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Problem-Project based Learning Model for Improving Entrepreneurship Character and Reasoning Ability of Mathematics Education Students

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ABSTRACT

Pembelajaran matematika belum mengintegrasikan karakter kewirausahaan dan kemampuan penalaran mahasiswa. Oleh karena itu, penelitian ini bertujuan untuk mengembangkan model Pembelajaran Berbasis Masalah-Proyek (PPjBL) untuk memperkuat kedua aspek tersebut dalam mata kuliah Pemrograman Linear. Model PPjBL dikembangkan menggunakan kerangka kerja penelitian dan pengembangan Borg & Gall melalui tahapan desain, validasi, dan uji coba di kelas. Sintaks akhir PPjBL terdiri dari enam tahap: orientasi masalah, pengorganisasian mahasiswa untuk pembelajaran, memandu investigasi, merancang proyek, mengimplementasikan proyek, dan mendokumentasikan laporan proyek. Hasil validasi pakar dan dua uji coba implementasi menunjukkan bahwa model ini praktis dan efektif. Karakter kewirausahaan mahasiswa meningkat dari tahap "mulai berkembang" menjadi "menjadi kebiasaan". Sementara itu, kemampuan penalaran matematika meningkat dalam kategori tinggi, menunjukkan kemajuan dalam formulasi masalah, konstruksi logis, deduksi, interpretasi, dan generalisasi. Temuan ini menegaskan bahwa PPjBL berfungsi sebagai kerangka kerja pedagogis transformatif yang mampu mengintegrasikan penalaran kognitif dan pengembangan karakter afektif melalui pengalaman belajar yang autentik. Model ini memposisikan mahasiswa sebagai pemecah masalah aktif melalui keterlibatan dengan permasalahan kewirausahaan di dunia nyata. Penelitian di masa mendatang dapat memperluas penerapan PPjBL ke domain matematika lain, dengan menekankan pengembangan pemikiran kritis, kreativitas, dan kolaborasi sebagai hasil terpadu dari pendidikan matematika holistik.

Mathematics learning has not yet integrated students' entrepreneurial character and reasoning abilities. In response, this study aimed to develop a Problem–Project Based Learning (PPjBL) model to strengthen both aspects within the *Linear Programming* course. The PPjBL model was developed using the Borg & Gall research and development framework through stages of design, validation, and classroom trials. The final PPjBL syntax consists of six stages: problem orientation, organizing students for learning, guiding investigation, designing the project, implementing the project, and documenting project reports. Results from expert validation and two implementation trials showed that the model is practical and effective. The entrepreneurial character of students improved from the "start to grow" to the "being habit". Meanwhile, mathematical reasoning ability increased in the high category, indicating progress in problem formulation, logical construction, deduction, interpretation, and generalization. These findings affirm that PPjBL serves as a transformative pedagogical framework capable of integrating cognitive reasoning and affective character development through authentic learning experiences. The model positions students as active problem solvers through engagement with real-world entrepreneurial problems. Future research may extend the application of PPjBL to other mathematical domains, emphasizing the cultivation of critical thinking, creativity, and collaboration as integrated outcomes of holistic mathematical education.

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INTRODUCTION

The government's efforts to overcome unemployment in Indonesia must receive support from various parties. Currently, the unemployment rate in Indonesia is increasing from year to year. Badan Perencanaan Pembangunan Nasional (Bappenas) has stated that unemployment is a major problem in economic development in Indonesia. The high unemployment rate of 6.18% per year also has an impact on poverty levels and regional and regional income inequality. The high unemployment rate can result in poverty and the emergence of crime which is certainly not expected by all parties (Triatmanto & Bawono, 2023). Unemployment and poverty are currently major problems for the Indonesian nation that cannot be solved. According to Badan Pusat Statistik (BPS) data in August 2019, the number of openly unemployed was 8.96 million people (7, 87%) of the total workforce of around 113.83 million people. Of the 8.96 million unemployed people, most of them are in rural areas (Pritadrajati et al., 2021). The latest data from the BPS in August 2024, the open unemployment rate reached 7.05 million people or 5.28% of the total workforce. When viewed from the educational background of the unemployed based on BPS data in February 2024, it was 27.09% with elementary education and below, 22.62% with junior high school education, 25.29% high school education, 15.37% vocational education and 9.63% diploma education to Bachelor's Degree (Badan Pusat Statistik, 2024).

One of the factors that causes a country to become developed is when the number of entrepreneurs in the country amounts to at least 2% of the population. Meanwhile, in Indonesia, only 0.18% of the total population, or 400,000 people, become entrepreneurs (Tiurina et al., 2022). Since 2009, the government has compiled an entrepreneurship-based curriculum that should be integrated into learning aimed at preparing the younger generation to be competitive and able to open a new business world. It is hoped that education can improve the quality of human resources by developing the creativity and entrepreneurial spirit of students (Bagis, 2022).

In this context, leaning mathematics plays a strategic role not only in developing students' cognitive abilities but also in cultivating their entrepreneurial character (Palácios et al., 2019; Palmér & Johansson, 2018; Udonsa, 2015). Mathematics is not merely a tool for numerical computation, but a means of developing analytical thinking, logical reasoning, creativity, and decision-making, all of which are essential foundations for entrepreneurship (Nacario & Pedraza, 2025; Onoshakpokaiye, 2021). Therefore, mathematics learning should be directed not only toward mastering abstract concepts but also toward forming an entrepreneurial mindset that enables students to identify opportunities, design strategies, and make optimal decisions based on quantitative reasoning.

One of the compulsory courses in the mathematics education program at universities is *Linear Programming*, which provides students with the knowledge and skills to model and solve optimization problems involving limited resources, such as maximizing profit or minimizing cost within real or simulated business contexts. This course not only strengthens students' understanding of mathematical modeling but also introduces them to decision-making situations that mirror entrepreneurial challenges. In solving linear programming problems, students are required to demonstrate strong mathematical reasoning ability, including the capacity to interpret relationships among variables, construct logical arguments, and justify their chosen solutions. Such reasoning processes are not merely mathematical exercises; they reflect cognitive skills essential for entrepreneurship, such as critical thinking, analytical judgment, and strategic planning (Guevara & Patel, 2021; Rosendahl Huber et al., 2020; Sikhosana & Motsepe, 2025). Mathematical reasoning ability, as a core component of mathematical competence, enables students to move from conceptual understanding to practical application and problem-solving in authentic contexts. Therefore, strengthening students' reasoning ability through mathematics learning, particularly in courses such as *Linear Programming* can serve as an effective pathway to fostering entrepreneurial character,

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bridging cognitive competence with creativity, innovation, and responsible decision-making in future professional or entrepreneurial endeavors.

However, conventional mathematics instruction tends to emphasize procedural fluency rather than reasoning and creativity. As a result, students often fail to connect mathematical ideas with real-world or entrepreneurial applications. To address this issue, innovative learning models such as the *Problem–Project Based Learning (PPBL)* approach are needed. This model combines problem-based and project-based learning principles to promote deeper reasoning, collaboration, and creativity while fostering entrepreneurial character through authentic, real-life projects.

