



Deconstruction Cognitive Strategies: Are Students Truly Engaging in Connective Thinking or Merely Memorizing Patterns?

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ABSTRACT

Kemampuan berpikir koneksi merupakan aspek esensial dalam pembelajaran matematika di pendidikan tinggi. Namun, mahasiswa cenderung mengandalkan hafalan pola daripada mengembangkan strategi berpikir konseptual, yang berdampak pada kegagalan menghadapi soal non-rutin. Penelitian ini bertujuan menganalisis karakteristik strategi kognitif mahasiswa dan mengidentifikasi faktor yang memengaruhi pilihan strategi tersebut. Menggunakan pendekatan kualitatif dengan teknik think-aloud, wawancara mendalam, dan analisis hasil kerja tertulis pada mahasiswa S1 Pendidikan Matematika. Hasil menunjukkan bahwa mahasiswa memulai penyelesaian melalui visualisasi dan eksplorasi pola, namun hanya sebagian mampu melakukan generalisasi logis. Refleksi terbukti menjadi pemicu utama transformasi strategi dari hafalan ke koneksi, meskipun masih terdapat construction hole dalam jaringan konseptual. Temuan ini memberikan kontribusi teoretis dalam penerapan skema Toshio dan implikasi praktis untuk desain pembelajaran yang menekankan refleksi dan koneksiitas konseptual.

Connective thinking skills are an essential aspect of mathematics learning in higher education. However, students tend to rely on memorising patterns rather than developing conceptual thinking strategies, which results in failure when faced with non-routine problems. This study aims to analyse the characteristics of students' cognitive strategies and identify the factors influencing their choice of strategies. Using a qualitative approach with think-aloud techniques, in-depth interviews, and analysis of written work from undergraduate mathematics education students. The results show that students begin problem-solving through visualisation and pattern exploration, but only a few are able to perform logical generalisation. Reflection was found to be the primary trigger for the transformation of strategies from memorisation to connectivity, although there were still construction holes in the conceptual network. These findings contribute theoretically to the application of Toshio's schema and have practical implications for learning design that emphasises reflection and conceptual connectivity.

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INTRODUCTION

Critical and connective thinking are two essential components of effective mathematics learning, particularly in higher education (Tasni, 2020). Critical thinking enables students to analyze, evaluate,



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and thoughtfully consider information, while connective thinking refers to the ability to link different mathematical concepts in order to solve more complex problems. These two types of thinking complement each other in shaping stronger problem-solving skill).

However, empirical evidence indicates that many students still rely heavily on memorizing patterns rather than constructing deep conceptual relationships when solving mathematical problems (Gultom, 2024; Tasni, Saputra, et al., 2020). This tendency leads to the emergence of construction holes in the cognitive process, characterized by students' failure to establish connections between new ideas—an essential component in addressing complex mathematical problems (Suharna et al., 2023).

The process of creatively connecting mathematical concepts often begins with an incomplete cognitive structure and involves reflection to adapt this structure to the problem at hand (Setyawati et al., 2021). Students face challenges in connecting concepts, applying mathematics, and solving problems (Suharna, 2020). Difficulties arise in forming conceptual connections, procedural connections, numerical connections, and generalized connections during the problem-solving stages (Tasni, Saputra, et al., 2020).

Mathematical connections are crucial for developing a deep understanding of mathematical concepts. Linking new concepts with prior knowledge—particularly with superordinate and convertible concepts—facilitates mathematical comprehension (Yang et al., 2021). The integration of conceptual and procedural knowledge is essential for effective mathematics instruction (Hurrell, 2021).

Networks that interrelate ethnomathematics, STEAM education, and global approaches can assist in analyzing mathematical connections in everyday practice (Rodríguez-Nieto & Alsina, 2022). Project-based learning enhances students' conceptual understanding by engaging them in natural contexts (Cruz et al., 2022). Mathematical resilience positively influences problem-solving abilities (Fatimah & Purba, 2021).

Cultural elements, such as traditional foods, can be utilized as contextual learning resources to connect mathematics with daily life (Sukendro et al., 2020). However, students may experience difficulties in understanding mathematical word problems, highlighting the need for more effective comprehension strategies (Aqsa et al., 2021).

These findings underscore the importance of connecting mathematical concepts across various contexts and approaches. Research has shown that students' ability to interrelate mathematical concepts is essential for achieving comprehensive understanding and for applying mathematics in real-life situations (Ahmad et al., 2019; Mayorova et al., 2021; Tasni & Susanti, 2017).

Nevertheless, many students still struggle to connect basic mathematical concepts with more advanced ones, which in turn hampers their problem-solving abilities. Studies indicate that students often find it difficult to bridge foundational mathematics with higher-level concepts, thereby impeding their capacity to solve problems effectively (Rodríguez-Nieto et al., 2023).

Contributing factors include inadequate learning practices, reliance on rote memorization, and difficulty in applying theoretical knowledge to real-world scenarios (Saha et al., 2024). Cognitive factors, such as domain-specific prior knowledge and text comprehension skills, also significantly influence students' mathematical problem-solving performance (Amalina & Vidákovich, 2023).

This poses a serious challenge in mathematics education at the university level, as students who are unable to connect these concepts tend to face difficulties when dealing with more complex and contextualized problems. Therefore, it is crucial to understand and deconstruct the cognitive strategies students employ in mathematical problem solving in order to identify whether they rely more on pattern memorization or on connective thinking.

Connective thinking, which leads to the integration of various mathematical concepts, is a strategy that can foster students' critical and creative thinking skills (Tasni & Nurfaidah, 2017). This study aims to explore and deconstruct students' cognitive strategies, as well as to identify the factors influencing their tendency to choose between connective thinking and reliance on pattern memorization.

Understanding these factors is essential for formulating recommendations that support the development of more effective connective thinking strategies in mathematics learning. To this end, the study employs a qualitative approach using a case study method, which allows for an in-depth investigation of students' thought processes in solving mathematical problems—both before and after the reflection process.

As higher education continues to evolve in complexity, mathematical problem-solving requires deeper and more contextual approaches (Slaney & Garcia, 2015). Previous studies have demonstrated that approaches such as problem-based learning, inquiry-based methods, and cognitive conflict strategies can enhance students' critical and connective thinking skills (Anggraeni et al., 2023; Hung & Lin, 2015).

