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Students' Verbal Thinking Structure in Solving Geometry Problems: Interaction Analysis on Procedural, Disputational, and Exploratory

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ABSTRACT

Penelitian ini bertujuan untuk mendeskripsikan struktur berpikir verbal siswa dalam menyelesaikan masalah geometri melalui analisis interaksi dalam aktivitas prosedural, disputational, dan eksplorasi. Penelitian ini menggunakan pendekatan kualitatif dengan metode studi kasus, melibatkan lima siswa kelas VIII yang dipilih berdasarkan kemampuan komunikasi verbal dan akademik mereka. Data dikumpulkan melalui rekaman video diskusi kelompok, yang kemudian ditranskripsikan dan dianalisis berdasarkan pola interaksi verbal siswa. Hasil penelitian menunjukkan bahwa dalam aktivitas prosedural, siswa cenderung mengikuti langkah-langkah penyelesaian tanpa analisis mendalam dan hanya bertukar informasi secara instruksional. Pada aktivitas disputational, terjadi perbedaan pendapat dan pertahanan argumen, yang mendorong siswa untuk lebih kritis dalam mengevaluasi solusi. Sementara itu, aktivitas eksplorasi memungkinkan siswa untuk mengajukan pertanyaan, menguji hipotesis, serta merefleksikan solusi secara lebih mendalam. Pola perkembangan interaksi ini mendukung teori Zone of Proximal Development (ZPD) Vygotsky, di mana interaksi sosial berperan dalam mendorong perkembangan kognitif siswa. Temuan ini mengindikasikan bahwa interaksi verbal dalam pembelajaran geometri tidak hanya membantu siswa memahami konsep, tetapi juga meningkatkan keterampilan berpikir kritis dan pemecahan masalah. Oleh karena itu, penelitian selanjutnya disarankan untuk memperluas cakupan subjek, mengkombinasikan metode kuantitatif dan kualitatif, serta mengeksplorasi peran guru dalam memfasilitasi transisi siswa dari tahap prosedural ke eksplorasi agar pembelajaran berbasis diskusi lebih efektif.

This study aims to describe students' verbal thinking structure in solving geometry problems through interaction analysis in procedural, disputational, and exploratory activities. This research used a qualitative approach with a case study method, involving five grade VIII students who were selected based on their verbal communication and academic abilities. Data were collected through video recordings of group discussions, which were then transcribed and analyzed based on students' verbal interaction patterns. The results showed that in procedural activities, students tend to follow the solution steps without in-depth analysis and only exchange information instructionally. In disputational activities, there are differences of opinion and defense of arguments, which encourage students to be more critical in evaluating solutions. Meanwhile, exploratory activities allow students to ask questions, test hypotheses, and reflect more deeply on solutions. This pattern of interaction development supports Vygotsky's Zone of

Proximal Development (ZPD) theory, where social interaction plays a role in promoting students' cognitive development. This finding indicates that verbal interaction in geometry learning not only helps students understand concepts, but also improves critical thinking and problem solving skills.

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INTRODUCTION

Students' verbal thinking structure is a cognitive pattern reflected in the use of spoken language when they communicate ideas and strategies in solving mathematical problems. It shows how students organize information, understand concepts, and construct solutions through discussion-based verbal interactions (Syarifudin et al., 2019; Ulger, 2015). In mathematics learning—particularly in geometry—verbal thinking structures are essential for linking visual and conceptual representations, enabling students to interpret relationships among geometric elements and articulate their reasoning clearly (Bhuvaneswari et al., 2021; González & DeJarnette, 2015; Sfard & Kieran, 2001). Verbal communication also helps students identify and correct reasoning errors, making classroom discussions an effective medium for strengthening both conceptual and procedural understanding.

Students' verbal interactions in mathematics discussions can be categorized into three main types of activities, namely procedural, disputational, and exploratory (Sánchez et al., 2013). Furthermore, Sánchez et al. (2013) pointed out that procedural activities refer to instructional communication or simply carrying out the steps of solving without in-depth analysis. Meanwhile, disputational activities occur when students argue or defend their arguments in group discussions, which often involve confrontation of ideas and debate. Exploratory activity is a more complex form of interaction, where students ask questions, test hypotheses, and evaluate and modify the solutions they develop. These three forms of interaction provide an overview of how students think verbally in solving geometry problems and how they collaborate in building mathematical understanding (Syarifudin et al., 2019).