Problem-Based Learning (PBL) is a learning model that uses problems as the basis for students to learn. Problem-based learning is done through the presentation of inquiry-oriented problems (Fassbender et al., 2022). The problems presented in PBL are problems in everyday life, and through these problems can stimulated to study these problems based on the knowledge and experience that students already have, so that from the experiences they have, students will form new knowledge and experiences (A. Triantafyllou, 2022). PBL trains students to solve problems that require creative thinking. PBL uses a lot of problem-solving as a learning activity and provides opportunities for students to think creatively, express critical ideas, and communicate the results of their work to friends. This model exposes students to decision-making, problem-solving, and various types of investigative activities (Amna Saleem et al., 2021; Sun, 2023). PBL develops thinking skills, social skills, and the ability to collaborate effectively (Erdem et al., 2025; Sasson et al., 2018). PBL challenges students to work together in a group to solve problems. This problem is used to provide a challenge to students about curiosity and initiative to complete a problem (Graaff & Kolmos, 2007; Kolmos, 2009; Taufikin, 2017). According to Bagis (2022), PBL can be an alternative in an effort to form a noble character, such as religious, responsible, hard work, independent, democratic, tolerant, caring, environmental, and socio-religious, love for the homeland and nation.

The project-based learning model, or project-based learning (PjBL), is a learning model that provides opportunities for teachers to manage learning in the classroom by involving project work. Project work in project-based learning is seen in the process, creativity, critical thinking, and student activities in the learning process, so that it will have an impact on increasing student learning outcomes (Alzaabi, 2022; Osunbunmi & Fang, 2022). The implementation of PjBL accommodates the development of students to acquire skills that will build a strong foundation for their future in the global economy (Condliffe et al., 2017), including leadership attitudes (King & Smith, 2020), creativity (Albar & Southcott, 2021; Hanif et al., 2019), critical thinking (Sasson et al., 2018), independent (Beneroso & Robinson, 2022), and self-efficacy (Samsudin et al., 2020). So, PjBL can be applied as a strategy for learning to instill entrepreneurship competencies in students.

Although previous studies have shown the benefits of PBL in promoting problem-solving and PjBL in enhancing creativity and project management skills, research that integrates both approaches to simultaneously develop *mathematical reasoning ability* and *entrepreneurial character,* particularly in mathematics education contexts, remains limited. Most prior research tends to treat reasoning as a purely cognitive construct, isolated from affective or character-based outcomes, while studies on entrepreneurship education often overlook the cognitive rigor and logical reasoning embedded in mathematical problem solving. This gap indicates a lack of integrative pedagogical models that bridge cognitive and character development through contextualized mathematical learning.

Therefore, there is a strong need to design and implement a learning model that not only engages students in problem analysis but also immerses them in project-based experiences that simulate real-world entrepreneurial challenges. The Problem–Project Based Learning (PPjBL) model is proposed as an innovative pedagogical framework that combines the analytical depth of PBL with the practical creativity of PjBL, thereby providing a holistic approach to enhancing both mathematical reasoning and entrepreneurial character among mathematics education students.

Based on the growing demand for university graduates who possess both strong mathematical reasoning ability and entrepreneurial character, as well as the challenges identified in conventional mathematics instruction that tend to emphasize procedural fluency over creativity and reasoning, there is a need for innovative pedagogical approaches in mathematics education. Such innovations should not only enhance students' learning outcomes but also cultivate their capacity for critical thinking, problem-solving, and entrepreneurship. Therefore, this study aims to develop an effective Problem–Project Based Learning (PPjBL) model designed to improve both the entrepreneurial character and mathematical

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reasoning ability of mathematics education students. Specifically, this developmental research seeks to: (1) examine the implementation of the PPjBL model in the learning process; (2) evaluate the validity, practicality, and effectiveness of the developed model; and (3) determine the extent to which the PPjBL model enhances students' entrepreneurial character and reasoning ability in mathematics learning.

METHOD

This research employed a developmental research design aimed at producing an innovative Problem–Project Based Learning (PPjBL) model accompanied by supporting learning media to enhance both the entrepreneurial character and mathematical reasoning ability of mathematics education students. To support the implementation of the model, a comprehensive set of learning materials was developed, including a syllabus, lesson plans, task plans, student worksheets, and assessment instruments aligned with the objectives of the PPjBL framework. The study was carried out in the Mathematics Education Study Program at Muhammadiyah University of Enrekang, involving fourth-semester students enrolled in the Linear Programming course as research participants. The model development process was systematically organized following the Research and Development (R&D) approach adapted from Borg and Gall, which comprises several sequential phases designed to ensure the validity, feasibility, and effectiveness of the developed learning model.

1. Preliminary Studies

This phase involved identifying mathematical content and competencies relevant to the research focus, particularly Linear Programming, which emphasizes optimization, resource allocation, and mathematical modeling. A needs analysis was conducted through interviews and open questionnaires to determine students' difficulties in applying mathematical reasoning to real-world or entrepreneurial problems.

2. Conceptual Model Design

Based on the findings, a conceptual framework for the PPjBL model was designed connected to entrepreneurial contexts. In this framework, students were required to apply mathematical reasoning through linear programming problems in MSMEs around student environments and business projects. The project explicitly required students to: (a) develop mathematical models for business optimization (maximizing profit or minimizing costs, determining production efficiency), (b) solve the model using quantitative methods using the graphical method or Simplex algorithm, and (c) interpret results within the context of entrepreneurial decision-making.

3. Validation by Experts and Revision

The PPjBL design and learning instruments were reviewed by experts in mathematics education and instructional design. Feedback from experts was used to refine both the learning model and the instruments assessing reasoning ability and entrepreneurial character.

4. Limited Trial

The revised PPjBL model was piloted in the Linear Programming course with a small group of fourth-semester students enrolled in the Mathematics Education Study Program. This limited trial aimed to evaluate the model's effectiveness in facilitating students' mathematical reasoning and entrepreneurial character development. During this phase, students engaged in a series of mathematics-based entrepreneurial projects that required them to apply quantitative analysis, optimization techniques, and mathematical modeling to authentic business contexts.

5. Final Revision

The results from the limited trial were carefully analyzed to improve the effectiveness of the PPjBL model in promoting both mathematical reasoning and entrepreneurial character. Data were collected through multiple methods to ensure the validity and depth of findings. First, interviews with experts in learning design and mathematical content were conducted to refine the conceptual and pedagogical aspects of the PPjBL model. Second, open-ended questionnaires were distributed to students to identify their learning needs, prior experiences, and difficulties in connecting concepts of Linear Programming with real-world entrepreneurial applications. Third, structured classroom observations were carried out to assess the quality of PPjBL implementation across its learning phases. Fourth, questionnaires were administered to measure students' entrepreneurial character, emphasizing dimensions such as achievement motivation, creativity, risk-taking, and independence as indicators of

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character development. Finally, project-based mathematical tasks and reasoning rubrics were employed to evaluate students' mathematical reasoning ability through project assignments.

The assessment instruments were expanded to include two main categories: (a) instruments for assessing students' entrepreneurial character, and (b) instruments for assessing students' mathematical reasoning ability in Linear Programming. Entrepreneurial character was assessed based on achievement, motivation, innovativeness, risk-taking, and independence in completing projects. Meanwhile, students' mathematical reasoning ability was assessed through their performance in the first and second projects using the assessment indicators presented in Table 1.