Optimizing thinking strategies in mathematics learning will help students develop a more holistic understanding of mathematical concepts and enhance their ability to build connections across various mathematical domains (Wang & Abdullah, 2024). Instruction that facilitates the development of these thinking abilities not only improves students' capacity to solve more complex mathematical problems but also prepares them to face the challenges of a professional world that increasingly demands strong analytical and problem-solving skills (Gultom, 2024).

Therefore, it is essential for educators to design instructional strategies that support the development of mathematical connections through approaches that encourage students to think critically, creatively, and contextually (Bhiih, 2024). Optimizing such thinking strategies lays a solid foundation for students to construct deep mathematical understanding and apply it in diverse situations (Scheiner, 2023; Wild & Neef, 2023).

Accordingly, this study focuses on advancing cognitive theory in mathematics education with the aim of optimizing students' connective thinking strategies through a more holistic approach (Aragón et al., 2024). The novelty of this research lies in the development of a conceptual model for the transformation of students' connective thinking strategies, which can be implemented in mathematics curricula at the higher education level. This model is expected to contribute significantly to the advancement of more effective and contextual mathematics teaching methods.

Mathematics education in higher education plays a strategic role in shaping students' logical, critical, and structured thinking. However, in practice, mathematics instruction often becomes confined to formal and procedural delivery, heavily oriented toward rote memorization rather than fostering conceptual and connective thinking skills (Shaw et al., 2020; Yusof & Tall, 1994).

In many countries, including Indonesia, students frequently rely on instant solution patterns when tackling mathematical problems, without truly understanding the relationships between the concepts they apply (Nilimaa, 2023). This condition potentially hinders the development of reasoning abilities and cognitive flexibility—skills that are crucial for solving contextual problems.

In the context of globalized higher education, connective thinking becomes increasingly important, as it encourages students to relate mathematical knowledge to real-life situations and across disciplines.

The phenomenon of weak connective thinking skills among university students has become a significant concern in mathematics education research. Several studies have indicated that reliance on procedural memorization leads to difficulties in constructing conceptual networks, which directly affects students' ability to solve non-routine or applied problems (Pambudi et al., 2020; Tasni, Nusantara, et al., 2020).

When students are unable to form comprehensive mathematical connections, they tend to fall into construction holes—points of failure in developing coherent conceptual understanding (Tasni & Nurfaidah, 2017). Furthermore, instructional approaches that overly emphasize final outcomes without encouraging reflection on the thinking process exacerbate this issue (Munir et al., 2023).

This research is particularly important, as there is still a limited number of studies that explicitly deconstruct how students' cognitive strategies are formed and how reflective processes can trigger a transformation toward more productive connective thinking patterns.

To gain a deeper understanding of students' thinking processes, this study adopts the conceptual framework of Toshio's cognitive schema. This schema divides the thinking process into four main stages: cognition, inference, formulation, and reconstruction (Tasni, Saputra, et al., 2020).

In the context of mathematical problem solving, each of these stages serves as a critical indicator for assessing the extent to which students can integrate concepts and reflect on their thinking processes. Toshio's theory enables a systematic analysis of construction holes that emerge in students' thinking strategies, while also providing a conceptual foundation for understanding the transformation toward connective thinking as a result of reflective intervention (García-García & Dolores-Flores, 2018).

Through this approach, the study seeks to reveal whether students are genuinely constructing conceptual connections or merely relying on memorized algorithms devoid of conceptual meaning.

This study aims to describe the characteristics of students' cognitive strategies in solving mathematical problems, analyze the extent to which they rely on connective thinking as opposed to pattern memorization, and identify the factors that influence this tendency. In addition, the study seeks to formulate data-driven recommendations for optimizing connective thinking strategies in mathematics instruction at the higher education level.

To achieve this, the research employs a qualitative approach through a case study of undergraduate students in mathematics education who exhibit basic and semi-productive patterns of connective thinking. Data were collected through observation, in-depth interviews, and document analysis using the think-aloud method.

The scientific contribution of this study lies in its effort to deconstruct students' thinking strategies using Toshio's schema-based approach, which has rarely been applied in depth within the context of higher education. Furthermore, this research provides empirical evidence on the effectiveness of reflection as a trigger for transforming thinking strategies from memorization to connective thinking, a topic that has not been extensively explored in previous studies ([Aras et al., 2022](#); [Rahmi et al., 2020](#); [Setiyani, Karimah, et al., 2024](#)).

This study also enriches the literature by presenting qualitative data drawn directly from student interactions while solving mathematical problems, offering deep insights into the cognitive dynamics involved in the learning process. The findings of this study are expected to serve as a foundation for the development of instructional strategies that not only emphasize procedural mastery but also foster connective thinking skills as a core component of mathematical literacy particularly essential in today's era of increasing complexity.

Toshio's cognitive schema theory serves as the primary foundation of this study, as it offers a systematic framework for understanding how students construct their thinking structures within the context of mathematical problem solving. The schema consists of four stages: cognition, inference, formulation, and reconstruction which sequentially represent the processes of understanding a problem, formulating a solution strategy, constructing new knowledge, and reflecting on the chosen solution ([Tasni, Saputra, et al., 2020](#)).

In the context of connective thinking, Toshio's theory is particularly relevant as it can identify construction holes that arise when students fail to establish conceptual relationships among mathematical components. This concept is not only cognitively significant, but also supports reflection and the reconstruction of learning strategies, which are central to understanding-based educational approaches.

Several previous studies have highlighted students' difficulties in productively constructing mathematical connections. For example, Tasni, Saputra, et al. ([2020](#)) found that many students failed to develop connection ideas even after undergoing a reflective process. They exhibited obstacles at various stages of Toshio's schema, such as difficulties in verifying information, designing solution strategies, and reevaluating solutions.

In another study, it was also found that failures in connective thinking were often caused by a lack of motivation and an inability to perform conceptual generalization ([Tasni & Susanti, 2017](#)). On the other hand, research by García-García and Dolores-Flores ([2021](#)) emphasized the importance of reflection in encouraging students to build idea connections, particularly when addressing non-routine problems. This context underscores that mathematics instruction focused solely on procedural outcomes is insufficient for developing mature connective thinking structures.

Although various studies have addressed the challenges of connective thinking, a significant gap remains in understanding how the transformation of cognitive strategies occurs at a deeper level, particularly in the context of mathematics education students in higher education. Most existing research focuses on the secondary school level or emphasizes cognitive outcomes without exploring the internal processes of students' thinking strategies ([Sumarmi et al., 2021](#); [Tarmizi & Bayat, 2010](#)).