Several studies have discussed the importance of verbal interaction in supporting mathematics learning. Webb (2014) examined students' verbal interactions in small groups and found that effective oral communication can improve students' understanding of mathematical concepts. Furthermore, (Barron, 2000, 2009) revealed that group discussions involving argumentation and exchange of opinions can accelerate problem solving and improve concept retention in students. In line with these findings, Mueller et al. (2016) and Pomés et al. (2020) that verbal interaction allows students to actively build and revise their understanding of a mathematical concept, especially in problem-solving-based discussions.

Other studies have also shown that students' verbal interaction patterns have significant implications for the development of their critical and reflective thinking skills. Watson & Chick (2001) identified that students' involvement in collaborative discussions can improve their ability to evaluate and construct more structured mathematical arguments. White et al. (2012) emphasized that the success of group discussions is highly dependent on how verbal interactions are managed, where good coordination can produce a positive impact on problem solving, while poor coordination can hinder the achievement of optimal solutions. Francisco & Maher (2005) found that verbal interactions in mathematical discussions are often cumulative, where students develop understanding gradually by incorporating ideas that arise during the discussion process.

Research on verbal interaction has also developed in the context of using technology in learning. Sangin et al. (2008) examined the effects of animation and snapshots in supporting verbal interaction in computer-based group discussions. The results showed that although the use of technology can improve the quality of discussion, the resulting verbal categories did not show significant differences compared to static text-based discussions. These findings suggest that the success of verbal interaction depends

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not only on the medium used, but also on the social dynamics and communication strategies applied by students in their groups.

Although various studies have examined students' verbal interaction in mathematics learning, there are still research gaps that need to be addressed. Most of the previous studies focused more on the effectiveness of verbal interaction in improving concept understanding without specifically identifying how students' verbal thinking structures are formed in procedural, disputational and exploratory activities. In addition, there are still few studies that deeply examine how these interaction patterns can lead to the development of problem-solving strategies in geometry. Therefore, further research is needed to explore how students construct and modify their understanding through verbal interactions in various stages of mathematical thinking.

Based on this background, this study aims to describe the structure of students' verbal thinking in solving geometry problems through analyzing the interactions that occur in procedural, disputational, and exploratory activities. By understanding these interaction patterns, it is hoped that this study can provide new insights into how oral communication can be used as a tool to improve students' conceptual understanding and problem solving in mathematics, especially in geometry.

METHOD

This research uses a qualitative approach with a case study method to analyze students' verbal thinking structures in solving geometry problems. Case studies were chosen because they allow researchers to explore in depth how verbal interactions occur in small groups as well as how students' thinking patterns develop in the problem-solving process. According to Creswell (2012) a case study is a research strategy that intensively examines a phenomenon within a specific scope, such as patterns of student interaction in mathematics learning activities.

The subjects in this study consisted of five 8th grade students at SMP Negeri 1 Monta who were selected based on certain criteria. These students have good verbal communication skills, so they can convey ideas clearly and actively participate in discussions. In addition, they had varying levels of academic ability, with three students categorized as high ability and two students as medium ability, according to the recommendations of their mathematics teachers. Another criterion was that these students were used to working in groups to solve math tasks, especially in geometry. By considering these factors, the five students were placed in a discussion group to collaborate in solving geometry problems, so that their verbal interactions could be analyzed in procedural, disputational, and exploratory contexts.

The main instruments in this study are: (1) Geometry Task: Students were given one geometry problem-based essay question, namely: "*If 100 equilateral triangles and 60 squares with equal side lengths are available, how many model spatial figures can be formed, including cubes, blocks, prisms, and pyramids*?" This problem is designed to encourage students to interact in developing solution strategies as well as communicating their thinking verbally; (2) Video Recording: During the discussion, students' interactions were videotaped to obtain authentic data related to their communication patterns and thinking structures; and (3) conversation Transcripts: Data obtained from video recordings were transcribed for further analysis based on the categories of procedural, disputational, and exploratory interactions.

