of Re, β , Ec, and Pr on Nu at $\eta=1$

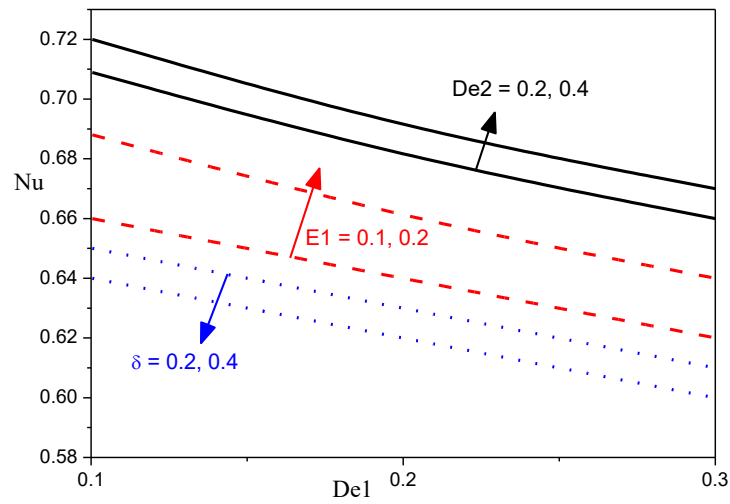


Figure 19. Inspiration of $De1$, $De2$, $E1$, and δ on Nu at $\eta=1$

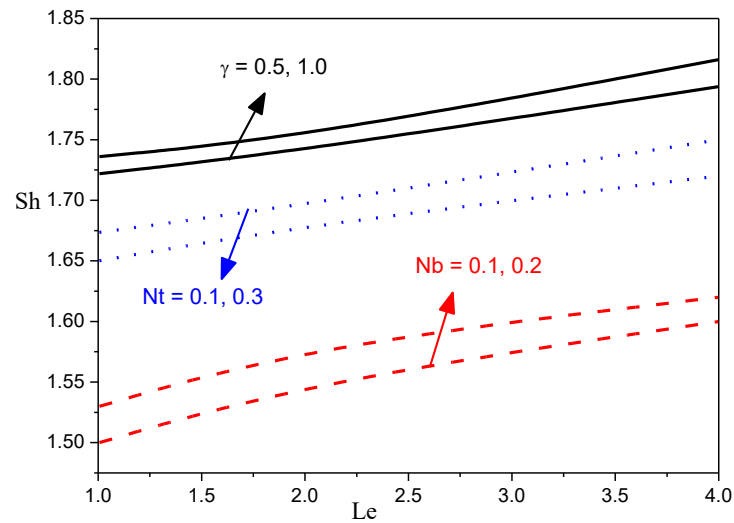


Figure 20. Control of Le , γ , Nb , Nt on Sh

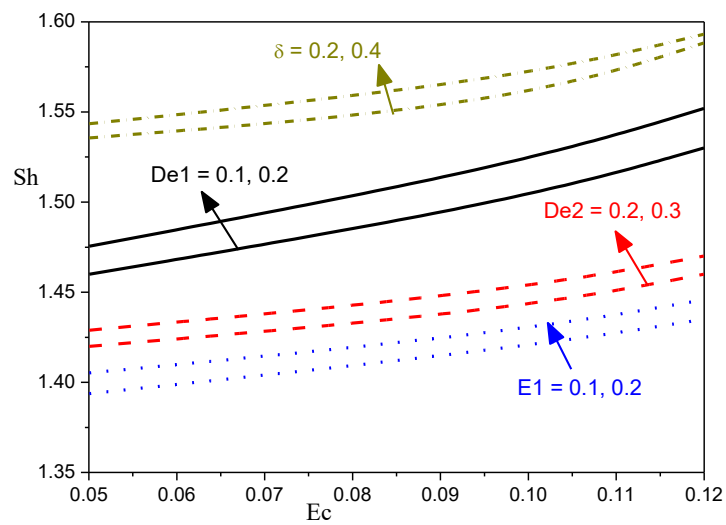


Figure 21. Encouragement of Ec , $De1$, $De2$, $E1$, δ on Sh

Conclusion

Velocity increases with buoyancy, Brownian motion, thermophoresis, space- and temperature-dependent heat sources, chemical reaction rates, magnetic parameters, and permeability but decreases with higher Deborah numbers, Eckert numbers, and Forchheimer parameters. Temperature rises with buoyancy, magnetic field, Forchheimer parameter, Brownian motion, thermophoresis, Eckert number, and Deborah numbers, while it decreases with increasing Lewis number, Prandtl number, activation energy, and temperature difference parameter. Nanoparticle concentration grows with buoyancy ratio, Casson parameter, chemical reaction rate/order, and mass-flux Deborah number (De2), but reduces with Grashoff number, magnetic field, slip, thermophoresis, Eckert number, Lewis number, and Prandtl number. Shear stress increases with magnetic parameters, slip, Deborah numbers (De1), Prandtl numbers, and thermal diffusivity, but decreases with Grashoff numbers, reaction rates, heat source parameters, and Deborah numbers (De1, De2). The Nusselt number improves with the Reynolds number, Casson parameter, and mass-flux Deborah number (De2), but declines with the Eckert number, heat-flux Deborah number (De1), and permeability/porosity. The Sherwood number increases with magnetic parameter, slip, Deborah number (De1), Prandtl number, and Lewis number, but decreases with Grashof number, reaction rate, Casson parameter, and mass-flux Deborah number (De2). Non-Darcy effects and Deborah numbers reduce heat transfer (Nu) while enhancing shear stress and mass transfer (Sh). Increasing energy activation decreases shear stress and heat transfer but increases mass transfer at the wall. Limitations include assumptions of steady laminar flow and neglect of nanoparticle aggregation; future work may extend to turbulent flow and experimental validation. Results have practical applications in optimizing heat and mass transfer in chemical processing, electronic cooling, and biomedical systems.

Author's Contribution

Sivaiah Sreeramula conceptualised the study, developed the mathematical model, and supervised the research. B. Tulasi Lakshmi Devi contributed to the formulation of the problem and assisted in the numerical analysis. Devika Dabke performed the computational simulations and data interpretation. P. Chandrakala (corresponding author) coordinated the research activities, contributed to writing and editing the manuscript, and handled correspondence. All authors reviewed and approved the final manuscript.

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