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Educators' Knowledge Transposition: the Algebra Devolution of Thinking

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ABSTRACT

Pengetahuan dalam pembelajaran matematika merupakan makna yang diproses secara situasional melalui transposisi. Proses ini dimulai dengan repersonalisasi (matematisasi konsep oleh pendidik) dan rekontekstualisasi (pemberian konteks matematis baru), sehingga menjadi pengetahuan yang bersifat aposteriori. Peran pendidik sangatlah fundamental karena berkaitan langsung dengan tahapan tersebut. Penelitian ini menggunakan metode fenomenologi hermeneutik, dengan data yang dikumpulkan melalui dokumentasi, observasi, dan dialog reflektif-argumentatif. Pengetahuan diciptakan secara kolektif melalui konvergensi pemahaman di antara para pendidik dan peneliti. Temuan penelitian mengungkapkan tiga konsepsi pengetahuan pendidik, yaitu pengetahuan untuk praktik, pengetahuan dalam praktik, dan pengetahuan tentang praktik, serta lima tantangan utama yang terkait dengannya. Lebih lanjut, ditemukan bahwa proses penerjemahan pengetahuan pendidik menghasilkan devolusi pemikiran aljabar dari pendidik ke siswa, khususnya di tingkat sekolah dasar. Temuan ini menekankan pentingnya pengembangan kurikulum dan pelatihan guru dalam mempromosikan pembentukan pemikiran aljabar awal sebagai bagian dari reformasi pendidikan matematika.

Knowledge in mathematics learning is meaning that is processed situationally through transposition. This process begins with repersonalization (mathematization of concepts by educators) and recontextualization (providing new mathematical contexts), so that it becomes aposteriori knowledge. The role of educators is fundamental because it is directly related to these stages. This research uses the hermeneutic phenomenological method, with data collected through documentation, observation, and reflective-argumentative dialog. Knowledge is created collectively through convergence of understanding among educators and researchers. The research findings revealed three conceptions of educators' knowledge, namely knowledge for practice, knowledge in practice, and knowledge of practice, as well as five key challenges associated with them. Furthermore, it was found that the process of educator knowledge transposition results in the devolution of algebraic thinking from educators to students, particularly at the elementary school level. These findings emphasize the importance of curriculum development and teacher training in promoting the formation of early algebraic thinking as part of mathematics education reform.

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INTRODUCTION

Educators are constantly challenged to make complex instructional decisions in order to facilitate meaningful mathematics learning. However, both educators and researchers face obstacles that limit the amount to which learning activities occur as planned (Sensevy, 2012; Chevallard & Bosch, 2020). These constraints underscore the crucial need for scholarly knowledge that is adaptable, practical, and context-sensitive, especially when it comes to converting academic mathematics into forms that learners can understand and apply (Abboud, Robert, & Rogalski, 2020).

Teaching is fundamentally interpretive and situational. According to Dewey (1930), instructors mediate comprehension by assisting pupils in developing new conceptual associations that are consistent with their cognitive structures. Reflective engagement with subject matter and instructional approaches is essential for effective teaching (Cochran-Smith & Lytle, 1999a; Santos-Trigo, Camacho-Machín, & Barrera-Mora, 2024). Within this dynamic, educators must reinterpret disciplinary content in ways that are both intellectually rigorous and instructionally suitable.

This shift is represented in the notion of didactic transposition, which was first proposed by Chevallard (1989) and expanded upon by Chevallard and Bosch (2020). Didactic transposition describes how mathematical knowledge progresses from the scholarly domain (a priori) to knowledge to be taught, knowledge taught in classrooms, and knowledge learned by students. Importantly, this process includes both repersonalization and recontextualization, which are steps in which abstract mathematical concepts are changed by social, cultural, and educational contexts (Bergsten et al., 2010).

Algebra makes a good case for learning didactic transposition. Despite being a fundamental mathematical subject, algebra is usually introduced late in students' education, leading to conceptual misconceptions (Kieran, 2007). According to research, introducing algebraic thinking early on, particularly in the primary grades, can improve students' ability to generalize, detect structures, and engage in symbolic reasoning (Blanton et al., 2011; Radford, 2018; Clements, Guss, & Sarama, 2024). Early algebraic interventions have shown that young students may engage in complex mathematical reasoning when supported by purposeful instructional design (Blanton et al., 2023; Stephens et al., 2021; Cai & Knuth, 2005).

However, this early integration is more than just a curriculum change; it is a pedagogical transformation. Kieran (2007) proposes numerous modifications required to promote algebraic reasoning in young learners, including prioritizing relationships over procedures, perceiving operations as reversible, emphasizing representations, and redefining the equal sign as a signal of equality rather than computation. Recent research has expanded on this by looking into the use of learning trajectories, balancing scales, and parity reasoning to promote relational understanding in various student populations (Blanton, 2022; Stephens et al., 2022; Blanton et al., 2024).

Despite the rising quantity of literature, the educator's critical role in information transfer is understudied. While many research focus on curriculum design or student learning results, few investigate how educators understand and change algebraic knowledge during instruction (Lundberg & Kilhamn, 2018; Postelnicu, 2017). Furthermore, most research ignores how reflective processes influence educators' changing understanding of mathematics and its didactic consequences.

This study fills that gap by presenting educators as active agents in the transformation of knowledge—mediators who use their professional judgment and reflective insight to reframe mathematical concepts for their students. Drawing on Cochran-Smith and Lytle's (1999b) professional knowledge paradigm, the study analyzes how educators use knowledge-for-practice, knowledge-in-practice, and knowledge-of-practice in their pedagogical decision-making. These categories help to expose the implicit, experienced, and theoretical knowledge that educators use in their transpositional work (Cochran-Smith & Lytle, 1993).

Schön's (1983) theory of reflective practice, which comprises reflection-in-action, reflection-on-action, and reflection-for-action, also influences the research (Killion & Todnem, 1991; Butke, 2006). These modes of reflection allow educators to critically assess and predict their teaching strategies, indicating how algebraic knowledge is applied across educational contexts.