This research was conducted through several systematic stages to analyze students' verbal thinking structure in solving geometry problems. The first stage was subject selection, in which five students who met the criteria were designated as research participants and grouped to take part in geometry problem solving-based discussions. Next, at the discussion implementation stage, students worked in small groups to discuss the solution of the given geometry task, while their verbal interactions were recorded for further analysis. After the discussion was completed, the transcription and categorization stage was carried out by converting students' conversations into written text and classifying them based on three main activities, namely procedural, disputational, and exploratory. Procedural activities reflect routine problem solving without in-depth analysis, disputational activities illustrate differences of opinion or debate in problem solving, while exploratory activities involve asking questions, hypothesis testing, evaluation, and in-depth reflection on the resulting solutions. The last stage is data analysis, where the categorized data is analyzed using qualitative descriptive method to

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identify the pattern of students' verbal thinking structure and understand how verbal interaction contributes to geometry problem solving.

Data analysis in this study was conducted through three main stages to understand students' verbal thinking structure in solving geometry problems. The first stage was data reduction, where transcripts of recorded conversations were selected by highlighting parts relevant to procedural, disputational, and exploratory activities. This process aims to filter out the most significant information in describing students' interaction patterns during discussions. Furthermore, the categorized data were presented in the form of narrative descriptions to clarify the interaction dynamics that occurred at each stage of problem solving. The presentation of this data allows the identification of students' communication patterns as well as changes in their way of thinking in group discussions. The last stage is conclusion drawing, where the results of conversation analysis are used to interpret the development of students' verbal thinking structure in solving geometry problems. Through these stages, the research can provide insights into how verbal interactions contribute to conceptual understanding.

Methods should be described with sufficient details to allow others to replicate and build on the published results. This section explains the research design, the reasons for the design, the research procedures applied, the population and research samples or participants, research instruments, data collection techniques, and data analysis techniques. The description should be in the past tense.

RESULTS AND DISCUSSION

A group of students solve geometry problems through discussion activities. This group consists of five students, each of whom is given the symbol S1, S2, S3, S4, and S5. In the group, S1, S3, and S5 include students with high ability, while S2 and S4 have moderate ability. The problem solving process begins with understanding the problem that has been given. S1 started the discussion by explaining the meaning and purpose of the problem. After some group members understood the contents of the problem, the discussion continued to the problem solution stage.

Based on video recordings and transcripts of conversations during the problem solving process carried out by the five students in this study, further analysis was carried out by the researcher. To analyze the interactions and problem-solving strategies that emerged in the discussion, the researcher used an analytical tool developed by Sánchez et al. (2013).

3.1 Verbal Thinking Structure in Procedural Activities

In procedural problem solving activities by a group of students can be seen as the following excerpt.

1. *S1: this loh ... there are 100 triangles and 60 rectangles to arrange cubes, beams, prisms, and pyramids (understand the problem while communicating to his friends)*
2. *S2: here's the picture re .... (responding from S1 by illustrating from the provided image on the problem of some triangles and square.*
3. *S1: yes yes then again (responded by conical settling about the cube)*
4. *S3: yes right.*
5. *S1: yes, we spend triangle and square to arrange cube, beam, prism, limas.*

Based on the transcript of the conversation, the verbal thinking structure of students in carrying out procedural activities can be identified. This process begins with the stage of understanding the problem, which is shown by S1 (line 1). Furthermore, S2 (line 2) responded through making a visual representation of the problem at hand. The next stage is clarification and confirmation, performed by S1 (line 4), as a form of equalizing perceptions of the ideas that develop. This thinking process then ended with the formulation of problem solving strategies, as stated by S1 (line 5). This thinking structure can be described as follows:

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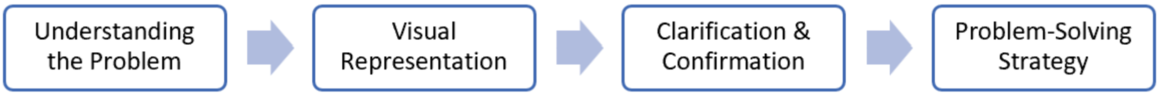

Figure 1. Students' Verbal Thinking Structure in Procedural Activities

In general, students' verbal thinking structures that occur in procedural activities are: (1) students tend to follow the steps of problem solving sequentially without in-depth analysis, (2) the interaction that occurs is more instructional in nature, where students remind each other about the information available and the procedures that must be carried out, (3) there is no debate at this stage, but only an exchange of information that supports mutual understanding, (4) students convey their ideas directly without providing deeper arguments or justifications, and (5) problem solving in this stage is more directed at execution without exploring alternative solutions.