Table 1. Indicator of Reasoning Ability

Indicator	**Description of Mathematical Reasoning**	
	Project 1	**Project 2**
Problem Formulation	Students identify decision variables, constraints, and the objective function (profit maximization or cost minimization) that represent the real business situation quantitatively.	Developing a simple business model based on given data and identifying key variables and objectives
Logical Construction	Students construct a mathematical model by expressing relationships among variables, formulate inequalities or equations, and justify the logical connections between real-world data and mathematical expressions.	Translating the business problem into mathematical form
Deductive Reasoning	Students apply the graphical or simplex method correctly, perform step-by-step calculations, and provide logical justification for each procedure or transformation made during the solution process.	Solving the mathematical model using appropriate optimization methods
Interpretation & Evaluation	Students interpret the mathematical results (optimal production, maximum profit), evaluate the feasibility of the solution, and explain how these outcomes influence business or entrepreneurial decisions.	Interpreting mathematical solutions in business decision-making contexts
Generalization & Communication	Students communicate their mathematical reasoning clearly in project reports and presentations, construct coherent arguments supported by quantitative evidence, and propose alternative strategies or solutions based on the model.	Presenting project reports and reflecting on learning outcomes

The collected data were analyzed using descriptive statistics, focusing on:

1. Feasibility of the PPjBL model,

Feasibility evidence from expert validation was summarized with means and percentage agreement for each dimension: supporting theory, syntax, social system, reaction principle, supporting system, accompanying impact, and language use. The PPjBL model is declared feasible if the average expert assessment is at least 1.76 to 3.25 after the model is revised according to the experts' suggestions. If the average expert assessment is 3.26 – 4.00, the learning model is declared very feasible for implementation.

2. Effectiveness of the PPjBL model

The effectiveness of the PPjBL model was evaluated with a mixed-methods approach that combined quantitative pre-post measurement of learning outcomes and qualitative process evidence. The evaluation focused on: (a) results of entrepreneurial character measurements before and after the PPjBL model was implemented for students; (b) performance on project tasks reflecting mathematical reasoning in Linear Programming; and (c) observed the quality and fidelity of the PPjBL model implementation across each instructional syntax.

Mathematical reasoning ability was measured based on a performance rubric with five indicators: Problem Formulation; Logical Construction; Deductive Reasoning; Interpretation & Evaluation;

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Generalization & Communication. Score of each project artifact and presentation on a 4-point scale (1 = Emerging, 2 = Developing, 3 = Proficient, 4 = Advanced). Item descriptors define observable evidence for each level (e.g., "identifies decision variables and formulates objective function with all relevant constraints" = level 4 on Problem Formulation). Total rubric scores and subscale scores were calculated for each student.

The PPjBL learning model is considered effective if it improves entrepreneurial character. This improvement is measured by comparing pre-test and post-test results. The test consists of 51 questions in a questionnaire with a 4-point scale: very appropriate to not appropriate. The test was adapted from research by Anwar & Saleem (2019), which measured entrepreneurial character based on a literature review: achievement motivation, innovativeness, risk-taking, and independence.

Observations were conducted to assess the quality and fidelity of the PPjBL model implementation across each instructional syntax. The observation process aims to ensure that every phase of the learning model is carried out in accordance with the designed procedures and pedagogical principles. Each session was observed using a structured observation sheet developed based on the PPjBL syntax.

RESULTS AND DISCUSSION

Results of Preliminary Studies

The preliminary research phase was conducted to examine the extent to which mathematics learning and the development of students' entrepreneurial character had been achieved through the Linear Programming course. This stage aimed to identify existing instructional gaps, analyze the quality of learning tools, and explore both students' and lecturers' needs for improving entrepreneurial character and mathematical reasoning ability. To ensure validity and depth, data were collected through triangulation of sources, including classroom observations, document analysis, simulation tests, and interviews with lecturers and students.

Classroom observations revealed that learning in the Linear Programming course remains largely theoretical, dominated by lectures and routine exercises with limited student engagement. The tasks given tend to emphasize procedural computation rather than reasoning or problem formulation. Students were rarely involved in exploring authentic or context-based problems that could connect mathematical models to entrepreneurial or decision-making situations in a business context. Consequently, mathematics was perceived as an abstract and formulaic discipline, rather than as a reasoning tool for analyzing and solving real-world problems. These findings highlight a significant gap between conceptual understanding and practical application, indicating that students' reasoning processes, such as logical construction, deductive reasoning, interpretation & evaluation, and generalization & communication, have not been adequately nurtured.

Parallel observations on entrepreneurial character revealed a similar pattern of limitation. Students enrolled in Entrepreneurship courses demonstrated low creativity in identifying business opportunities, limited innovation in product design, and weak performance in achieving targeted profits. Most business products were replicas of existing ideas rather than novel solutions. These findings suggest that students lacked the analytical and reasoning skills necessary for evaluating alternatives, forecasting outcomes, and making rational business decisions based on quantitative data. Thus, mathematical reasoning could serve as a bridge to strengthen logical decision-making within entrepreneurial contexts.

The document analysis of learning tools (syllabus, lesson plan, task plan, student worksheet, and assessment instruments) further confirmed these gaps. Although the syllabus follows a formal structure, it does not yet emphasize higher-order reasoning outcomes. Lesson plans, while systematically organized, still focus on lecturer-centered delivery and do not specify inquiry-based or student-centered learning steps. Moreover, there are no explicit activities designed to foster students' reasoning processes, such as model construction, logical argumentation, and reflective analysis. Assessment instruments are limited to traditional written tests that measure procedural knowledge without evaluating key reasoning indicators such as the ability to justify solutions, connect representations, or interpret mathematical results. This implies that both the content and the assessment framework fail to cultivate deep mathematical reasoning.

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A content analysis of student textbooks and worksheets provided additional insights. The results indicated that (1) no lecturer-developed textbooks are available that integrate contextualized mathematical reasoning or entrepreneurial content; (2) less than 15% of the material involves higher-order thinking components; (3) exercises are largely procedural, with predictable and non-creative solutions; (4) technology integration for modeling or simulation is minimal; and (5) student worksheets consist mainly of isolated problems without promoting exploration, reflection, or interdisciplinary thinking. Collectively, these materials do not encourage students to use mathematics as a reasoning framework for analyzing data, optimizing outcomes, or making evidence-based entrepreneurial decisions.

The entrepreneurial character simulation test, consisting of 51 items measuring achievement motivation, innovativeness, risk-taking, and independence, produced an average score of 130, with a maximum of 148 and a minimum of 114, far below the maximum possible score of 204. These results quantitatively confirm that students' entrepreneurial character remains underdeveloped, particularly in terms of innovativeness and independent thinking traits that are closely associated with the ability to reason, analyze, and evaluate alternative solutions.

The needs analysis, conducted through interviews with lecturers and students, provided deeper qualitative insights. Lecturers emphasized the importance of regular FGDs (Focus Group Discussions) to revise syllabi and develop learning strategies that integrate mathematical reasoning and entrepreneurship. They identified the necessity of: (1) incorporating reasoning-based learning approaches; (2) designing assessments that measure logical justification and model interpretation; (3) embedding real-world problems that require decision-making; and (4) creating technology-assisted learning resources to visualize optimization and reasoning processes. Lecturers also noted time constraints and the lack of research-based teaching materials as major barriers to innovation in instructional design.

From the student perspective, learning was perceived as monotonous, overly theoretical, and disconnected from real-life or entrepreneurial experiences. Students expressed that learning activities did not provide meaningful challenges requiring reasoning or creative problem-solving. They also reported limited feedback and recognition for reasoning efforts, the absence of core textbooks or digital references, and minimal use of technological tools in learning. Importantly, students highlighted their desire for learning experiences that connect mathematics with practical reasoning, business decision-making, and real-world problem solving.