Few studies have utilized authentic data—such as written work, think-aloud recordings, and reflective interviews—to map the transformation of students' cognitive strategies in mathematical problem solving from a holistic perspective. Furthermore, there is limited research that explicitly examines the effectiveness of reflection as a trigger for the reconstruction of cognitive schemas based on Toshio's framework.

This article positions itself to address that gap by deconstructing students' thinking strategies based on qualitative field data, including verbal recordings, written work, and reflective responses analyzed through a cognitive schema approach. By focusing on students categorized as exhibiting basic and semi-productive connective thinking, the study targets an underexplored area: the transitional space between procedural thinking and productive connective thinking.

This study also expands our understanding of how reflection and metacognitive awareness can facilitate the reconstruction of cognitive strategies—an area that has not been comprehensively examined in the context of higher education (Mathews, 1997; Moreira-Párraga & Alcívar-Castro, 2022).

In terms of methodological approach, previous studies have tended to employ quasi-experimental or quantitative correlational designs to assess the relationship between cognitive styles and mathematics learning outcomes (Awaliya & Masriyah, 2022; Setyana et al., 2019). While valid for population-level generalizations, these approaches do not allow for an in-depth exploration of the contextual and internal thinking processes of students.

In contrast, qualitative approaches using case study strategies—such as those employed by Tasni et al. (2019)—have proven effective in revealing the complex and non-linear dynamics of cognition. This underscores the importance of using descriptive-qualitative methods based on authentic data to analyze the transformation patterns of cognitive strategies within Toshio's schema framework. Such an approach not only maps students' difficulties but also identifies opportunities for the effective development of connective thinking.

Based on the theoretical and empirical review, a conceptual synthesis can be formulated to guide the direction of this study. Students' cognitive strategies in solving mathematical problems should not be understood as fixed entities, but rather as dynamic structures that can be transformed through processes of reflection and reconstruction.

Toshio's schema is employed as a framework to map this dynamic process, while connective thinking is positioned as the ideal form of an integrated cognitive structure. This study will empirically examine how students transition from procedural memorization patterns to productive connective thinking networks through reflective analysis of rich and complex qualitative field data.

METHOD

This study employs a qualitative approach with a case study strategy to explore in depth the cognitive strategies used by students in solving mathematical problems, particularly in the context of connective thinking and the tendency to rely on pattern memorization. This approach was chosen because it enables the contextual and holistic exploration of the subjects' thinking processes and meaning-making through narrative and reflective qualitative data. The case study method is considered relevant for understanding the dynamics of individual thinking in real and complex situations, as demonstrated in previous mathematics education research that emphasizes the importance of examining thinking processes through micro-contexts and in-depth interpretation (Maulyda et al., 2025; Rosida & Masduki, 2023; Yusrina et al., 2023).

The data sources in this study consist of primary data obtained directly through the active participation of the research subjects. The types of data include transcripts from think-aloud protocols, in-depth interviews, and students' written work in solving reflective mathematical problems. These data were collected from undergraduate mathematics education students who had previously been identified as exhibiting tendencies toward basic and semi-productive connective thinking. The focus of the data is directed toward cognitive narratives during the problem-solving process and post-solution reflection, serving as the basis for identifying transformations in thinking strategies. This approach is supported by recent studies that emphasize the importance of qualitative data such as think-aloud protocols and in-depth interviews for revealing analytical thinking processes and metacognitive strategies in mathematical problem solving (Musodiqoh & Jaelani, 2024; Sari & Lutfi, 2023; Setiyani et al., 2025).

Data collection was carried out using three main methods: (1) direct observation of subjects during problem solving using the think-aloud technique, (2) in-depth interviews conducted before and after the reflection process to explore metacognitive aspects and awareness of cognitive strategies, and (3) document analysis of students' written work. The instruments used included a think-aloud protocol guide, a reflection-based interview guide, and a mathematics problem-solving worksheet. The think-aloud technique enabled the researcher to capture students' thought processes in real time, while the

reflective interviews provided space for subjects to interpret and reconstruct their own thinking experiences. This approach aligns with the findings of (Athukorala & Fernando, 2024), who demonstrated that think-aloud protocols are effective in revealing students' metacognitive skills in mathematical problem solving and show a moderate correlation with task-based metacognitive questionnaires. In addition, research by (Ikhwani et al., 2023) highlighted the importance of in-depth interviews and document analysis in identifying students' metacognitive awareness, regulation, and evaluation during numeracy problem solving—particularly when linked to reflective and impulsive cognitive styles. Thus, this combination of methods enables strong data triangulation for comprehensively understanding the dynamics of students' thinking.

The inclusion criteria for data in this study include: (1) active undergraduate students enrolled in the Mathematics Education program, (2) having completed a problem-solving course or an equivalent subject, (3) exhibiting tendencies toward basic or semi-productive connective thinking based on initial identification through diagnostic testing, and (4) willingness to participate in all phases of the research. Meanwhile, the exclusion criteria include: (1) students who demonstrate extreme dependence on algorithmic memorization without conceptual connections, and (2) inability to coherently complete the think-aloud procedure or participate in interviews. These criteria are designed to ensure that the data truly represent the dynamic transition of cognitive strategies, which is the central focus of this study.

A similar approach in defining inclusion and exclusion criteria has been applied in prior research. For instance, (Moreira-Párraga & Alcívar-Castro, 2022), in their systematic review, applied strict inclusion criteria to identify instructional interventions relevant for students with learning difficulties in mathematics, excluding studies that did not explicitly assess the use of cognitive strategies. (Utama & Budiarto, 2024), in their literature review on PBL in mathematics education, established exclusion criteria that omitted studies which did not address higher-order thinking aspects such as problem solving and critical thinking. Appropriately defining inclusion and exclusion criteria is essential to ensure the validity and relevance of data in mathematics education research.

The unit of analysis in this study is the individual student's thinking process during mathematical problem solving, with observation units consisting of verbal expressions, problem-solving actions, and reflective narratives that emerge during and after task completion. The subjects were selected through purposive sampling based on variation in thinking strategies, while still aligning with the category of basic to semi-productive connective thinking. This selection is consistent with qualitative research strategies that prioritize depth of case understanding over statistical generalization. A similar approach was employed by (Tririnika et al., 2024), who used a qualitative case study design to analyze students' mathematical problem-solving skills through observation, interviews, and document analysis. Likewise, (Sirmacı & Uyar, 2024) applied a case study method involving observation and mathematical modeling activities over six weeks to explore middle school students' problem-solving strategies. Their study highlighted how students naturally employed specific strategies, such as formulating equations and inequalities, when facing contextual problems.