The results showed that in procedural activities, students tend to follow the solution steps without in-depth analysis. The interaction that occurs is more instructional, where they only exchange information without questioning or evaluating the thinking used. This is in line with the research of Kumpulainen & Mutanen (1999), which revealed that in the early stages of problem solving, students focus more on task execution than critical exploration of concepts. Furthermore, Rojas-drummond et al. (2008), Littleton & Mercer (2013), and Seidl (2022) explained that procedural activities often occur in situations where students simply carry out instructions without deep reflection on the concepts they use. In this study, students simply repeated the procedures they had previously learned without considering other possible solutions. This phenomenon indicates that in the early stages of the discussion, students were still in the zone of limited understanding and had not developed more flexible thinking in solving the problem.

However, the results of this study also indicate that although procedural activities do not encourage debate or exploration, this stage is still important in building a foundation for students' understanding before they move on to more complex thinking activities. In line with the findings of Heron et al. (2023) and Saleh et al. (2007), early interactions in the study group serve as a foundation for the development of more in-depth discussions in later stages.

3.2 Verbal Thinking Structure in Disputational Activities

The following is an excerpt from the transcript of students' verbal thinking activities to solve goemetry problems.

6. *S5: How does the pyramid drawn from square (inquire about drawing composed of square)*
7. *S1: Like this (show the picture of the answer)*
8. *S2: this is it? (see the answer shown by S1)*
9. *S3: Where the hell ..*
10. *S4: Oh ... (see also)*
11. *S5: How is that? Is it not like this? (S5 shows the image he has written)*
12. *S2: like this is okay.*
13. *S1: Same, the important will tercamnar limas are composed of the third and rectangular.*
14. *S4: Like this (explains to s1 about the answer from him).*
15. *S2: Like this (asserts to s5).*
16. *S1: Like this loh (also affirmed),*
17. *S5: Oh .... (got confused)*
18. *S4: Like this, inikan rectangle and triangle, which you write it two triangles that combine into quadrilateral. (explain)*
19. *S1: Yes ... (and want to participate and hear the explanation from s4)*
20. *S5: I understand ... (follow the affirmation of s4)*
21. *S1: Actually not we arrange with all triangle or there is square can still (follow asserted in different way)*

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Based on the transcript of the conversation, the verbal thinking structure of students in carrying out disputational activities can be identified. The initial stage begins with reflective questions that provoke concept exploration by S5 (line 6), which is referred to as Initial Inquiry. The second stage is initial visualization and attempts to validate understanding through concrete representations carried out by S1 (line 7), S2 (line 8), and S3 and S4 (lines 9 and 10). The third stage is characterized by the emergence of differences in understanding, where students negotiate meaning, as done by S5 (line 11), S2 (line 12), and S1 (line 13). The fourth stage is a process of explanation and clarification from peers, as shown by S1, S2, S3, S4, and S5 (lines 14-20). The last stage is elaboration and generalization, which is done by S1 (line 21), by trying to expand understanding through exploring other alternatives in the preparation of pyramids. This thinking structure can be described as follows:

Figure 2. Students' Verbal Thinking Structure in Disputational Activities

In general, students' verbal thinking structures that appear in Disputational activities are: (1) there are differences of opinion between students regarding problem solving strategies, (2) students defend their respective arguments and submit justifications based on their understanding, (3) interactions tend to be competitive, where students respond to each other and try to convince other group members of the solutions they propose, (4) discussions that occur can lead to better understanding when students begin to connect relevant concepts, (5) agreement in the group is only reached after all members are satisfied with the arguments submitted and agree on a common solution.

At the disputational stage, it was found that students began to show differences of opinion and defend their arguments in the discussion. They not only conveyed ideas directly, but also began to provide justifications and challenge the thinking of their groupmates. This is in line with the research of Barron (2000) and (2009) can improve deeper understanding of concepts. Weber et al. (2008), Lloyd & Murphy, (2023), and Mata-Pereira & da Ponte, (2017), also explained that when students challenge each other's arguments in mathematical discussions, they are encouraged to understand the principles behind the answers they give. In this study, students who initially only received information passively began to question and test their own ideas. This suggests that disputational interactions play a role in shaping students' critical thinking skills and analytical abilities.