Taken together, these findings illustrate an urgent need for an innovative instructional model that simultaneously promotes mathematical reasoning and entrepreneurial character through integrated, project-oriented learning. The results underscore that reasoning ability should not be treated merely as a computational or procedural skill, but as a higher cognitive process encompassing problem formulation, logical construction, justification, interpretation, and generalization, all of which align naturally with entrepreneurial thinking. Therefore, the development of a Problem–Project Based Learning (PPjBL) model becomes essential to transform mathematics learning into an authentic, inquiry-driven, and application-oriented process. This model is expected to bridge the gap between mathematical theory and entrepreneurial practice, positioning mathematics as both a tool for analysis and a framework for innovation and reasoning in real-world contexts.

Conceptual Model

The product design is developed according to the description of the results of the preliminary study stage. The learning model products developed in this study refer to basic learning theory (Piageat's Cognitive Development Theory, Ausubel's Meaningful Learning Theory, Vygotsky's Social Learning Theory, Jhon Dewey's Learning Experience Theory, and Bruner's Discovery Learning Theory, etc.), and previous research that examined the entrepreneurial character and learning model of PBL and PjBL. Furthermore, linking between theories is carried out so that it becomes a learning model building that meets the component requirements of a learning model, including: syntax, principles, reactions, social systems, and support systems. This product design is still theoretical in nature, so it is necessary to obtain further assessment from experts through validation activities.

The learning model product developed is Problem Project-based Learning or PPjBL with the PPjBL learning step sequence described as follows: (1) Provide orientation in the form of problems;

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(2) Organizing students to study; (3) Guiding individual and group investigations; (4) Designing the project to be executed; (5) Implement the project; (6) Documenting and reporting project findings.

Results of Validation by Experts

The product of the PPjBL learning model was validated by three experts: a learning science expert, a material expert, and a media expert. The validation results of the product design are presented in Table 2.

Table 2. Results of Validation of the Draft Conceptual Model by Experts

Aspect	Experts			Average	Criteria
	1	2	3		
Supporting theory	4.00	4.00	3.75	3.92	Valid
Syntax	4.00	4.00	4.00	4.00	Very valid
Social system	4.00	4.00	3.50	3.83	Valid
Principle reaction	4.00	3.75	4.00	3.92	Valid
Supporting system	4.00	3.50	3.75	3.75	Valid
Instructional and accompaniment impact	4.00	4.00	3.75	3.92	Valid
Language use	4.00	4.00	4.00	4.00	Very valid

The draft conceptual model that had been assessed by experts was further developed into a prototype of the PPjBL learning model. The prototype development was outlined in the form of learning device prototypes consisting of: (1) syllabus; (2) lesson plan; (3) task plan; (4) student worksheets; and (5) assessment instruments. This prototype was designated as Prototype 1. The validation results of Prototype 1 by experts, conducted to examine its compliance with pedagogical, grammatical, and learning technology theories, are presented in Table 3. The pedagogical system theory validation was carried out by three mathematics education experts.

Table 3. Results of Validation of Pedagogic Theory

Aspect	Experts			Average	Criteria
	1	2	3		
Completeness of model requirements	3.67	3.83	4.00	3.83	Valid
Theoretical basis of learning	2.25	3.75	3.88	3.21	Valid
Model development	2.25	3.75	4.00	3.33	Valid
Role of the lecturer	3.75	4.00	4.00	3.92	Valid
Class atmosphere	3.40	3.50	4.00	3.63	Valid
Presentation completeness	3.00	4.00	4.00	3.67	Valid
Language	4.00	4.00	4.00	4.00	Very valid

Table 3 above shows the results of expert validation from the pedagogical theory aspect in learning media prototype I. The minimum score on the basic aspects of learning theory and aspects of model development indicates that both aspects still need improvement. The model still needs to strengthen the basic theory on which it is developed. Model components, such as reaction principles, support systems, social systems, and accompanying impacts, are visible and integrated with the implemented learning steps. Thus, the implementation of the PPjBL model is reflected in the learning media used. The assessment results of the Prototype 1 learning media are presented in Table 4.

Table 4. Validation Score of Learning Media

Learning Media	Experts			Average	Criteria
	1	2	3		
Syllabus	3.27	3.40	3,73	3.47	Valid
Lesson plan	3.38	3.31	3.69	3.46	Valid
Students' worksheet	3.44	3.41	3.50	3.45	Valid
Task plan	3.69	3.43	3.74	3.62	Valid
Assessment instruments	3.65	3.27	3.63	3.52	Valid

Result of First Trial

Prototype 2 is the outcome of the expert validation stage and has been revised based on expert advice, then through the limited trial stage. Limited trials were carried out 2 times. The first trial was

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carried out from March 24 to May 12, 2025, for fourth-semester Mathematics Education students who took the Linear Program course. The purpose of the trial phase is to see the consistency of the implementation of the PPjBL syntax. During this phase, students engaged in a series of mathematics-based entrepreneurial projects that required them to apply quantitative analysis, optimization techniques, and mathematical modeling to authentic business contexts.

In the first project, students analyzed real-life cases of micro, small, and medium enterprises (MSMEs) in their local communities. They collected and organized real data on resource availability (such as raw materials, labor hours, and capital investment), identified relevant decision variables, and formulated the objective function and system of constraints. Through this process, students modeled optimization problems aimed at either maximizing profit or minimizing production cost. To solve the formulated models, they utilized GeoGebra and POM-QM for Windows, enabling both graphical and simplex-based approaches to linear optimization. Students then interpreted their mathematical results to provide actionable recommendations for improving the efficiency and profitability of the analyzed MSMEs.

The practicality assessment was conducted at the end of the eighth learning meeting, considering that over the eight meetings the lecturer was able to provide an objective evaluation of the practicality and ease of implementation of the PPjBL learning model in the classroom. The results of the practicality assessment by the supporting lecturers are presented in Table 5.

Tabel 5. Observation Results of Learning Implementation

Syntax of PPjBL Model	Syntaxes Implementation (%)
Problem orientation	80
Organizing students to learn	100
Individual and group investigations	80
Create a project design	85
Execute the project	100
Document and report project findings	100
Average	90.83

Based on the average percentage of learning implementation of 90.83%, it can be concluded that the consistency criteria for implementing the PPjBL model are in the very good category. This shows that, in general, the PPjBL learning model, along with the learning tools, is easy to apply in class. The PPjBL learning model is said to be effective if it has been able to achieve its development goals, namely, increasing entrepreneurial character and reasoning ability. This increase is seen by comparing the results of the pre-test and post-test.

Entrepreneurship Character Development

The test consisted of 51 questionnaire items using a four-point scale ranging from very appropriate to not appropriate. The pretest and posttest results are presented in Table 6.

Table 6. Pretest-Posttest Score Entrepreneurship Character on Limited Trial

Test	Achievement Motivation	Innovativeness	Risk Taking	Independence
Pretest	2.76	2.41	2.21	2.82
Posttest	3.24	3.11	3.07	3.12

The categories of students' entrepreneurial character on a four-point scale are presented in Table 7.