The data analysis technique was conducted using a descriptive-qualitative approach with thematic analysis, guided by the stages of Toshio's schema: cognition, inference, formulation, and reconstruction. The analysis process involved data reduction, categorization, pattern identification, and narrative interpretation to uncover construction holes and the forms of transformation in cognitive strategies. All analytical procedures were conducted manually and verified through source and time triangulation to ensure data credibility. The researcher cross-checked findings from observations, interviews, and written documents to establish construct validity in the analysis.

During data processing, a schematic coding framework was applied to systematically and transparently organize the subjects' thinking patterns. This approach aligns with the thematic analysis guidelines developed by Braun and Clarke, which emphasize six core steps: familiarization with the data, initial coding, searching for themes, reviewing themes, defining and naming themes, and writing the report (Byrne, 2022; Campbell et al., 2021). To enhance the credibility and validity of the findings, triangulation techniques were employed by integrating multiple data sources and analytical methods (W. Xu & Zammit, 2020). Furthermore, the use of systematic coding techniques helped organize and interpret qualitative data effectively (Cernasev & Axon, 2023).

RESULTS AND DISCUSSION

Based on the analysis of observation data, think-aloud protocols, and in-depth interviews with the research subjects, six main themes were identified that represent students' cognitive strategies in solving mathematical problems. Each theme reflects the dynamics of students' thinking in constructing connective schemas as well as their tendencies toward pattern memorization during the problem-solving process involving origami ornament-based tasks.

1. Dominance of Visual Identification as the Initial Step in Problem Solving

Students employed visual identification strategies as the initial step in solving mathematical problems based on origami ornaments by redrawing lower-level ornaments (levels 1–4) to identify the constituent plane shapes, such as triangles, rhombuses, trapezoids, and hexagons. This step was crucial for constructing a cognitive schema that supports better problem comprehension. For example, subject S6 stated during the interview, as illustrated in Figure 1 below:

<i>P</i>	<i>:</i>	<i>How did you determine the number of triangles with side length of one unit at each level of the origami ornament?</i>
<i>S6</i>	<i>:</i>	<i>So first, I drew the four-level origami ornament, then I observed the pattern of unit triangles formed in the one-level to four-level origami ornaments.</i>
<i>P</i>	<i>:</i>	<i>And what did you do next?</i>
<i>S6</i>	<i>:</i>	<i>Then I determined the number of triangles with side length of one unit in the one-level to four-level origami ornaments, and then I wrote them down in the form of a number sequence.</i>
<i>P</i>	<i>:</i>	<i>Why did you write them in the form of a number sequence?</i>
<i>S6</i>	<i>:</i>	<i>Because I would use the number sequence pattern to determine the next terms. Ma'am.</i>

Figure 1. Interview Transcript with Subject S6 at the Initial Stage of Solving the Connection Problem-Solving Test (TPMK) Problem

Figure 2. The following is an illustration of the re-drawing steps of the ornament by Subject S6 from the document “S6’s Problem-Solving Test Before Reflection”.

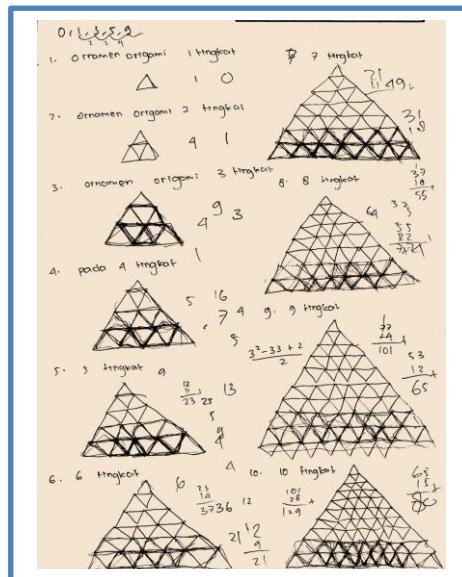


Figure 2. Redrawing of the Origami Ornament at Each Level by Subject S6 as an Initial Step in Pattern Identification

This visual identification strategy is fundamental in the cognition of mathematical problem-solving. Wang and Abdullah (2024) assert that deep learning models based on visual analysis can identify cognitive strategies through visual patterns, underscoring the significance of this process in decision-making. Furthermore, Yusrina et al. (2023) emphasize the crucial role of visual representation and pattern analysis in solving complex mathematical problems. A study by Hery Murtianto et al. (2022)

also emphasized that effective visual organization reduces cognitive load, thereby enhancing the effectiveness of the mathematics learning process. Research by Kurban (2024) indicates that training based on cognitive load theory can improve the spatial visualization skills of pre-service mathematics teachers through visual organization and structured examples. This is supported by a study by Rizos and Gkrekas (2024), which found that visual pattern recognition is strongly associated with students' success in mathematical problem-solving, particularly through spatial and growth patterns. Conceptually, Wang and Abdullah (2024) argue that visual perception forms the foundation of conceptual thinking, as pattern recognition relies on the brain's ability to cognitively and semantically construct and organize visual objects.

2. Pattern Exploration Strategy and Structured Inference

The research subjects consistently employed a number pattern exploration approach by arranging sequences of numbers based on empirical data from the origami ornament illustrations to identify quantitative patterns. Once a pattern was identified, they calculated successive differences until a constant pattern was observed in the third row, which served as the basis for applying a second-degree arithmetic formula. For example, Subject S1 arranged a number sequence and discovered that a constant pattern emerged in the third row, allowing for the formulation of a general equation. The interview conducted with Subject S1 is presented in Figure 3 below:

P	: <i>How did you find the general formula for the n-level origami ornament that shows the number of triangles with a side length of one unit based on the number pattern you arranged?</i>
S1	: <i>Here... I observed the differences between the first and second terms, Ma'am. That is, 4 minus 1 equals 3, then 9 minus 4 equals 5, 16 minus 9 equals 7, and 25 minus 16 equals 9. So the second sequence of numbers is 3, 5, 7, 9.</i>
P	: <i>Then, how did you find the third sequence "2, 2, 2"?</i>
S1	: <i>The method is the same as for the previous sequence, Ma'am. I found it by calculating the differences between the terms.</i>
P	: <i>Why did you calculate the differences between terms up to the third sequence?</i>
S1	: <i>Hmn... because of that, Ma'am... I wanted to determine the values of m_0, m_1, and m_2.</i>
P	: <i>So why did you use a second-degree arithmetic formula to determine the general formula for the n-level origami ornament?</i>
S1	: <i>Because the number sequence that was formed spans two levels, Ma'am.</i>

Figure 3. Interview transcript with Subject S1 during the use of arithmetic concepts in the process of determining the number of unit triangles based on the emerging number pattern

The following is an illustration of the sequence arrangement and pattern calculation by Subject S1 prior to reflection, as presented in Figure 4 below.