As a result, this study investigates how educators create meaning in elementary mathematics classrooms through the reflective transposition of algebraic knowledge. It adds to the discipline by

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conceptualizing algebraic thinking as devolved knowledge—knowledge that is constantly shaped through instructional negotiation. By shedding light on how educators mediate this devolution through personal and contextual lenses, the study enhances both the theoretical discourse and practical implementations of didactic transposition in mathematics education.

METHOD

This study employs qualitative methods based on both interpretative and critical paradigms, with a particular emphasis on hermeneutic phenomenology as a philosophical lens. Ricoeur's (1985) works provide as the foundation for hermeneutic phenomenology, which emphasizes the inseparability of experience and interpretation. It recognizes that understanding a phenomenon—such as an educator's knowledge transposition—requires a thorough examination of how that knowledge is constructed, experienced, and contextualized within living teaching practices. The interpretive paradigm views the educator not just as a subject of study, but also as a co-constructor of meaning, whose reflective narration shows the complex processes that underpin instructional judgments.

The ontological focus of this study is on the nature of educators' professional knowledge as it is transposed across didactic moments: from scholarly mathematical knowledge to knowledge to be taught, and from knowledge to be taught to taught knowledge, as originally theorized in Chevallard's Didactic Transposition Theory (Chevallard & Bosch, 2020). In parallel, the study uses Pino-Fan et al.'s (2015) Didactic-Mathematical Knowledge paradigm to describe the interconnected nature of educators' epistemological views, instructional decisions, and mathematical understanding as they are enacted in the classroom.

From an epistemological standpoint, the research focuses on how meaning is produced around mathematical information under the constraints of curriculum, power relations, and pedagogical norms. The study investigates how educators' reflective interpretations reveal tensions between intended knowledge and actual classroom practice, and how these tensions shape or obstruct the devolution of algebraic thinking—the transformation of complex algebraic ideas into cognitively accessible learning opportunities.

The study also incorporates critical pedagogical approaches, particularly those developed by Freire (as referenced in Suryadi, 2019), who sees education as an unfinished emancipatory process. This perspective allows the study to investigate how structural influences, including as curricular mandates and standardized tests, moderate or impede the educator's agency in transforming knowledge. Thus, the critical paradigm provides a framework for interpreting educators' acts as fundamentally political and value-laden choices, rather than simply technical implementations.

Data were gathered using a combination of document analysis, classroom observation, and semi-structured interviews with ten elementary mathematics teachers. The document analysis focuses on the national curriculum texts and instructional materials created by each participant. Classroom observations were carried out during online synchronous teaching sessions using Zoom to capture real-time didactic exchanges. Each interview lasted 60-90 minutes and was structured as a reflective-argumentative dialogue, allowing educators to clarify their teaching rationales, decision-making processes, and underlying knowledge structures.

The data analysis followed the principles of hermeneutic phenomenology, which included the following stages:

1. Repeatedly read transcripts and review videos to immerse in the data.
2. Horizontalization involves recognizing significant statements without premature categorization.
3. Group statements to identify reoccurring themes linked to transposition, reflection, and knowledge production.
4. Developed textural and structural descriptions to describe the event.
5. Explore how educators perceive and apply algebraic knowledge.

Throughout the investigation, special emphasis was placed on the creation of learning obstacles, specifically how they are caused not just by student misunderstandings but also by educators' didactic decisions and recontextualization practices. These findings are critical for understanding how the transposition process, when filtered through individual instructor views and institutional contexts, can both facilitate and impede the development of students' algebraic thinking.

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Finally, this methodology is carefully connected with the research goal: to discover how educators' reflective interaction helps to the transfer and devolution of algebraic knowledge. It adds to the study's theoretical contribution by highlighting the epistemic, didactic, and sociopolitical components of teacher knowledge in early algebra instruction.

RESULT AND DISCUSSION

1. Reflection on action

This study situates educators' reflective activity within Schön's (1983) concepts of Reflection-in-Action and Reflection-on-Action. during Reflection-in-Action refers to educators' real-time cognitive adjustments during teaching, Reflection-on-Action is a retrospective critical study of prior teaching experiences to guide and improve future practice. This study focuses mostly on Reflection on Action as a technique of investigating how mathematics educators interpret and modify their knowledge to improve algebraic thinking instruction.

The concept of knowledge transposition, which is based on Chevallard's Anthropological Theory of Didactics (1999), is central to this reflection. Knowledge transposition describes the process by which educators personalize and recontextualize scientific or a priori mathematical information into a posteriori teaching knowledge.

To analyze educators' thoughts, this study used a tripartite framework proposed by Eraut (1994) and expanded further by Cochran-Smith and Lytle (1999). The first domain, knowledge for practice, includes formal and theoretical information, as well as research findings, which serve as the foundation for teacher preparation and instructional planning. The second category, knowledge in practice, refers to the contextual and experiential knowledge that emerges in real time within the intricacies of classroom interactions. Finally, knowledge of practice is a reflective and metacognitive dimension that emerges from critical interaction with and analysis of one's teaching experiences, enabling educators to construct new understandings and inform future pedagogical decisions.

These domains comprise a cyclical reflective process in which information for practice inspires instructional design, knowledge in practice manifests as performed teaching, and knowledge of practice arises from reflective evaluation, altering the knowledge foundation for future teaching episodes. This cycle operationalizes the process of knowledge transposition in mathematics education by demonstrating how educators constantly revise and recontextualize their knowledge in response to the demands of practice.

1.1. Knowledge for practice

Knowledge for practice refers to the formal, theoretical knowledge acquired through academic study and empirical research that serves as the core resource for teaching (Fenstermacher, 1994). It symbolizes the intellectual knowledge that educators bring to instructional design and practice. This knowledge is required for effective personalization and recontextualization during instruction, according to Chevallard's (1999) approach. Several key elements of Knowledge for Practice arose during thoughtful dialogues, shedding light on its function and challenges in knowledge transposition.