Table 7. Category of Student Entrepreneurship Character

Test	Average	Category
Pretest	2,55	Start to grow
Postest	2,95	Start to grow

The results of the first limited trial showed that the average learning outcomes were 90.83%. The results of the effectiveness test showed that the PPjBL learning model of 0.28 was in the weak category. Even though there was an increase in the average pre-test and post-test scores, the level of increase was still in the low category. So that the PPjBL model has not significantly increased the

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entrepreneurial character of students. Therefore, it is necessary to improve the implementation of the PPjBL model in the final trial.

Mathematical Reasoning Ability

The measurement of mathematical reasoning ability was conducted using pretest and posttest instruments consisting of problem-based tasks aligned with reasoning indicators, as presented in Table 8.

Table 8. Pretest and Posttest Score Mathematical Reasoning Ability on First Trial

Indicator	Pretest Mean	Posttest Mean	n-Gain	Category of Improvement
Problem Formulation	2.05	2.70	0.33	Moderate
Logical Construction	2.00	2.45	0.23	Low
Deductive Reasoning	1.90	2.45	0.26	Low
Interpretation & Evaluation	2.00	2.50	0.25	Low
Generalization & Communication	1.85	2.65	0.37	Moderate
Total Average	1.96	2.55	0.29	Moderate

The findings from the first trial showed that students' mathematical reasoning ability improved modestly after the implementation of the PPjBL model. The overall mean increased from 1.96 (developing) in the pretest to 2.63 (approaching proficient) in the posttest, indicating progress but also revealing that students have not yet reached the expected level of mathematical reasoning proficiency.

Classroom observations during the first trial showed that the implementation of the PPjBL model had begun to foster students' reasoning processes, yet its impact was not evenly distributed among all participants. In several project groups, only a few students actively contributed ideas, identified problems, or constructed mathematical arguments, while others tended to follow passively. This uneven participation reduced opportunities for collective reasoning, discussion, and justification of ideas that are essential for developing mathematical reasoning ability. Students focused more on finishing project tasks than on analyzing mathematical models or interpreting quantitative data.

These findings indicate that although the PPjBL model has the potential to develop reasoning-oriented learning, its implementation still needs improvement. The main challenge lies in designing learning steps that explicitly guide students to reason, explain, and reflect on the mathematical process they are engaged in. Therefore, the refinement of the model in the next stage should emphasize structured scaffolding to stimulate reasoning dialogue, the use of reflective questions that encourage deeper analysis, and feedback mechanisms that allow students to evaluate their reasoning process during project work.

Based on the results of observations during the implementation of the PPjBL model, it was found that: (1) the implementation of syntax and other model components had not been fully executed as intended; (2) the research focus was still directed toward the product development phase, so continuous refinement of the learning strategies within each syntax was needed; (3) the developed learning model had not yet been consistently implemented across learning sessions; and (4) the improvement in entrepreneurial character remained in the low category; and (5) the enhancement of mathematical reasoning ability was only at a moderate level, showing that while students had begun to formulate problems and interpret results more effectively, they still encountered challenges in constructing logical arguments, making valid deductions, and generalizing solutions. The limited improvement suggests that the model has not yet fully supported students in connecting abstract mathematical reasoning with contextual entrepreneurial situations.

Several improvements to the PPjBL model in the first trial were made to be applied on the second trial. Researchers found a deficiency in the first trial of the problem orientation stage, namely, students did not interact directly with problems in the business world. Therefore, lecturers provide opportunities for students to interact directly with business practitioners, especially Small and Medium Enterprises (SME), explore SME problems, and solve problems based on real data in the second trial. In the investigation phase in the first trial, students still had difficulty in the investigation process when given a new problem. Students have difficulty doing spontaneous exploration and formal generalization. So that in the second trial of this stage, the lecturer gave problems with a gradual level of difficulty, guided, and facilitated students in carrying out spontaneous exploration activities and formal generalizations.

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Figure 1. Student Research Activities in the Second Trial of PPjBL Learning Model

Improvements at the stage of designing and implementing the project also need to be done because the researchers found that not all students were actively involved in project design. So that the improvements made by researchers for the second trial, namely, the lecturer acted as a mediator in the discussion. Lecturers need to explain the objectives of the project implementation and instructions for implementing the project so that students understand the importance of the project they will be working on. This activity motivates students to be actively involved in designing and implementing projects. So that all students actively carry out the project, the lecturer directs that each member has a responsibility in implementing the project. Each group reports on the progress of the project implementation during class learning. The lecturer also provides the opportunity for each group member to report the results of their work through the division of tasks.

Weaknesses at the stage of reporting project findings, some students have not been able to present their work properly. So, lecturers need to provide instructions on how to report project results, product assessment indicators, and project presentation evaluation indicators. At the evaluation and action-taking stages, the researcher found a problem: students were not active in expressing opinions about the learning experiences they had during the implementation of the PPjBL model. Therefore, the researcher took steps to improve this stage by providing opportunities for students to express their opinions regarding the implementation of the PPjBL model in the form of a questionnaire. The model for improving the results of the first limited trial is hereafter referred to as Prototype 3, which will be applied to the second trial.

Result of Second Trial

In the second trial, students were tasked with designing their own business venture based on a feasible product or service idea. Within this scenario, they were required to plan a production strategy under limited capital and resource constraints. The project required students to determine the optimal production combination that would yield the maximum revenue or the minimum cost. They developed mathematical models that represented the trade-offs between cost, production capacity, and expected profit, and then solved these models using linear programming methods. Beyond computation, students were also required to present comprehensive project reports that linked their quantitative findings to entrepreneurial decision-making, demonstrating how mathematical reasoning supports innovation, strategic planning, and responsible business management.

The second trial was conducted by implementing the PPjBL Prototype 3 Model based on the evaluation and reflection results of the first trial. It was carried out in the same class through seven face-to-face meetings, from 19 May to 30 June 2025. The results of the practicality assessment by the supporting lecturers are presented in Table 9.

Table 9. Observation Results of Learning Implementation

Syntax of PPjBL Model	Syntax Implementation (%)
Problem orientation	95
Organizing students to learn	100
Individual and group investigations	90
Create a project design	95
Execute the project	100
Document and report project findings	100
Average	96.67

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Based on the average percentage of learning implementation of 96.67%, it can be concluded that the consistency criteria for implementing the PPjBL model are in the very good category. This shows that, in general, the PPjBL learning model, along with the learning media, is easy to apply in class. The effectiveness of the PPjBL model is shown through the comparison of the scores of the posttest of the limited trial (posttest 1) and the posttest of the final trial (posttest 2) of entrepreneurship character and reasoning ability.

Entrepreneurship Character Development

The scores for each entrepreneurial character in the second trial are presented in Table 10.

Table 10. Pretest-Posttest Score Entrepreneurship Character on Final Trial

Test	Achievement Motivation	Innovativeness	Risk Taking	Independence
Post-test 1	3.24	3.11	3.07	3.12
Post-test 2	3.38	3.21	3.16	3.24

Meanwhile, the categories of students' entrepreneurial character on a four-point scale are presented in Table 11.

Table 11. Category of Student Entrepreneurship Character

Test	Average	Category
Post-test 1	2,95	Start to grow
Post-test 2	3.41	Being habit

An increase in student entrepreneurial character scores based on an N-Gain score of 0.60. This means that the level of increasing the entrepreneurial character of students is in the medium category, so that the effectiveness of the PPjBL model in improving the entrepreneurial character is in the good category. The results of the second trial phase showed that the practicality assessment and effectiveness test of the PPjBL learning model had reached the predetermined indicators, and the average entrepreneurial character score had increased.