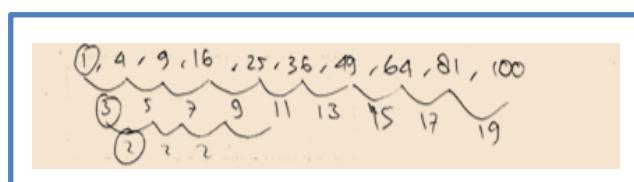


Figure 4. Sequence arrangement and difference calculation of number patterns by Subject S1 as the basis for a second-degree arithmetic formula

These results are consistent with the findings of (Yusrina et al., 2023), who stated that students with a reflective cognitive style are capable of recognizing numerical patterns and generalizing formulas through systematic and careful steps. Similarly, (Dewi et al., 2023) found that reflective students are able to identify numerical patterns and solve numeracy problems through a systematic and meticulous thinking process. Reflective students demonstrate strong abilities in understanding problems, designing solutions, applying strategies, and conducting re-evaluations.

In addition, (Viator et al., 2020) emphasized that a reflective-analytic thinking style, including the use of linear strategies in pattern analysis, plays a significant role in enhancing the quality of inference and decision-making in pattern-based problem-solving contexts. (Nasution, 2018) also supports the importance of verifying quantitative patterns as a foundation for the formal development of mathematical verification strategies. Furthermore, (Marzuki et al., 2022) explained that pattern exploration strategies based on empirical data enhance cognitive flexibility in completing both sensorimotor and cognitive tasks (Zulkarnain et al., 2023). Meanwhile, (Si & Huan, 2024) highlighted the role of structured cognition in integrating visual and numerical data to form effective problem-solving strategies.

3. Reflection as a Trigger for Strategy Correction and Reconstruction

The findings indicate that some subjects made errors during the initial stage of problem-solving (TPMK I), but were able to correct these mistakes after engaging in reflection during the TPMK II stage. For instance, Subject S5 acknowledged an error in determining the number of geometric shapes and corrected it by rechecking the pattern in the illustration and the calculation results. The interview with Subject S5 is presented in Figure 5 below:

P	: After working on the TPMK II question, what were you able to correct in the first part regarding triangles with a side length of one unit?
S5	: Hmm... actually, the data I gathered was almost the same, Ma'am, from the 1-level to the 10-level origami ornament. The only mistake was when I determined the number of unit triangles in the 7-level origami ornament.
P	: So, what strategy did you use to correct that error?
S5	: First, I drew the 4-level origami ornament, Ma'am... then I determined the number of unit triangles in the 1-level to 4-level origami ornaments.
P	: After determining the number of unit triangles from level 1 to level 4, what did you do next?
S5	: I wrote each piece of data I obtained into a number sequence, Ma'am, and then calculated the differences until I found a constant difference in the third row.

Figure 5. Interview Transcript with Subject S5 in Identifying Errors after the Reflection Process

An illustration of error correction and strategy reconstruction by Subject S5 after the reflection process is presented in Figure 6 below, which shows improvements in the pattern diagram and calculation steps.

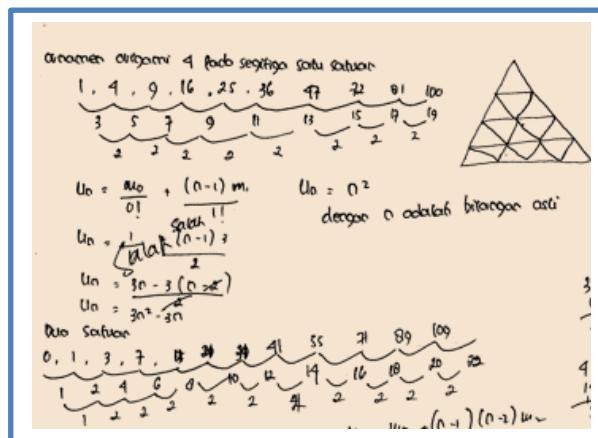


Figure 6. Correction and Reconstruction of Problem-Solving Strategy by Subject S5 after Reflection

The reflection carried out by the students supports the cognitive reconstruction process necessary for developing more mature and effective thinking schemas. This aligns with research by Setiyani, Karimah, et al. (2024), which shows that students with a reflective style actively correct errors and re-examine their steps in the problem-solving process. Moreover, Setiyani et al. (2025) assert that reflection contributes to the enhancement of cognitive flexibility, enabling strategy adaptation according to the demands of a problem. Salido et al. (2020) also found that reflective strategies strengthen metacognitive abilities in correcting and reorganizing mathematical problem-solving strategies. Lebih jauh, (Jung &

Park, 2024) meneliti mekanisme reflektif dalam proses pemecahan masalah desain dan menyimpulkan bahwa refleksi berulang secara siklik sangat penting untuk perbaikan strategi dan hasil akhir pekerjaan kognitif Terakhir, (Setiyani, Karimah, et al., 2024) menekankan bahwa proses refleksi meningkatkan kesadaran kognitif yang berperan sebagai motor penggerak perubahan strategi pemecahan masalah.