First, scholarly knowledge production in university mathematics education, while important, sometimes lacks clear integration with classroom mathematical contexts. Educators had difficulty connecting advanced courses like Abstract Algebra and Real Analysis to the school curriculum, highlighting a crucial gap in knowledge transfer. The following are selected excerpts from interview transcripts with educators.

(1) *…Throughout my university years, I studied Abstract Algebra and Real Analysis, but I frequently struggled to see how these subjects were related to school-level mathematics…*

(2) *…My lecturer required us to prove theorems rather than just memorize them. However, at the time, I did not recognize the importance of such practices in teaching…*

In subjects, such as Algebraic Structure, Linear Algebra, Linear Programming, Real Analysis, Number Theory, Numerical Methods, Calculus, and College Algebra, lecturers develop situations that facilitate the formation of mathematical objects as concepts, rules or conjectures, evidence, problem, or solution to the problem. The thought process experienced by students is intended to produce scholarly

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knowledge. In the Capita Selecta subject, lecturers and students already understand the material, although with different experiences. However, the variety of learning processes experienced by students causes variations in the concept depiction. Thus, one of the objectives of this subject is to encourage school mathematics material to become scholarly knowledge.

The researchers explored to what extent scholarly knowledge formation in university affects the educator's knowledge transposition effectiveness. At least three issues were revealed in the educator's scholarly knowledge formation. The gap between prior and new knowledge was not appropriately managed; thus, new knowledge could not be mastered perfectly. Concept mastery was considered a priority; hence, deep thinking processes are considered less significant. Furthermore, most advanced mathematics subjects cannot be interpreted because of their sophisticated application. Ways of understanding, in this case, mental objects, are considered very useful by educators, while a broader mastery of mathematics should be mastered for teaching. However, not all mathematics is mastered in-depth; some are not stored in long-term memory.

Mathematics learning requires a complete prior knowledge to form new knowledge ideally. When mastery of concepts is considered essential, many take shortcuts and do not go through a long and deep-thinking process. Several major concepts often question the benefits of knowing and using this knowledge in teaching. This condition does not represent the interactive character of educator knowledge and overrides the issue of knowledge in use (Feiman-Nemser, 2012).

Scholarly knowledge needs to be built through an in-depth process. Suryadi (2019) suggested a zemi approach in constructing scholarly knowledge in mathematics lectures, i.e., an approach that emphasizes individual processes through repersonalizing certain mathematical materials and through exposition and reflective lecture activities gradually and iteratively. This approach provides both individual and group benefits. Individually, each student gains the experience of in-depth learning on how a mathematical object is constructed, either through historical tracing or through repersonalization and recontextualization.

School mathematics lectures that can successfully construct students' scholarly knowledge on certain mathematical materials can be applied to other subjects, such as curriculum analysis and study planning, mathematics education seminars, or thesis preparation (Suryadi, 2019). In the subjects of intermediate and advanced mathematics, a student's scholarly knowledge construction is considered complete when a mathematical object is successfully constructed in a formal form (WoU) through (re-)depersonalization and (re-)decontextualization processes. Collectively, students gain the experience of mutually reinforcing social interactions through a gradual and iterative process of exposition and reflection in class discussions.

To understand the construction process, each student may conduct a reference search from textbooks, historical studies, or material repersonalization. Each student can describe the selected material as scholarly knowledge in which the parts presented have sufficient mathematical arguments. The more profound and comprehensive the student's scholarly knowledge makes the greater chance to help other parties (their students) learn the material.

Second, implicit ways of thinking—mathematical reasoning habits acquired via advanced study—are a crucial but often overlooked component of knowledge for practice. Educators recognized that long-term engagement in mathematical problem-solving changed their cognitive methods in subtle but fundamental ways. The following are selected excerpts from interview transcripts with educators.

(1) *...At the time, I had no idea I was learning to think like a math researcher. However, this has had a significant impact on my teaching, particularly in explaining the logic behind mathematical concepts...*

(2) *...Learning advanced courses such as Real Analysis or Abstract Algebra involved a long, often unnoticed process that, in retrospect, significantly shaped my current way of thinking...*

In advanced subjects, the student's thought process is similar to mathematics researchers, which leads to depersonalization and decontextualization to produce scholarly knowledge. Likewise, in intermediate subjects, the student's thought process is similar to the mathematics researchers; however, the lecturer already understands the material and the thought process possibilities experienced by students. Students will experience re-depersonalization and re-decontextualization in these situations

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and eventually form scholarly knowledge. One of the goals of school subjects is to make school mathematics a scholarly knowledge through repersonalization and recontextualization (Suryadi, 2019).

Students' experience building mathematics as scholarly knowledge through three types of mathematics (advanced, secondary, school) has a strategic role because they are prepared to become educators who will help others learn mathematics. Thus, every possible thought process in the learning process must be familiarized through real experience during academic education in the undergraduate mathematics education program.

Third, didactic designs and pedagogical tactics learned throughout university studies equip educators with a variety of teaching options, expanding their instructional repertoire. Likewise, the lecturer's unique teaching style and various anticipations in interacting with students provide a deep pedagogical role. The following are selected excerpts from interview transcripts with educators.

(1) ...During my university studies, my lecturers frequently used unconventional examples. In retrospect, this has been extremely beneficial, as it has provided me with a variety of alternative approaches for explaining concepts to my students today...

(2) I learned a lot not only from the course content, but also from how my lecturers presented it, answered questions, and navigated impasses during class discussions...

Intermediate and advanced mathematics subjects have similar characteristics, i.e., the material is new for students but not for the lecturers. However, lecturers can develop new mathematical cases in advanced subjects, as mathematics researchers usually face. In this situation, the interaction between lecturers and students occurs naturally, similar to researchers when dealing with new mathematical problems. The difference is that lecturers have practical and discursive experience and knowledge gained through research experience. This condition becomes a valuable experience for students because the lecturer-student interactions can form a comprehensive thinking experience. It can trigger various mental actions and develop a thought process based on mental actions, commonly known as Ways of Thinking. Furthermore, decontextualization and depersonalization produce the simplest and most elegant mathematical descriptions that form the flow of mathematical understanding, known as Ways of Understanding. This experience is essential for students because new mathematical problems not yet familiarized by an educator can occur at a school level, especially when the mathematical case is a new problem for both educators and students (Suryadi, 2019).