Mathematical Reasoning Ability

During the second trial, the refined PPjBL model was implemented to provide students with greater opportunities to think critically, reason logically, and make meaningful connections between mathematics and entrepreneurship. Students were challenged to design their own business by planning production strategies under limited capital and resource constraints. This required them to determine the most efficient production combination to maximize revenue or minimize cost through the construction and analysis of mathematical models using linear programming techniques, as summarized in Table 12.

Table 12. Pretest and Posttest Score Mathematical Reasoning Ability on Final Trial

Indicator	Pretest Mean	Posttest Mean	n-Gain	Category of Improvement
Problem Formulation	2.70	3.75	0.81	High
Logical Construction	2.60	3.70	0.79	High
Deductive Reasoning	2.45	3.55	0.71	High
Interpretation & Evaluation	2.75	3.60	0.68	Moderate
Generalization & Communication	2.65	3.70	0.78	High
Total Average	2.63	3.66	0.75	High

Observation results during the final trial showed that students better demonstrated and integrated performance in completing their entrepreneurial projects using the concept of linear programming. Compared to the limited trial, students' activities reflected deeper engagement with both the mathematical reasoning process and the entrepreneurial decision-making context.

Each group was assigned to design and run a small-scale business. During the project, students were required to plan a production strategy with limited capital is Rp. 100.0000,-. Observations revealed that students actively discussed how to allocate raw materials, determine the number of products to be produced, and calculate the optimum point.

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Throughout the project meetings, most groups showed significant improvement in reasoning and collaboration. Students no longer depended solely on procedural steps; instead, they began to analyze why a particular solution was optimal. In several groups, discussions became more analytical. Students questioned whether their model accurately represented the business problem and compared several production alternatives before deciding on the best plan. They also checked the sensitivity of their results, asking "what if" questions to predict changes in profit if prices or resources shifted. This indicated that students had begun to apply mathematical reasoning not merely as a computation tool but as a strategic framework for decision-making.

While executing the project, students managed their mini ventures while monitoring costs and outcomes. They updated their models based on real data, recalculating profit margins and adjusting production quantities to maintain efficiency. This iterative process fostered critical reflection, as students learned that mathematical models must be adapted to dynamic business realities. Their ability to interpret quantitative results and relate them to actual decision-making situations showed a stronger connection between reasoning and entrepreneurship.

Group presentations at the end of the project further confirmed this progress. Students explained how they used linear programming to determine optimal production levels and justified their decisions using quantitative evidence. Many groups also included visual representations of their models, such as graphs of feasible regions or tables of constraints and profits, which helped them communicate reasoning processes clearly. Students could link mathematical results with managerial considerations like cost efficiency, product mix, and resource utilization.

Nevertheless, observation notes also recorded a few remaining challenges. Some groups still struggled to verify the accuracy of their mathematical models, particularly when translating contextual assumptions into linear equations. A small number of students required additional guidance in interpreting dual values or understanding the implications of slack variables in production constraints. However, compared to the limited trial, the proportion of students facing such difficulties had significantly decreased.

Overall, the second trial revealed that the PPjBL model had successfully encouraged students to engage in authentic reasoning processes while applying mathematics to real business challenges. Students were able to connect quantitative modeling with entrepreneurial decision-making, demonstrating that mathematical reasoning can support innovation, efficiency, and reflective judgment in business contexts. These findings suggest that the refined PPjBL model effectively integrates reasoning and entrepreneurship, positioning mathematics not only as an academic discipline but as a meaningful tool for intelligent and responsible action in real-world problem-solving.

Students Responses

A total of 96.6% stated that the application of the PPjBL model design provided benefits for learning the Linear Program courses. The complete results are: (1) 100% of students admit that learning increases achievement motivation; (2) 100% of students admit that learning fosters innovative character; (3) 97% of students admit that learning provides the experience to dare to take risks; (4) 98% of students admit that learning provides an opportunity to be autonomous; (5) 100% of students admit that learning encourages higher order thinking skills; (6) 100% of students admit that learning encourages interaction and communication with lecturers, friends, and the community; (7) 100% of students admit that learning provides the maximum possible opportunity for self-actualization; (8) 95% of students admit that learning encourages the use of ICT as a means of exploring and presenting information; (9) 100% of students admit that learning encourages more creative and independent; (10) 98% of students admit that learning encourages more creativity and skill in creating creative ideas/products; (11) 100% of students admit that learning makes them aware of local potential that can be explored as a source of creative economy; (12) 97% of students admit that learning can improve communication and collaboration skills; (13) 98% of students feel that learning to train always reviews and reflects on the performance that has been achieved; (14) 95% of students feel that the evaluation of learning carried out by lecturers is comprehensive, both in terms of process and results.

Accompanying Impact Achievements

The results of the effectiveness test show the implementation of the design with the fulfillment of all bills. The PPjBL learning design has a positive impact and all project bills are in a good category as visualized in Figure 2.

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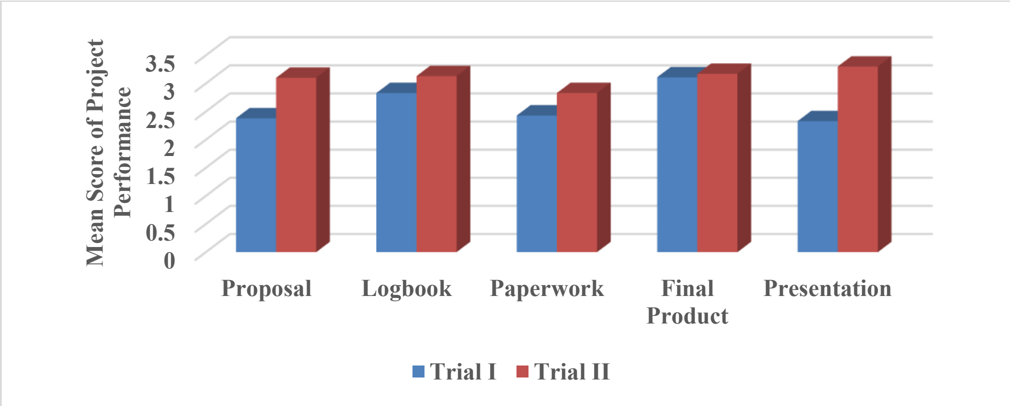

Figure 2. Comparison of Student Project Performance in Trial I and Trial II

The impact of the PPjBL model accompaniment is also seen in the students' presentation skills during the completion of the project. The presentation ability assessment was carried out 3 times during the project completion process. The impact of other accompaniments is also seen in self-assessment. Self-assessment to determine the cohesiveness of the group and the extent to which the applied design is perceived to increase active participation, encourage collaboration, and promote habits of reflection. The self-assessment was conducted twice in the middle and at the end of the project, with a recap of the results as follows.