However, one of the challenges that arise in the disputational stage is the tendency of students to defend their arguments competitively, which can hinder collaborative discussions. Francisco & Maher, (2005), Lowrie, (2017), and Reinking & Yaden Jr, (2021), suggest that in some cases, disputational interactions can lead to stagnation of understanding if not facilitated properly, as students focus more on defending their opinions rather than seeking solutions together. This condition shows that without proper direction, disagreements in discussions can actually become a barrier in building deeper understanding. Therefore, the role of the teacher becomes very important in directing the discussion to remain productive, by ensuring that the debates that occur are constructive and encourage students to explore various perspectives in solving problems collaboratively.

3.3 Verbal Thinking Structures in Exploration Activities

This activity is characterized by critical and exploratory activities by group members which include planning, hypothesis testing, evaluation, and experimenting. The following are the verbal thinking activities of students in the exploration category in solving geometry problems.

22. *S2: If 20 square to construct the beam how?*
23. *S1: Four (pedestal and top), four (front and back), and two (left and right). (counting the side of the block, if the square is used to arrange the side of the beam)*
24. *S4: Yeah right (while also counting the sides of the beam)*
25. *S5: Yeah wes (come drawing)*
26. *S3: One, two, ..., ten (counting the square that can be used to arrange the beam image)*

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27. *S1: here this ... (recalculate how many squares are needed to arrange the blocks)*
28. *S2: Oh so (while resuming the picture)*
29. *S5: Like this picture (while showing the image)*
30. *S1: Yes like that, but given dashed (drawing dashed line)*
31. *S2: Okay ... (while drawing and sometimes while looking at the answer from S1, naming how to give different beam name to S1)*
32. *S3: This is yes, (while writing the numbers 1, 2, ..., 10 on the beam indicating the number of squares)*
33. *S4: Okay ... (drawing blocks by releasing 1, 2, ..., 10 on the beam indicating the number of squares)*
34. *S5: Like this yes, (he wrote the letters ABIJ, IJDM, ... which is in the image Beams)*

Based on the transcript of the conversation, we can identify the students' verbal thinking structure in carrying out the exploration activity. The first stage is idea exploration, which is characterized by a question from S2 (line 22): "How many sides are needed to form a block from a square?" The second stage is the systematic calculation of the sides of the block by S1 (line 23), which is then followed by S4 and S5 (lines 24 and 25) by performing calculations and preparing drawings. At this stage there was also collective validation by all group members. The third stage is the revisualization of ideas through various individual approaches by S1, S2, S3, and S5 (lines 26-29). The fourth stage is naming and symbolic representation carried out by S2, S3, S4, and S5 (lines 30-34). The last stage is collective validation and the preparation of the final representation, where all subjects (S1-S5) produce different but still valid drawings of the blocks. This thinking structure can be described as follows:

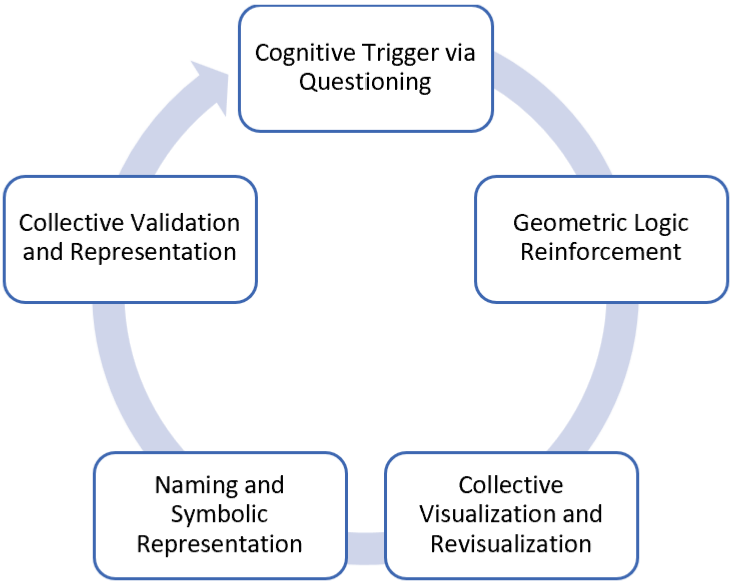

Figure 3. Students' Verbal Thinking Structure in Exploration Activity

In general, students' verbal thinking structures that appear in exploration activities are: (1) students begin to ask questions to dig deeper into the concepts they are discussing, (2) there are planning activities and hypothesis testing of the proposed solutions, (3) students show reflective thinking by evaluating their answers and comparing them with other alternatives, (4) discussions are more collaborative, where students not only defend their opinions but are also open to ideas from other group members, and (5) problem solving in this stage often results in a broader and deeper understanding because students actively connect the mathematical concepts they have learned before.