Fourth, mathematical attitudes such as systematic reasoning, critical thinking, and rigorous justification emerge indirectly through university experiences, and are an important component of knowledge for practice. These affective and dispositional factors influence educators' problem-solving strategies and instructional styles The following are selected excerpts from interview transcripts with educators.

(1) *...I think that because my university education constantly required logical and structured thinking, I now approach problem-solving in a more systematic manner...*

(2) *...Building mathematical proofs during my university years, where I was expected to justify rather than merely accept claims, helped shape my habit of thinking critically...*

Attitude can be defined as a tendency towards specific patterns of behavior or responses to certain types of emotional feelings in specific domains such as those related to mathematics (MaaB & Schloglmann, 2009). Wittgenstein (in Wright, 1990) suggested that mathematics develops critical, systematic, logical, and creative thinking skills. Mathematics has excellent potential to provide various abilities and attitudes required by humans to live intelligently in their environment and adequately manage the world's issues (Bell 1978, National Research Council 1989, Souviney 1994). Mathematics can develop attitudes, such as meticulous, critical, efficient, painstaking, consistent, and holding to the universal truth.

Fifth, educators acknowledged the value of theoretical knowledge regarding learning theories, curriculum analysis, and lesson planning gained through coursework and thesis research in developing

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effective teaching strategies. This dimension promotes alignment between student needs and instructional content. The following are selected excerpts from interview transcripts with educators.

(1) *...I recall that we were taught how to evaluate the curriculum and create organized lesson plans during my undergraduate thesis. When I started teaching, that experience was really helpful because it made me feel more prepared and able to think methodically...*

(2) *...My perspective was widened by my university experience, particularly when I studied learning theories, which demonstrated that teaching effectively necessitates matching the needs of the students with the content. I didn't understand until much later how important that information is to making wise choices in the classroom...*

The student's basic knowledge of transposition develops theoretically through school mathematics, curriculum analysis, and mathematics education seminars. It is also developed through research approaches, in this case, through thesis research. These subjects allow students to generate new knowledge by building belief in the truth of a phenomenon that is the focus of the study. This process is carried out through four philosophically systematic stages, i.e., perceptual, memorial, introspective, and a priori beliefs (Audi, 2010).

Students must produce valid perceptual data such as written test results, observations, interviews, and document analysis at the perceptual stage. Data analysis skills developed through the research methodology subject is used to interpret and analyze data to produce information. After obtaining all information from the results of data analysis, further analysis is carried out to look for patterns of relationships between information to produce initial propositions that form the basis of the new knowledge that will be generated. During the memorial stage, students use a theoretical framework that has been developed during the mathematics education seminar lectures and supported by thesis writing to examine the initial findings in greater depth. At this stage, students can build stronger justifications for the newly generated knowledge and increase their confidence that the findings have a reliable value. The subjective understanding of the theoretical framework used to build justification allows students, as researchers, to proceed to the introspective stage. At this stage, students are encouraged to conduct in-depth discussions on the knowledge that is believed to be valid using the results of previous studies from other parties. Thus, students can strengthen their beliefs and describe the findings based on the results of previous studies. The description can explain new findings that strengthen, complement or contradict previous findings. Based on the third stage, students will eventually formulate general conclusions, a manifestation of the a priori belief stage (Suryadi, 2019).

Together, these features illustrate how knowledge for practice serves as the scholarly framework for educators to personalize and recontextualize, indicating the first level of information transposition important to algebraic thinking training. These findings directly address the study question of how educators translate formal mathematical knowledge into pedagogic aids through reflective practice.

1.2. Knowledge in practice

The underlying assumption of Knowledge in Practice is that education is fundamentally uncertain, and that spontaneous skills emerge situationally, built in response to the particular realities of everyday classroom and school life (Cochran-Smith & Lytle, 1999a). This viewpoint emphasizes the importance of educators' practical knowledge, arguing that educators learn most successfully when given opportunity to evaluate and reflect on the implicit information embedded in proficient practice. Such reflection occurs continually during expert educators' decision-making processes, where many considerations and arguments are weighed in real time. To improve teaching practice, educators must strengthen, clarify, and express the implicit knowledge that is inherent in their experience and professional wisdom.

The educator's ability to reflect in action, as well as adjust and rearrange ongoing teaching tactics while they are in use, is an important component of Knowledge in practice. Four major impediments to knowledge in action have been identified through reflective interactions with educators.

First, educators frequently feel mentally unprepared when teaching for the first time. Classroom realities are rarely ideal or fully anticipated, and beginner educators are unlikely to have encountered or

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studied these settings previously. The following are selected excerpts from interview transcripts with educators.

(1) *...I recall that we were taught how to evaluate the curriculum and create organized lesson plans during my undergraduate thesis. When I started teaching, that experience was really helpful because it made me feel more prepared and able to think methodically...*

(2) *...My perspective was widened by my university experience, particularly when I studied learning theories, which demonstrated that teaching effectively necessitates matching the needs of the students with the content. I didn't understand until much later how important that information is to making wise choices in the classroom...*

This condition is referred to as the epistemology of practice (Schön,1983). The notion that knowledge in practice goes hand in hand with increasing recognition within the educational community that formal research does little to address education's immediate and significant problems.

Russell (2018) explained that Schön's idea of professional knowledge in action is similar to what educational researchers refer to as practical knowledge. Practical knowledge is educators' knowledge about classroom situations, the dilemmas faced in carrying out purposeful actions, the complexities of interactive teaching, and thinking-in-action.