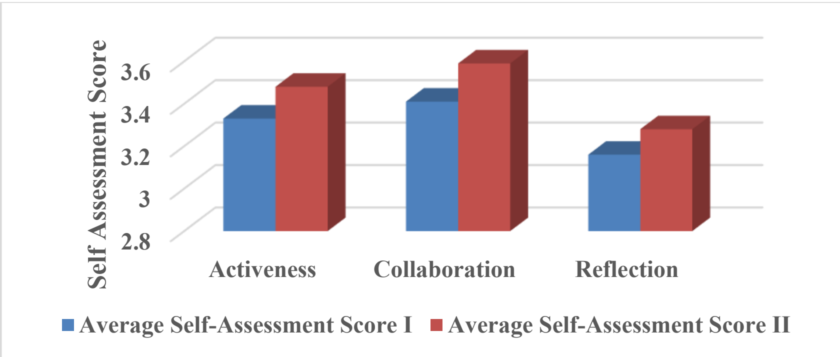

Figure 3. Self-Assessment Score on Trial I and Trial II

Figure 2 shows that the activity, collaboration, and reflection activities of groups and individuals are perceived to be increasing. It means that students have a spirit of cooperation, are more active, compact, and coordination between students is well established. Learning mathematics has experienced a paradigm shift. Learning that initially only focused on mastering concepts that had implications for improving learning outcomes began to shift towards learning that equips students with entrepreneurial character. However, this paradigm has not been supported by significant pedagogical changes. This phenomenon is in line with the opinion of Mwasalwiba (2010) that there has been a paradigm shift, so that it changes the perspective on entrepreneurship education, targets, and general goals, but pedagogical evolution has not occurred. The slow pace of pedagogical evolution is thought to be strongly related to the readiness of lecturers, approach dilemmas in transforming entrepreneurial values, creativity, and innovativeness that are difficult to teach, and difficulties in measuring impacts comprehensively. The problem of implementing entrepreneurship education for students in terms of learning design choices, capacity factors, and the capabilities of the administering institution (Bé et al., 2005; Boldureanu et al., 2020).

Discussion

The implementation of the Problem–Project Based Learning (PPjBL) model in the Linear Programming course produced five main products: (1) a general guidebook for designing PPjBL learning; (2) a syllabus and lesson plan; (3) an assignment plan; (4) student worksheets; and (5) test and project assessment instruments. Together, these components form a coherent and integrative learning package aimed at developing students' entrepreneurial character and mathematical reasoning

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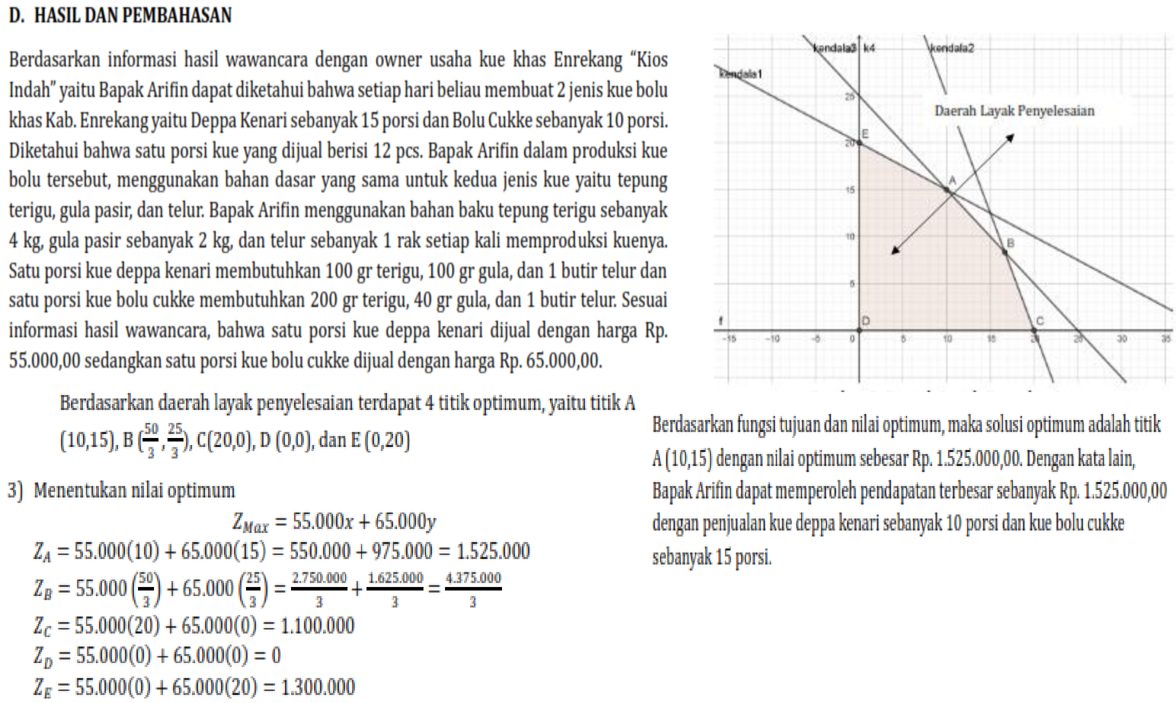
ability. The development process was grounded in the needs of both lecturers and students, ensuring contextual relevance and theoretical alignment. Consistent with constructivist learning theory of Piaget and Vygotsky (Mugambi, 2018), the design emphasizes the creation of meaningful learning experiences where knowledge is actively constructed through reflection, collaboration, and real-world engagement. The PPjBL framework was refined iteratively following Borg and Gall's (1983) model, emphasizing validation, limited trials, and continuous improvement to enhance both pedagogical quality and practical applicability.

The observed increase in entrepreneurial character during the second trial (N-Gain higher than in the first trial) indicates the model's effectiveness in fostering authentic, experience-based learning. In the initial implementation, students' engagement was still limited, as they were more accustomed to teacher-centered instruction. However, during the final trial, project-based activities allowed students to collect real-world data from Micro, Small, and Medium Enterprises (MSMEs), conduct interviews with entrepreneurs, and identify key challenges in production and marketing. Such exposure enabled them to link abstract mathematical concepts, particularly optimization and constraints to actual entrepreneurial contexts. As Kolb's Experiential Learning Theory (Kolb & Kolb, 2017) suggests, authentic experiences followed by reflection enhance both cognitive and affective learning outcomes, allowing students to internalize entrepreneurial values such as independence, innovation, and persistence.



D. HASIL DAN PEMBAHASAN

Berdasarkan informasi hasil wawancara dengan owner usaha kue khas Enrekang "Kios Indah" yaitu Bapak Arifin dapat diketahui bahwa setiap hari beliau membuat 2 jenis kue bolu khas Kab. Enrekang yaitu Deppa Kenari sebanyak 15 porsi dan Bolu Cukke sebanyak 10 porsi. Diketahui bahwa satu porsi kue yang dijual berisi 12 pcs. Bapak Arifin dalam produksi kue bolu tersebut, menggunakan bahan dasar yang sama untuk kedua jenis kue yaitu tepung terigu, gula pasir, dan telur. Bapak Arifin menggunakan bahan baku tepung terigu sebanyak 4 kg, gula pasir sebanyak 2 kg, dan telur sebanyak 1 rak setiap kali memproduksi kuenya. Satu porsi kue deppa kenari membutuhkan 100 gr terigu, 100 gr gula, dan 1 butir telur dan satu porsi kue bolu cukke membutuhkan 200 gr terigu, 40 gr gula, dan 1 butir telur. Sesuai informasi hasil wawancara, bahwa satu porsi kue deppa kenari dijual dengan harga Rp. 55.000,00 sedangkan satu porsi kue bolu cukke dijual dengan harga Rp. 65.000,00.