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The results showed that at the exploration stage, students began to ask questions, test hypotheses, and evaluate the solutions they discussed. The interaction that occurs is more reflective and collaborative, where students are open to new ideas and try to connect the concepts they have learned before. This finding supports Andrews et al. (2020), Hennessy et al. (2023); and Gillies (2016) research, which states that exploration activities are the most effective form of communication in learning because it allows students to construct their understanding through in-depth dialog. In addition, Sfard & Kieran (2001) also found that in exploratory discussions, students are more likely to develop more flexible and creative problem solving strategies.

Furthermore, Sangin et al. (2008), Hwang et al. (2018), and Wu et al. (2021s) revealed that explorative interaction can significantly improve students' conceptual understanding. In this study, students who previously only followed procedures began to question the concepts they used and tried to find alternative solutions. This shows that explorative discussions not only help students understand geometry concepts, but also train them to think more critically and independently in solving problems.

However, it should be noted that not all students can easily move to the exploration stage in mathematical discussions. Some students still need guidance to build courage in asking questions, developing hypotheses, and exploring various possible solutions. This difficulty can be caused by lack of confidence, limited understanding of concepts, or lack of experience in discussions that encourage exploration. This finding is in line with Webb (2014), Gillies (2019), and Ding et al. (2018) research, which emphasizes that a supportive learning environment and positive social interactions play an important role in encouraging students to be actively involved in exploratory discussions. Therefore, teachers need to create an inclusive learning atmosphere, provide directed guidance, and encourage supportive interactions so that all students have the opportunity to participate in the exploration process optimally.

The following is a chart that illustrates the Verbal Thinking Structure of Students in Solving Geometry Problems based on Procedural, Disputational, and Exploratory activities:

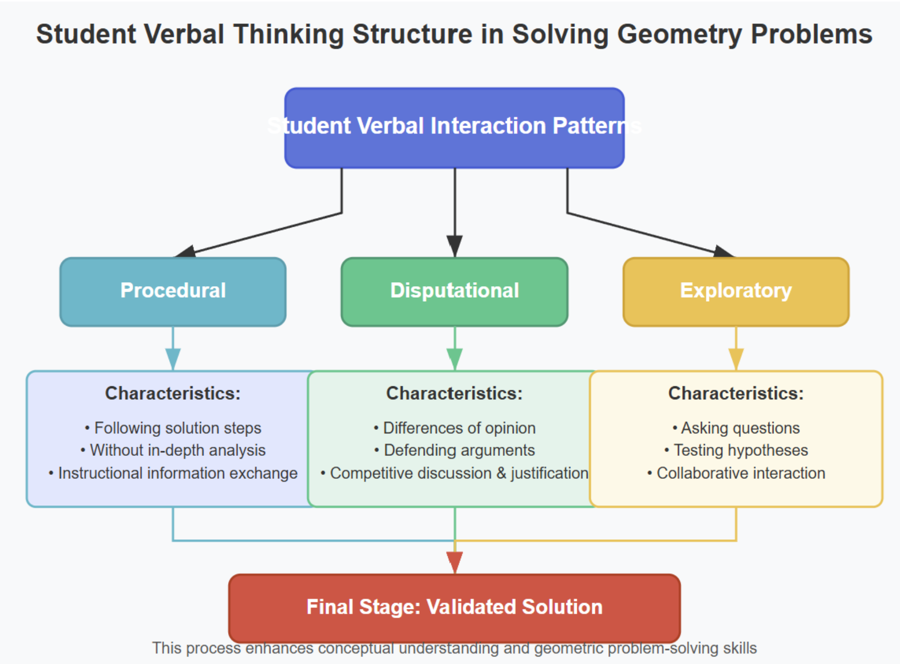

Figure 4. Student Verbal Thinking Structure in Solving Geometry Ploblems

3.4 Development Pattern of Students' Verbal Thinking Structure

The results of this study show that students' interaction patterns develop gradually from procedural to disputative to exploratory activities. At the beginning of the discussion, students tend to follow the procedural solution steps. As their understanding of the given problem increased, students

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began to question and challenge the ideas that emerged in the disputational discussion. In the final stage, they begin to explore various possible solutions collaboratively.