Fenstermacher (1994) defined practical knowledge as knowledge obtained from experience as an educator, which differs from the knowledge obtained from previous studies. Practical knowledge includes doing something at the right place and time and observing and interpreting a series of events related to one's course of action. Richardson (1994) argued that there is certain freshness and practicality of the knowledge needed by educators, which does not always have to be in line with law-like statements from formal research. Such a concept explores how educators discover knowledge in action and deliver knowledge explicitly through deliberation and reflection.

Second, many educators continue to struggle with classroom control. The following are selected excerpts from interview transcripts with educators.

(1) ...Managing the classroom proved to be much more difficult than I had anticipated. Enforcing discipline is only one aspect of it; another is constantly adjusting to the class's shifting dynamics...

(2) ...After a number of teaching experiences, I realized how important it is to create a welcoming environment in the classroom. It's not only a technical issue; students' emotional health must also be taken care of...

The classroom should allow students to actively participate in a rich and rewarding mathematical practice to enhance their mathematical capacity (Henningsen & Stein, 1997). A comprehensive understanding of mathematics and its learning is critical to the success of a mathematics class; meanwhile, classroom management is one of the critical aspects that contribute to a positive learning atmosphere. Learning is almost impossible without a well-organized and disciplined classroom. Classroom management requires a positive learning environment that will impact each student's life, including awareness of the student's emotional aspects, planning, time management, and discipline.

An educator must begin each class with a well-thought-out lesson plan. This principle refers to Vygotsky's (1962) Social Learning Theory, which examined how the social environment affects learning. Vygotsky suggested that learning occurs through students' interactions with peers, educators, and other experts. Thus, educators can create learning environments that maximize students' interaction through discussions, collaborations, and feedback.

Third, the formation of didactic contracts between educators and students is frequently inadequate or underdeveloped. The following are selected excerpts from interview transcripts with educators.

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(1) *...Sometimes I think that instead of trying to think for themselves, students are just passively waiting for answers. Maybe I haven't been able to get everyone to agree that they should be in charge of finding solutions to the issues yet...*

(2) *...At first, I believed that giving someone a task and not much direction would be enough. I quickly discovered, though, that students do not always take charge of their education; intentional methods are needed to cultivate a sense of accountability...*

Brousseau (2006) defines a didactic contract as a conscious social agreement made during the learning process to transfer responsibility from educator to student (the devolution process). This contract serves as the framework for the educational "game," requiring constant monitoring and adaptability if the desired devolution does not occur. The classroom thus becomes a dynamic place of transactions, with instructors constantly assessing and adjusting the balance of responsibility to foster active student engagement.

Fourth, overcoming pupils' learning obstacles is an extra and challenging problem. Building knowledge from core principles is tough, and correcting misunderstandings that have already taken hold is even more challenging. Reflective dialogues found that students frequently struggle with algebra in a number of critical areas, including algebraic meaning, writing algebraic expressions, algebraic thinking processes, and algebraic procedures with justification.

First, algebraic meaning. Many students may not initially understand the importance of algebraic symbols, and they frequently misapply variables, resulting in problem-solving errors. The following are the interview transcripts with the educators.

(1) *... In high school, I asked what the result of 2x plus 3y is, well some still answered 5xy. Then I continued, I gave an elementary school question: Can you add two apples with three mangoes? They said no, it is still two apples and three mangoes. Well, that is similar; we cannot add different variables. So, another example is 2x plus 3x; some still answer with 5x squared, so 2 added to 3 because there are 2 xs, so it is squared. So I came back to the elementary concept, what is the result of two mangoes plus three mangoes? Ok, it is not square of five mangoes, right? Oh yes, that is right, the students answered ...*

(2) *... About variable naming, from variables, constants. Constant begins with c, so I asked what is c? I do not know, Ma'am. The formula says ax plus b equals c, but if it does not have a variable, it is called constant, right? Maybe there is still a lack of understanding regarding math vocabulary...*

Second, writing algebraic expressions. Students frequently have trouble converting word problems into mathematical expressions. The following are interview transcripts with the educators.

(1) *... Algebra has many concepts, and it has applications. The students sometimes have difficulties interpreting word problems or problems with application into mathematical operations. For example, there are five chicken feet and two duck feet, a total of 15, right? The students are asked how many chickens and ducks are there. That is where they experience difficulties. Usually, students have difficulties formulating the mathematical operations from application problems. They can solve the problem if the mathematical operation is ready. However, there are numerous applications. That is where they have difficulties. It happens in middle school and high school. ...*

(2) *... For example, in middle school, there is problem such as 2x equals 4. We usually solve it using x equals 4/2. However, some students still have difficulties using that method. But if we say: if 2x equals 4, it is similar to 2 times what equals 4, they can answer immediately. It means that the number is equal to 2. They still have difficulties expressing it in writing. Writing x equals 4/2, x equals 2. It is a part that they still have difficulties...*

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Third, algebraic thinking process. One of the essential elements in studying algebra is the thought process. Students must solve problems through algebraic thought processes, not only by calculating or following existing procedures. The following are interview transcripts with the educators.

(1) *... I used to teach seven graders. It was the first time they studied formally, and they had difficulties in logical thinking at school; however, they did not have a problem with calculations. But logical thinking. So they had no problem with the procedure, but the thought process needed more logic ...*

(2) *... So, for example, m plus 4 equals 6. To find m, both subtracted by 4. That is usually the problem in middle school. In high school, they learn more advanced algebra, but they still have difficulties in operations like that. I often find issues like that. They should be taught the basics in grade seven ...*

Fourth, algebraic procedures with justification. Students sometimes fail to understand the reasoning behind algebraic procedures, limiting their ability to defend answers. The following are interview transcripts with the educators.