4. Dualistic Tendencies between Connective Thinking and Pattern Memorization

Data analysis reveals a dualistic tendency among the research subjects in their mathematical problem-solving strategies. Some subjects demonstrated productive connective thinking by being able to explain the logic behind formulas and patterns, as well as validate the formulas by comparing predicted results with empirical data. For instance, S2 and S6 actively tested the validity of the formulas by comparing their predictions against visual data and tables. In contrast, other subjects such as S7 and S3 tended to rely on quick strategies based on memorized patterns without engaging in deep reflection on the conceptual logic. This condition is illustrated in excerpts from the interview results presented in Figure 7 below:

P	:	<i>Are you confident in the accuracy of the formula and the data you obtained in the table?</i>
S2	:	<i>Hmm... yes, Ma'am, it seems correct because I tried calculating it for n = 5, and the result matched when I used the formula and also calculated it directly using the number sequence pattern, Ma'am — I got 10.</i>

Figure 7. Interview Transcript with Subject S2 during the Verification of Their

The following is a table illustrating the comparison between formula-based predictions and empirical data by Subject S6

Pertanyaan:		Berdasarkan informasi soal, lengkapilah tabel di bawah ini					
No.	Tingkatan Ornamen origami	Banyaknya segitiga pada ornamen origami dengan panjang sisi	Banyaknya belah ketupat	Banyaknya trapesium sama kaki yang memuat	Banyaknya polygon segi-enam yang memuat		
1.	Ornamen origami 1 tingkat	1	0	0	0		
2.	Ornamen origami 2 tingkat	4	1	3	3	0	
3.	Ornamen origami 3 tingkat	9	3	9	10	1	
4.	Ornamen origami 4 tingkat	16	7	18	21	3	
5.	Ornamen origami 5 tingkat	25	13	30	36	6	
6.	Ornamen origami 6 tingkat	36	21	45	55	10	
7.	Ornamen origami 7 tingkat	49	31	63	78	15	
8.	Ornamen origami 8 tingkat	64	43	84	105	21	
9.	Ornamen origami 9 tingkat	81	57	108	136	28	
10.	Ornamen origami 10 tingkat	100	73	135	171	36	
Ornamen origami n-tingkat		n^2	$\frac{0+1+2+3+\dots+(n-1)}{2}$	$\frac{3n^2-3n}{2}$	$2n^2-3n+1$	$\frac{n^2-3n+2}{2}$	

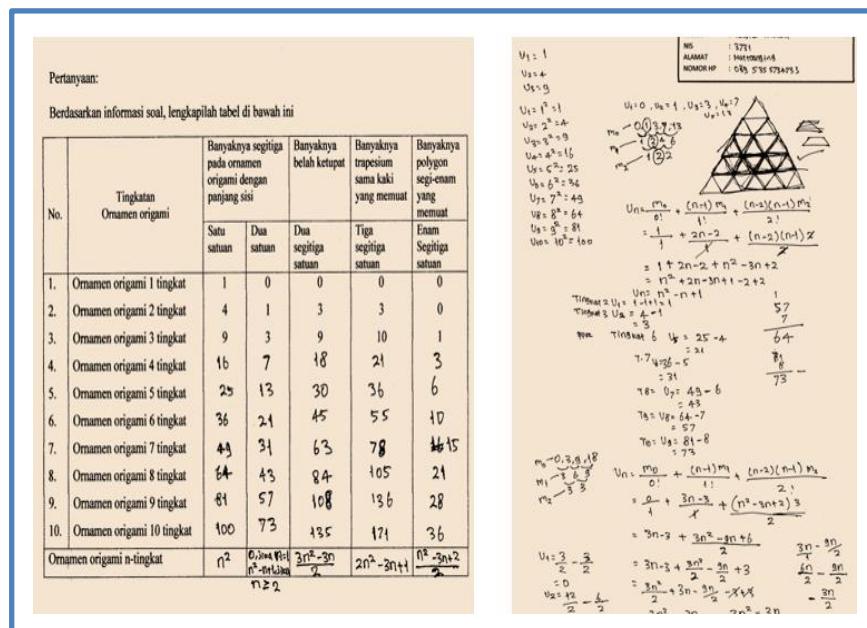


Figure 8. Cross-Verification between Formula-Based Predictions and Empirical Data by Subject S6

This dualistic phenomenon aligns with dual-process theory, which distinguishes between fast, pattern-based cognitive processing (System 1) and slow, reflective processing (System 2). Diederich (2024) state that a balanced combination of these two systems enhances problem-solving performance, particularly in complex problems (Diederich, 2024). In addition, (Kramer & Baumann, 2024) explain that cognitive prompting, which involves hierarchical and reflective thinking processes, can improve problem-solving accuracy in both artificial intelligence and human models.

These findings are also supported by research from (Exacta et al., 2024), which shows that reflective and impulsive cognitive styles influence the degree of misconceptions and the use of strategies in mathematical problem-solving. Furthermore, (Salido et al., 2020) emphasize the importance of

reflection and critical verification in effective connective thinking strategies. Lastly, Si and Huan (2024) assert that the integration of visual and numerical processing is essential in developing reflective and comprehensive problem-solving strategies.

5. Uncertainty Toward Formula Generalization as an Indication of a Construction Hole

Although some subjects were able to construct formulas for the origami ornament patterns, not all of them felt confident in the validity or generalizability of those formulas across all ornament levels. For instance, Subjects S4 and S7 were unable to explain the logic or justification behind the formulas they used. This condition indicates cognitive stagnation, referred to as a construction hole, where gaps in the cognitive schema network hinder the ability to apply formulas comprehensively. An excerpt from the interview with Subject S4 is presented in Figure 9 below:

P	<i>: ...How did you determine the values of m_0, m_1, m_2?</i>
S4	<i>: For the values of m_0, m_1, m_2, I chose the first term from each sequence, Ma'am.</i>
P	<i>: Did you experience difficulty in finding the formula... which led you to try identifying the general arithmetic sequence formula repeatedly?</i>
S4	<i>: Yes, Ma'am... I had difficulty finding the appropriate formula.</i>
P	<i>: Have you found a solution to derive the correct formula?</i>
S4	<i>: Hmm... it seems there's no suitable formula, Ma'am... (The subject appeared to give up and chose to write the obtained formula in the table even though they were aware that the formula was not accurate.)</i>

Figure 9. Interview Transcript with Subject S4 in the Process of Formula Generalization

The following is a screenshot of the formula constructed by Subject S4, which illustrates the doubt and difficulty experienced during the generalization process, as presented in Figure 10 below:

$$J_n = \frac{0 + (n-1)2(n^2 - 3n + 2)}{2}$$

$$= \cancel{0} 2n^2 - 2n + 2n^2 - 3n + 2$$

$$= \cancel{2n^2} - n \quad \times$$

$$U_n = \frac{0 + (n-1)2 + (n^2 - 3n + 2)}{2}$$

$$= \cancel{2n^2} + 2n^2 - 6n + 9$$

$$= \cancel{2n^2} - 4n + 2 \quad \checkmark$$

Figure 10. Formula Constructed by Subject S4 with Uncertainty in Validation and Generalization