This pattern of development is in line with Vygotsky (1978) theory of Zone of Proximal Development (ZPD), which states that students experience cognitive development through social interactions that challenge them to think more complexly. In the context of this study, the verbal interactions that occurred in the group provided opportunities for students to move from limited understanding to broader understanding through discussion and joint exploration. In addition, this pattern also corroborates the research results of Kumpulainen (2009), Nagabandi et al. (2018), and (Bassett et al., 2015), which show that interactions in learning groups develop dynamically based on changes in time and the complexity of the tasks given. In other words, students need time and experience to be able to move from one stage of thinking to the next.

3.5 Implications for Mathematics Learning

Based on the findings of this study, there are some important implications for mathematics learning:

- Discussion-based learning needs to be designed in such a way that students can move from the procedural stage to the exploration stage gradually.
- Teachers act as facilitators who help students develop critical thinking skills by providing challenges that are appropriate to their abilities.
- Verbal interaction in the group can be used as a formative assessment tool to measure students' understanding and identify areas that still need improvement.

The application of scaffolding strategies Vygotsky (1978) can help students who still have difficulty in asking questions or exploring new ideas.

CONCLUSION

The results of this study show that students' verbal thinking structure in solving geometry problems develops through three main stages, namely procedural, disputational, and exploratory. At the procedural stage, students tend to follow the solution steps without in-depth analysis, while at the disputational stage, they begin to defend arguments, challenge groupmates' thinking, and seek justification for the proposed solutions. The exploratory stage becomes the most complex form of interaction, where students actively ask questions, test hypotheses, and reflect deeply on the solutions they discuss. This development shows that verbal interactions in study groups not only help students understand geometry concepts more deeply, but also improve their critical thinking and problem-solving skills. This pattern of thinking development is in line with Vygotsky's Zone of Proximal Development (ZPD) theory, which states that social interaction plays an important role in promoting students' cognitive development. In addition, the results of this study support the findings of Mercer, (1996) who assert that verbal interaction in learning groups develops gradually and dynamically. Therefore, learning strategies that encourage exploratory discussions need to be applied more systematically in mathematics learning, where teachers act as facilitators who provide scaffolding to help students move from the procedural stage to the exploration stage. Thus, discussion-based mathematics learning not only serves as a means of understanding concepts, but also as a vehicle for forming reflective, analytical, and innovative mindsets in solving problems. Future research should investigate students' verbal thinking across levels, the role of technology, and teacher strategies in fostering conceptual understanding, critical thinking, and problem solving.

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The authors acknowledge the use of **ChatGPT (GPT-5, OpenAI, https://chat.openai.com/)** to assist in **refining the English structure, improving clarity, and enhancing the coherence of the introduction section**. The prompts used include *"Revise the introduction for conciseness and academic tone"* and *"Identify redundancy in paragraphs and suggest improvements."*

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The output from these prompts was used to **edit and refine the wording of the manuscript without generating or altering the research data or results**. While the authors acknowledge the usage of AI, they maintain that they are the sole authors of this article and take full responsibility for the content therein, as outlined in **COPE recommendations**.

Data availability

The datasets generated and analyzed during the current study are available from the corresponding author on reasonable request.

Ethics, Consent to Participate, and Consent to Publish: This study received ethical approval from **STKIP Taman Siswa Bima, Indonesia**. Participation in this research was voluntary, and informed consent was obtained from all participants. The authors confirm that the study was conducted in accordance with institutional and ethical research guidelines.

Author Contribution

N: Conceptualization, Methodology, Formal Analysis, Writing, Original Draft.

S: Data Curation, Investigation, Writing, Review & Editing, Validation, Visualization, Writing – Review & Editing.

Conflict of Interest

The authors declare that there is no conflict of interest.

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