(1) *... Actually, students already understand the elementary concepts, possibly from theorems. However, what are the sources? Why did it emerge? What is the source? Such as: when it moves sections, multiplication changes to division, from division to multiplication, addition, and subtraction from positives to negatives, the negatives change to positives. Possibly everyone already knows the concept. How did it emerge? Where did it come from? Maybe they do not know that yet. The reason is critical. How will they teach students if they do not have a reason? So, building from the bottom up is challenging. Then how to teach others, because we think we know, but only the formula without understanding the reasons. ...*

(2) *... For example determining the x value in 2x + 3 = 5. Mistakes often happen. For example, 2x equals -2, so x should be one. So the section over there often do not understand why it is the way it is. We often move sections, right? Sometimes they get mixed up because they do not understand why they have to move sections. There is a reason. Both sections have to be subtracted with 3 because they know it must be moved. So, it is because they do not understand the reasons. Therefore, they are overwhelmed by high school mathematics because they cannot justify the basics. ...*

1.3. Knowledge of practice

The third concept's application reflects a critical, political, and intellectual picture. It exemplifies pedagogical creation by challenging classroom and school practices, debating what constitutes expert knowledge, probing underlying assumptions, and clearly and thoroughly incorporating family and community life into the curriculum. Educators consider teaching for transformation as a lifelong professional undertaking (Cochran-Smith & Lytle, 1993).

This concept focuses on educators and stakeholders working together to investigate their assumptions, teaching techniques, curriculum creation, and school and community policies and practices. The educators' learning process begins with critical reflection, questioning, and identifying each other's experiences, assumptions, and beliefs. This is consistent with Freire's (2015) assertion that educators require a thorough understanding of their subject matter, as well as Knoblauch's (1988) assertion that education researchers acquire meta-knowledge about knowledge generation.

At least five issues related to practice knowledge were discovered during the reflective dialogue with educators. **First,** a culture of argumentative reflection among colleagues is largely lacking. Educators indicated a scarcity of comprehensive professional discussions within their schools. These are a few chosen quotes from educator interview transcripts.

(1) *...To be honest, there aren't many in-depth conversations about teaching and learning at my school. The majority of our discussions in the staff room are quite informal...*

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(2) *...Instead of talking about teaching methods or students' learning challenges, we often concentrate more on administrative duties….*

Second, workplace conditions, whether supportive or unsupportive, significantly influence educator development. Some educators are stuck in unfavorable environments, but others see problems as opportunities to lead change. Hargreaves (1994) suggested that altering school culture is impossible without systems that promote collaboration and collegialityThese are a few chosen quotes from educator interview transcripts.

(1) *…My development has been greatly impacted by my work environment. Here, I face obstacles that I use as opportunities, but I also see coworkers who are stuck in an unhelpful environment…*

(2) *...It is true that without a framework that encourages cooperation, changing a school's culture is challenging. In order to exchange experiences, we try to establish forums for discussion…*

Assessing practice through oral investigations is based on in-depth conversations about students' work, observations and reflections of educators, curriculum materials and practices, and documents and artifacts about the classroom and school. The analysis of these various data sources is mainly conducted orally and constructed through social interactions. The 'conversation' metaphor describes the learning process as a sustainable conversation in which participants articulate an emerging new consciousness from a particular perspective. There is a strong community image in the knowledge about this practice, i.e., the image of educators and other parties constructing knowledge through sharing conjoining understandings via face-to-face interactions from time to time.

Third, educators critically evaluate university and school mathematics curricula. They underlined the significance of incorporating in-depth core mathematics subject, such as algebra, geometry, and numbers, as well as opportunities for argumentative reflection at the end of each session. These are a few chosen quotes from educator interview transcripts.

(1) *...I understand the value of exploring fundamental concepts in university-level mathematics education curricula, like algebra and geometry instruction. To improve understanding, each lecture should end with an argumentative reflection…*

(2) *…The relationship between structural and functional reasoning in the context of the school mathematics curriculum needs to be examined in greater detail. Many subjects need more in-depth investigation, and we must highlight the key elements…*

Some educators argued the need to examine the connection between the structural and functional flow of thought in the school mathematics curriculum. Material is abundant, and certain parts need to be studied in-depth. It is also necessary to emphasize the essential and less important sections. In addition, it is necessary to examine the given problems to focus not only on the answers but also on the process.

Mathematics developed by experts (scholarly knowledge) is an a priori and formal knowledge; therefore, it needs to be transposed into curriculum material (knowledge to be taught). Transposition is carried out by repersonalization and recontextualization until it becomes an a posteriori knowledge. This transpositional process is challenging because educators are responsible for the mathematics content and need to consider the connection between structural concepts (structural trajectory) and the flow of thought (functional trajectory) that students may experience.

Fourth, educators carefully evaluate their didactic designs. The process usually starts with identifying goals, consulting reference materials, planning didactic sequences, and choosing instructional methods. Approaches range from explanatory approaches at senior levels to representations

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with teaching aids and visual materials in lower classes. These are a few chosen quotes from educator interview transcripts.

(1) *...I start by establishing goals and going over references when creating an instructional plan. I use a variety of strategies, including teaching aids for lower-level classes and explanations for advanced classes...*

(2) *...I understand that teaching and learning are very different. Concepts must be understood by students in a way that inspires didactic and pedagogical thought...*

Educators understand the significant distinction between teaching and learning. This awareness draws their attention to didactic and pedagogical factors, necessitating a shift from adidactic to didactic situations in order to effectively enhance student understanding. Adidactic situations (Brousseau, 2006) are contexts in which students operate in accordance with specific norms or aims without being explicitly aware of the underlying knowledge being taught. In many cases, the teacher does not explicitly transfer knowledge but rather fosters a devolution process in which students engage in problems and activities that indirectly lead to idea construction. In contrast, in a didactic situation, pupils actively interact with the core goals of learning—the formal concepts and reasoning to be acquired.

The transpositional relationship between adidactic and didactic contexts is crucial in understanding practice. The transposition process is transitioning from an adidactic context, in which students encounter the "game" or challenge informally, to a didactic situation, in which conceptual understanding is stated and validated. This interaction allows students to connect common experiences with formal mathematical knowledge.