This phenomenon is supported by (Syamsuddin et al., 2020), who explain the differing characteristics of reflective thinking among prospective mathematics teachers, where cognitive gaps or stagnation hinder the development of comprehensive problem-solving strategies. Furthermore, Sutarto, (Sutarto, Toto Nusantara, Subanji, 2016) emphasize the importance of understanding how local conjectures are formed as a crucial part of pattern-based problem-solving learning, particularly in the initial stages leading to symbolic and formal generalization. In addition, (Putri & Indrawatiningsih, 2023) show that developing problem representations through reflective processes can help overcome construction holes and thinking impasses in mathematical problem-solving. P. Xu and Salado (2022) add that systematic verification and correction processes can minimize errors and enhance confidence in applying mathematical formulas. (Maulana, 2024) states that metacognitive strategies—including awareness, planning, regulation, and evaluation of thinking—play a crucial role in helping students address numeracy challenges. Students with high cognitive awareness can adapt strategies and prevent cognitive stagnation, thus fostering a more coherent and reflective thinking process.

6. Strategy Transformation Through the Process of Data and Formula Verification

The verification process serves as a crucial stage in strengthening students' connective thinking schemas. Most subjects conducted cross-checks between the predicted results from formulas and the visual and tabular data. For example, Subject S6 carefully verified data accuracy through recalculation and comparison between the diagram model and the formula results for the pattern of triangles with side lengths of two units. This condition can be observed in the interview excerpt presented in Figure 11 below:

P	:	Are you confident that you followed the correct solution process and obtained accurate data?
S6	:	Yes, Ma'am... because I've rechecked the accuracy of the data I obtained, both through direct calculations based on the pattern in the diagram and by using the formula, and the results matched... moreover, every piece of data I obtained was the same as the data I got in TPMK I, Ma'am.

Figure 11. Interview Transcript with Subject S6 during the Data Verification Process

The following is an illustration of the cross-verification table created by Subject S6 in the document "S6's Problem-Solving Test after Reflection."

No.	Tingkatan Ornamen origami	Pertanyaan: Berdasarkan informasi soal, lengkapilah tabel di bawah ini			
		Banyaknya segitiga pada ornamen origami dengan panjang sisi		Banyaknya belah ketupat sama kaki yang memuat Tiga segitiga satuan	Banyaknya trapesium segi-enam yang memuat Enam segitiga satuan
		Satu satuan	Dua satuan		
1.	Ornamen origami 1 tingkat	1	0	0	0
2.	Ornamen origami 2 tingkat	4	1	3	3
3.	Ornamen origami 3 tingkat	9	3	9	10
4.	Ornamen origami 4 tingkat	16	7	18	21
5.	Ornamen origami 5 tingkat	25	13	30	36
6.	Ornamen origami 6 tingkat	36	21	45	55
7.	Ornamen origami 7 tingkat	49	31	63	76
8.	Ornamen origami 8 tingkat	64	43	84	105
9.	Ornamen origami 9 tingkat	81	57	108	136
10.	Ornamen origami 10 tingkat	100	73	155	191
Ornamen origami n-tingkat		$Un = n^2$		$Un = \frac{2}{3}n^2 - \frac{1}{3}n + 1$	$Un = 2n^2 - 2n + 1$
$Un = n^2$ V $Un = \frac{2}{3}n^2 - \frac{1}{3}n + 1$ V $Un = 2n^2 - 2n + 1$ V $Un = n^2 - n + 1$					
$Un = n^2$ V $Un = \frac{2}{3}n^2 - \frac{1}{3}n + 1$ V $Un = 2n^2 - 2n + 1$ V $Un = n^2 - n + 1$					
$Un = n^2$ V $Un = \frac{2}{3}n^2 - \frac{1}{3}n + 1$ V $Un = 2n^2 - 2n + 1$ V $Un = n^2 - n + 1$					

Figure 11. Cross-Verification Process between Formula-Based Predictions and Visual Data by Subject S6

This is in line with (P. Xu & Salado, 2022), who explain that the integration of formal verification and correction activities can improve accuracy in designing mathematical strategies. In addition, (Salido et al., 2020) emphasize that reflective ability and mathematical communication strengthen verification strategies and the reconstruction of thinking schemas. A study by (Ria, 2021) also indicates that cross-verification strategies enhance cognitive flexibility and adaptability in completing complex cognitive tasks. Furthermore, (Lavy & Shriki, 2023), in their study on cognitive load, state that structured checking and verification processes can reduce cognitive load and increase the effectiveness of mathematics learning. Lastly, (Restini et al., 2023) assert that metacognition and repeated monitoring are crucial for optimizing cognitive control during the problem-solving and result verification processes.

The results of this study provide a concrete illustration of how students' cognitive strategies develop in solving contextual mathematical problems. All identified themes reflect variations in the depth of thinking strategies, ranging from procedural to reflective-connective, and demonstrate that reflection-based interventions can trigger a shift from rote patterns to more productive connective thinking structures. Recent research confirms that reflective processes in mathematics learning can enhance higher-order thinking skills and foster deeper conceptual connections among students (Rahmi

et al., 2020). Additionally, cognitive scaffolding oriented toward reflection has proven to be an effective strategy for improving students' problem-solving quality in mathematics learning contexts (Aras et al., 2022). Other studies also indicate that the development of reflective thinking styles contributes to a cognitive shift from memorization-based patterns to more productive connective thinking structures within mathematics education (Setiyani et al., 2023). Thus, these findings support the importance of reflective interventions in facilitating the cognitive transitions essential for deep and effective mathematical understanding (P. Xu & Salado, 2022).

The main findings of this study indicate that students' cognitive strategies in solving mathematical problems span a spectrum from pattern memorization to productive connective thinking. The transformation of these strategies is mediated by a systematic reflection process, as evidenced by the shift from procedural errors to the construction of pattern-based formulas and logical justifications. These findings directly address the first and second research questions, namely, the characteristics of cognitive strategies and the extent to which students rely on connective thinking over pattern memorization. Furthermore, this study successfully identifies several factors influencing these strategic tendencies, such as clarity of visualization, the ability to generalize, and metacognitive awareness in verifying solutions.