Berdasarkan daerah layak penyelesaian terdapat 4 titik optimum, yaitu titik A (10,15), B $\left(\frac{50}{3}, \frac{25}{3}\right)$, C(20,0), D (0,0), dan E (0,20)

3) Menentukan nilai optimum

$$Z_{Max} = 55.000x + 65.000y$$
$$Z_A = 55.000(10) + 65.000(15) = 550.000 + 975.000 = 1.525.000$$
$$Z_B = 55.000\left(\frac{50}{3}\right) + 65.000\left(\frac{25}{3}\right) = \frac{2.750.000}{3} + \frac{1.625.000}{3} = \frac{4.375.000}{3}$$
$$Z_C = 55.000(20) + 65.000(0) = 1.100.000$$
$$Z_D = 55.000(0) + 65.000(0) = 0$$
$$Z_E = 55.000(0) + 65.000(20) = 1.300.000$$

Berdasarkan fungsi tujuan dan nilai optimum, maka solusi optimum adalah titik A (10,15) dengan nilai optimum sebesar Rp. 1.525.000,00. Dengan kata lain, Bapak Arifin dapat memperoleh pendapatan terbesar sebanyak Rp. 1.525.000,00 dengan penjualan kue deppa kenari sebanyak 10 porsi dan kue bolu cukke sebanyak 15 porsi.

Figure 4. Excerpt from Student Report: Linear Programming Model Analysis of the Traditional Cake Business "Kios Indah" in Enrekang

At the project development stage, students were tasked with designing their own business ventures under limited capital and resource constraints. Using linear programming, they modeled production problems to determine the optimal combination of products that would yield maximum profit or minimum cost. This process required them to make decisions based on data, construct logical relationships among variables, and evaluate trade-offs as core components of mathematical reasoning. Simultaneously, this activity strengthened entrepreneurial attributes by encouraging creative problem solving, risk assessment, and strategic planning. In this way, PPjBL functioned as a bridge between entrepreneurial learning theory (Nacario & Pedraza, 2025), which emphasizes action-oriented learning, and mathematical reasoning theory, which highlights the processes of formulating, representing, and justifying (Altindis, 2025).

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Figure 5. Business Project Results

The improvement in students' reasoning ability during the second trial further supports the pedagogical strength of the PPjBL model. The pretest–posttest comparison showed significant gains across all reasoning indicators from problem formulation to generalization and communication, shifting overall performance from the "developing" to the "proficient" level. Students demonstrated greater fluency in translating contextual business problems into mathematical models, constructing logical arguments to justify their solutions, and evaluating their results within entrepreneurial contexts. These findings align with Kim & Pegg (2019), who argue that reasoning develops most effectively when students engage in problem situations that require explanation, justification, and reflection. Through PPjBL, reasoning was not taught as an isolated skill but as an integral component of meaningful project work (Himmi et al., 2025).

A critical insight from this study is the reciprocal relationship between entrepreneurial character and mathematical reasoning ability. Rather than functioning as separate outcomes, these two domains mutually reinforced each other within the PPjBL environment. The necessity to make data-driven decisions in business contexts encouraged students to reason mathematical, while mathematical reasoning, in turn, provided cognitive tools for strategic entrepreneurial decision-making(Manikandan & CP, 2025; Opstad, 2018). When students realized that mathematical models could directly support business innovation and efficiency, they began to perceive mathematics as a tool for empowerment rather than abstraction. This synergy reflects the view of 21st-century competency frameworks (OECD, 2024), which emphasize creativity, critical thinking, and entrepreneurial mindset as interconnected elements of higher-order learning.

Observations from the final trial showed that students demonstrated stronger initiative, accountability, and collaboration during project completion. Each group actively participated in data collection, model formulation, analysis, and presentation of results. The introduction of structured scaffolding through guiding questions, progress checkpoints, and formative feedback helped sustain reasoning-focused dialogue and reflective thinking throughout the learning process. The lecturer's role shifted from a transmitter of knowledge to a facilitator of inquiry, guiding students to construct their own understanding while fostering autonomy and responsibility(Junainah et al., 2015; Shpeizer, 2019). This transformation reflects a shift toward student-centered pedagogy, where learning outcomes emerge from active engagement and collaboration rather than passive reception.

In summary, the PPjBL model effectively bridges mathematics and entrepreneurship by embedding mathematical reasoning within meaningful, real-world business contexts. Students not only improved their reasoning competencies: problem formulation, logical construction, deduction, and generalization, but also developed entrepreneurial character, such as achievement motivation, innovativeness, risk-taking, and independence. The final trial demonstrated that authentic, project-based experiences supported by structured scaffolding and reflective dialogue can foster deep learning that integrates cognitive, affective, and behavioral dimensions. Thus, PPjBL serves as a pedagogical innovation that promotes not only academic understanding but also real-world problem-solving capacity and entrepreneurial mindset in higher education.

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CONCLUSION

This study successfully developed and validated a Problem–Project Based Learning (PPjBL) model to enhance both entrepreneurial character and mathematical reasoning ability among mathematics education students in the Linear Programming course. The final version of the PPjBL model comprises six syntactical stages: (1) problem orientation through contextual situations; (2) organizing students for inquiry-based learning; (3) guiding individual and group investigations; (4) designing project implementation plans; (5) executing the project; and (6) documenting and reporting outcomes. These phases form a coherent learning sequence that integrates problem identification, project execution, reflection, and presentation into a unified pedagogical cycle.

PPjBL serves as a transformative pedagogical framework capable of integrating cognitive reasoning and affective character development through authentic learning experiences. The model aligns with experiential learning perspectives, positioning students as active problem solvers who construct meaning through engagement with real-world entrepreneurial challenges. Moreover, the lecturer's role transitions from a knowledge transmitter to a facilitator of reasoning and reflection, thereby fostering a more student-centered and participatory classroom culture.

Practically, the PPjBL model offers a viable strategy for higher education institutions to bridge mathematics and entrepreneurship, cultivating graduates who possess not only analytical precision but also creativity, innovation, and resilience as essential competencies for the 21st-century workforce. Future research may extend the application of PPjBL to other mathematical domains or interdisciplinary fields, emphasizing the cultivation of critical thinking, creativity, collaboration, and entrepreneurship as integrated outcomes of holistic mathematical education.

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AI Acknowledgment **[Required: Please choose from the two statements]**

The authors declare that generative AI or AI-assisted technologies were not used in any way to prepare, write, or complete this manuscript. The authors confirm that they are the sole authors of this article and take full responsibility for the content therein, as outlined in COPE recommendations.

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Ethics, Consent to Participate, and Consent to Publish declarations: The researchers obtained an official letter of approval to conduct the research from the Institute for Research and Community Service (LPPM) of Muhammadiyah University of Enrekang and the Office of Cooperatives, SMEs, Manpower, and Transmigration. After receiving approval from the Language Development Center of Muhammadiyah University of Enrekang, the manuscript was critically assessed and approved by the LPPM of Muhammadiyah University of Enrekang for study compliance (Number: 32/LP2M/UM/EKG/VI/2025).

Author Contribution

P S : Conceptualization, Writing - Original Draft, Editing and Visualization;
S D : Writing - Review & Editing, Formal analysis, Methodology, and Project Administration;
S N : Validation and Supervision

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Conflict of Interest

The authors declare that there is no conflict of interest.

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