The overall game has three main steps. First, an adidactic situation is provided for students to practice the rules to participate in the game. A game in an adidactic situation has a primary goal that is easily recognizable from the student's point of view. Second, students are jointly asked to find ways to improve their actions to make it easier to achieve the goals. Third, students are directed to develop justification for the conclusions they make. At this last stage, concepts are taken or imported from specific fields of science, such as mathematics. The three stages are known as action, formulation, and validation situations.

Fifth, assessing learned knowledge recognizes the range of cognitive abilities. Some students learn by memorization, but others understand deeper reasoning. These are a few chosen quotes from educator interview transcripts.

(1) *...Students build their knowledge based on their cognitive level. While some people only memorize, others understand the underlying logic of mathematical ideas...*

(2) *...Students and teachers should participate in the learning scenario as part of a group activity. This method helps students comprehend scientific ideas and see how applicable they are to everyday situations...*

By referring to the term situation in the theory of didactical situation and turning it into an activity, the adidactical situation is a daily activity with the closest objective that provides an end-in-view for students. The didactic situation is an activity with the primary goal. By distinguishing the closest goal (end-in-view, adidactic) with the main goal (didactic) of an activity and asking empirical questions on how the relationship between the two goals establish continuity in the joint work of educators and students, it is intended to explain that the two activities are integral and have equal steps. Scientific concepts are never decontextualized, but are reinterpreted in their use (re-contextualization) and valued for their importance to their various consequences. The closest goal, along with the series of activities, is not just a game that directs students to learn the primary goal (conceptual knowledge), but also to teach students how to handle daily activities that are of value to them and their society, as trivial as possible, that may escape mutual attention.

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The situation is not foreign for educators and students because it is a condition where educators and students become part of joint activities. It means that the scientific concepts used are considered valuable in terms of their consequences to improve the achievement of the immediate goals of the activity and not just how well these goals direct scientific understanding. Didactics aims to teach student academic understanding and direct students to become literate in a broad sense. This is the importance of end-in-views having an authentic connection with students' lives.

In summary, knowledge of practice is built through collaborative reflection, curriculum criticism, didactic design, and a focus on how students interact with knowledge—moving from implicit, context-based understanding (adidactic) to explicit conceptual mastery. The interaction of these didactic situations is essential to the transposition process and required for meaningful mathematical education.

2. Reflection For Action

The reflective-argumentative dialogue examined educators' experiences and future goals for mathematics teaching practice. Researchers assessed the predicted outcomes collaboratively and identified four repeating themes across cases, corroborated by exact quotes from interview transcripts. These patterns indicate shared principles as well as modest differences in how educators think on their responsibilities and goals.

First, all educators highlighted the need of lifelong learning as both a professional and moral necessity. While motivation varied significantly across people, the concept remained similar. These are a few chosen quotes from educator interview transcripts.

(1) ...As educators, I think we need to continue to learn throughout our lives. We must be ready to react appropriately to the ever-changing societal trends and the circumstances of students...

(2) ...My love of learning motivates me to keep working toward self-actualization. We have a big duty to educate the country's children and to address global issues…

Educators must be prepared with various learning and student conditions, including the advancement in society (Zein, 2016). From time to time, educators are very passionate about self-actualization to educate the nation's children and face the world's challenges. Therefore, educators are required to improve their professionalism in handling existing problems. Continuous learning should be conducted. Personally, educators continue to learn mathematics content and methods to teach mathematics in various constantly changing situations and conditions and use the latest technology. Changes in learning and teaching patterns are inseparable from educators' roles (Collie et al., 2011; Thien et al., 2014; Zacharo et al., 2018).

Second, educators expect to continue doing research in the future. The nature and scope of the taught material are explained through research involvement to interpret and theorize the educator’s work. These are a few chosen quotes from educator interview transcripts.

(1) *...I intend to carry out more research in the future. We can develop the theoretical underpinnings that guide our practice and articulate and interpret the material we teach through research…*

(2) *…We can learn more about teaching and how we can help bring about positive social change through research…*

McEwen commented that it is too narrow to see the practice as merely practical. A more generative conception is teaching as praxis, i.e., the idea that teaching involves a dialectical relationship between the processes of developing theory critically and action (Britzman, 2020). The point is that educators are constantly theorizing and negotiating between the classes they teach and school life as they seek to connect daily work and the efforts towards equality and social justice movements.

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Third, educators are expected to carry out reflective dialogues and cooperate in conducting investigations with colleagues in the community. Implicitly, in the construction of the investigation is the educator's learning process throughout his professional life. Learning from teaching through inquiry throughout the professional career assumes that novice and experienced educators must engage in similar intellectual work. These are a few chosen quotes from educator interview transcripts.

(1) *...I hope that partnerships like this can continue so that we can both continue to grow and learn from each other…*

(2) *...For new teachers like myself, I want this community to be a safe place where we can ask questions and learn without worrying about being judged…*

When educators work together in a research community, they encounter a 'common search' for meaning regarding their professional life. The research community has several major cultural dimensions, with time being the most critical dimension. When educators come together as researchers, they need time to review and maintain group cohesion from time to time. When the community advancement is not rushed, and when community members are committed to complex work from time to time, the ideas generated will grow, mutual trust is built, and participants feel comfortable to question or suggest sensitive issues and dare to reveal self-secrets. Over time, the research community shapes its history and culture. They also encounter commonalities of discourse and share experiences that serve as tests and procedures that provide structure and form for the continuous experience.

Fourth, the devolution of algebraic thinking from educators to students is expected to occur since elementary school. Students learn to see algebraically because appropriate learning environments have been designed and laid out according to certain mathematical and pedagogical ideas. These are a few chosen quotes from educator interview transcripts.