Several recent studies support these findings. (Rahmi et al., 2020) emphasize that systematic reflective processes promote the development of structured and critical mathematical thinking skills. (Aras et al., 2022) demonstrate the effectiveness of cognitive scaffolding in facilitating the shift from pattern memorization to conceptual understanding through critical reflection in mathematics learning. (Setiyani et al., 2023) report that students with strong reflective abilities tend to develop productive connective thinking strategies through pattern recognition and mathematical generalization.

In addition, (P. Xu & Salado, 2022) highlight the importance of metacognitive awareness in the solution verification process, which helps students minimize errors and strengthen the construction of mathematical formulas. (Scheibe et al., 2023) also emphasize the role of metacognition and repeated monitoring in optimizing cognitive control and decision-making in mathematical problem-solving. Within the framework of Toshio's schema theory, each subject exhibited distinct thinking dynamics across the stages of cognition, inference, formulation, and reconstruction (Dumestre, 2016). Subjects who reached the reconstruction stage generally demonstrated reflective ability by re-evaluating the logic of the formulas they developed, comparing them with empirical data, and revising previous reasoning errors. These findings align with recent studies emphasizing the importance of reflection as a process of clarification, connection, and the generation of new ideas based on prior experience and knowledge in mathematical problem-solving. For example, (Setiyani et al., 2023) explain that critical reflection facilitates the development of adaptive and creative problem-solving strategies. (Salido et al., 2020) likewise affirm the significance of a reflective cognitive style in generating effective strategies for tackling complex problems.

This article makes a significant contribution by addressing the gap in understanding the transition of students' cognitive strategies from memorization to connectivity through qualitative field analysis. (Tasni et al., 2020) identified that difficulties in connective thinking often arise from a lack of data verification and the inability to construct numerical connections and comprehensive generalizations. This study adds a transformational dimension, showing that guided task-based reflection can trigger strategic corrections and shape new, more conceptual thinking patterns. (Susanti, 2021) also emphasizes the importance of reflective strategies in correcting algorithmic errors and reconstructing thinking structures through self-evaluation.

Theoretically, this article expands the application of Toshio's schema by presenting empirical evidence that each stage in the schema can be identified through subjects' verbal and written analyses, and that these stages are dynamic, not necessarily linear, and can be reconstructed through reflection-based interventions. This finding supports the view of (Syamsuddin et al., 2020), who argue that a reflective-generative cognitive style enables students to construct problem-solving strategies based on a combination of new information and prior experience. Lebih lanjut, studi terkini oleh (Setiyani, Waluya, et al., 2024) menguatkan bahwa proses reflektif dan rekonstruktif yang dinamis sangat penting dalam pengembangan skema kognitif mahasiswa sehingga memungkinkan mereka untuk mengatasi stagnasi berpikir (construction hole) dan membangun koneksi kognitif yang lebih kokoh.

The study by (Aras et al., 2022) also demonstrates the effectiveness of instructional interventions that utilize reflection to correct faulty thinking and strengthen mathematical knowledge construction

through cognitive scaffolding. Furthermore, (Rahmi et al., 2020) emphasize the critical role of metacognitive monitoring in the reflective process as a primary mechanism for detecting and addressing construction holes in mathematics learning. (P. Xu & Salado, 2022) assert that systematic reflection-based interventions can effectively facilitate cognitive reconstruction and overcome conceptual thinking barriers among mathematics students.

Nevertheless, this study has limitations that must be acknowledged proportionally. First, the number of participants was limited and selected through purposive sampling, making broad generalization of the results unfeasible. Second, the analysis was conducted manually and interpretatively, although triangulation was employed. Third, the mathematical task context was visual and based on ornamental patterns, which may have influenced the dominance of visual-spatial strategies. Future studies should broaden the scope of mathematical tasks to include other types of problems, such as algebra or analytic geometry, in order to test cross-context validity. In this regard, further research may benefit from utilizing qualitative coding software tools (e.g., NVivo or ATLAS.ti) to optimize thematic analysis consistency (Setiyani et al., 2023).

The implications of this study can be directed toward lecturers and curriculum developers in designing learning activities that facilitate reflective processes, such as through tasks with structured scaffolding and formative feedback. (Aras et al., 2022) demonstrated that scaffolding in reflective thinking can bridge the stages of evaluation and reasoning, thereby strengthening the construction of new, more logical knowledge. In addition, Problem-Based Learning (PBL) approaches have also been shown to effectively enhance students' reflective and connective thinking skills (Yu & Zin, 2023). Therefore, the findings of this study may serve as a reference for formulating mathematics learning policies that emphasize the development of conceptual understanding and higher-order thinking skills.

Conclusion

This study successfully identified the characteristics of students' cognitive strategies in solving mathematical problems, revealing a spectrum ranging from pattern-memorization-based strategies to productive connective thinking. Through a qualitative case study approach, it was found that students typically begin problem-solving by visually identifying and exploring numerical patterns; however, not all of them reached a stable level of conceptual generalization. The reflection process played a significant role in driving strategic transformation, especially among those who were able to verify data and evaluate the alignment between formulas and empirical results. Indications of a construction hole emerged when students encountered difficulties in explaining the logical validity of the formulas they constructed. The analysis also showed that not all students were able to fully reconstruct their problem-solving strategies despite receiving reflective stimuli, indicating the need to strengthen reflective capacity as an integral part of mathematics instruction.

Theoretically, this study reinforces the relevance of Toshio's schema in mapping the dynamics of students' cognitive strategies and affirms that connective thinking is not a spontaneously emerging construct but one that is formed through a continuous reflective process. Practically, these findings contribute to the development of instructional models that emphasize the importance of verification, generalization, and reflection as key components in building meaningful mathematical understanding. This article also proposes concrete indicators for detecting shifts in students' thinking strategies and identifying potential weak points that may hinder the formation of conceptual connection networks.

The implications of these findings include the need to integrate reflective activities into the instructional design of university-level mathematics, particularly through approaches that enable students to reconstruct their own thinking strategies. For future research, cross-context testing with different types of mathematical problems and topics is recommended to enhance the external validity of the analytical model used. Additionally, exploring the influence of cognitive styles and learning backgrounds on students' capacity for connective thinking may serve as a direction for further theoretical and practical development.

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Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Author Contribution

L.P: Conceptualization, Methodology, Formal Analysis, Writing, Original Draft.

S.K: Data Curation, Investigation, Writing, Review & Editing.

M.I: Validation, Visualization, Writing – Review & Editing.

Conflict of Interest

The authors declare that there is no conflict of interest.

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