(1) *...It is our hope that algebraic thinking will start to shift from teachers to students as early as elementary school. For students to view mathematics through an algebraic lens, a supportive learning environment is crucial…*
(2) *...Students' understanding of algebra can be significantly impacted by well-designed learning experiences. This is an essential step in developing a solid mathematical foundation...*

Despite agreement on its relevance, participants identified deficiencies in the current curriculum. This observation supports Kilpatrick's (2011) statement that successful curricular transformation requires modifying students' learning experiences, not just the subject list. The fact that all participants expressed this issue independently indicates that it is both a shared worry and a systemic obstacle, rather than an isolated viewpoint.

In summary, while educators' thoughts were personal, they converged on shared professional ideals such as continual learning, research participation, community collaboration, and a dedication to early algebra. Their differences are more nuanced than directional. These findings not only verify previous theoretical frameworks, but also show how they are experienced and contested in everyday professional environments.

Conceptualizing Educator Knowledge and Its Role in Developing Early Algebraic Thinking

The previous section highlighted three interrelated concepts of educational knowledge: knowledge for practice, knowledge in practice, and knowledge as practice (Cochran-Smith & Lytle, 1999). These frameworks demonstrate not only the complexities of teaching, but also the dynamic and reflexive nature of knowledge transposition, especially in context of early algebraic thinking.

In the first notion (knowledge for practice), educators are viewed as recipients of expert-generated knowledge. Mastery of mathematical subject and pedagogical philosophy is seen as necessary for effective instruction. From this viewpoint, transposition begins with the internalization of formal knowledge—developed by researchers and policy-makers—and is then adapted by educators for classroom usage. However, this top-down procedure frequently overlooks classroom reality and instructors' interpretive agency (Clandinin & Connelly, 1995; Schön, 1995; Berthoff).

The second notion (knowledge in practice) emphasizes situational and experienced learning. Educators use implicit knowledge gained through regular interactions and reflection experiences. Transposition takes on a dialogical form in this context, with educators reinterpreting and reconstructing formal knowledge through classroom interactions, pedagogical innovation, and reflective conversations with peers. Our findings demonstrated that educators saw the devolution of algebraic thinking as a gradual co-construction of meaning, rather than a straightforward transfer of knowledge from instructor to student, with algebraic ideas emerging spontaneously from well-designed arithmetic challenges. Reflective supervision, peer coaching, and lesson analysis are techniques for surfacing and refining this tacit knowledge, allowing instructors to intentionally devolve mathematical reasoning to pupils (Schön, 1987; Dewey, 1930).

The third notion (knowledge as activity) takes a step further, characterizing educators as knowledge generators. Through collaborative inquiry and study, they create theoretical and practical insights that are grounded in their own classroom environments. This bottom-up approach to transposition promotes a more transformative devolution of algebraic thinking, in which students internalize algebraic structures and representations through classroom experiences designed to encourage abstraction, generalization, and symbolization. In this view, educators purposefully create learning settings in which thinking patterns can transition from procedural arithmetic to relational algebraic reasoning, primarily through discourse, pattern detection, and symbolic manipulation (Cochran-Smith & Lytle, 2009; Wells, 1999).

Our findings show that instructors who engage in reflective-argumentative discussion begin to rethink early algebra as a mode of thinking that can and should be developed beginning in the early years, rather than a sophisticated topic reserved for later grades. As a colleague said, "It is our hope that algebraic thinking will start to shift from teachers to students as early as elementary school." This phrase exemplifies didactic devolution, which involves intentionally situating students as active participants in building algebraic meaning rather than simply imparting knowledge.

This reconfiguration has immediate ramifications for curriculum and pedagogy. Educators recognized the insufficiency of current instructional materials, which rarely provide explicit opportunity for improving algebraic reasoning. To quote Kilpatrick (2011), "if the curriculum is a list of topics, there is no change, but if the curriculum is a series of experiences that learners have, then the changes can be significant." In practice, educators in this study saw didactic transposition as an intentional rearrangement of instructional experiences aimed at gradually devolving algebraic thinking through assignments that connect arithmetic operations to algebraic structures.

Thus, the term "algebraic thinking devolution" in this study refers not only to a shift in classroom responsibility—from teacher explanation to student reasoning—but also to a pedagogical transformation based on teachers' changing conceptualizations of their roles as knowledge mediators and curriculum designers. This approach is consistent with global discussions on early algebra (e.g., Cai & Knuth, 2011; Kieran et al., 2016), and it implies that persistent reflective inquiry among educators is critical to ensuring meaningful and equitable access to algebra for all students.

CONCLUSION

This study shows that knowledge is co-constructed through shared thoughts and conversations between educators and researchers. Through reflective-argumentative discourse, we traced educators' experiences, assessed their pedagogical practices (reflection on action), and investigated how knowledge about early algebra is transferred and devolved from teachers to students. This research showed three conceptualizations of educator knowledge—knowledge for practice, in practice, and of practice—that collectively shape how educators think, perform, and reflect when assisting students' algebraic development.

A major finding was that educators were able to explain their tacit knowledge of early algebra and convert it into explicit, practical information for themselves and others. Their shared experiences laid the groundwork for improving didactic transposition and creating learning settings that encourage students' algebraic thinking beginning in elementary school. This contributes to a shift away from the traditional belief that algebra is only appropriate for secondary levels.

However, this study has one critical weaknesses. It does not directly assess students' mathematical comprehension (acquired information) as perceived or observed by educators. The findings are based

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exclusively on instructors' accounts, with no supporting student-level data. Future research should look into how students perceive and absorb algebraic thinking in order to strengthen and validate the teacher-centered perspective offered here.

This study has various practical implications. First, professional development programs should include ongoing reflective inquiry and collaborative learning to assist instructors in making latent mathematical knowledge apparent. Second, teacher training should focus on creating learning situations that teach algebraic thinking to kids as early as possible. Third, the findings can help to shape alternative curricula, classroom-based evaluations, and instructional frameworks that emphasize early algebra as a developing and accessible mathematical realm for all students.

In essence, educators are more than just curriculum conveyors; they are knowledge creators and learning designers. Their role in transposing and devolving algebraic thinking is critical to developing a generation of students who view mathematics as a relevant, linked, and empowering discipline from an early age.

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