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Mathematical induction proofing: Procedural fluency reviewed from the creative thinking level of mathematics students

Pradipta Annurwanda[1,a,*], **Rizki Nurhana Friantini**[1,b]

[1]Department of Mathematics Education, STKIP Pamane Talino, Jln. Afandi Rani, Ngabang, Landak, Kalimantan Barat, 79357, Indonesia.

E-mail: [a] pradipta@stkippamanetalino.ac.id, [b] r.friantini@stkippamanetalino.ac.id.

* Corresponding Author

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ABSTRACT

Creativity in performing mathematics proof was assumed to be directed by the procedural fluency. This article examines the procedural fluency in proof based on the students' creative thinking level of mathematics. Subjects were selected pusposively to join the test and interview as the main instruments. Of the 36 students who took the test, 5 students were selected appropriate at each level of creative thinking skills to be followed with interviews.. The data were analyzed following data condensation, data presentation, and conclusion withdrawal as suggested by Miles, Huberman, and Saldana (2014). The results showed that very creative, creative, and quite creative students could demonstrate procedural fluency because they could use mathematical induction proof procedures correctly and modify the procedure in the correct rules although less creative students lacked completeness in performing mathematical induction proof procedures. Students of the lower creativity groups had less procedural fluency because they were unlikely to understand the use of mathematical induction proof procedures and found difficulties to apply mathematical induction proof procedures properly or even no attempt was made to modify procedures to solve problems.

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INTRODUCTION

Mathematics is a systematic and structured and interrelated science between one concept and another. To solve problems in mathematics, of course, it is a requirement to be able to be systematic and structured. Problems should be challenging enough for students to solve in order for students to increase their knowledge and understanding (Winata et al., 2020). According to Polya, problems are divided into 2 i.e. problems to find and problems to prove. The problem to proof can be solved by using proof to decide whether a particular statement is true or false (Friantini, 2014). Proof according to Morris is defined as a deductive argument that uses valid inference rules, axioms, definitions, and previously proven conclusions. In general, proof is necessary to validate a particular statement or argument through various different forms and means that are most importantly valid or convincing (Imamoglu & Yontar Togrol, 2010). Proof is an important activity in the study of mathematics as explained by Varghese that the term proof is an important component in mathematics. Further explanation about this term is this term stands as a very important tool and becomes the root of mathematics and shows the solution of all unknown problems (Sirmaci, 2012). There are various kinds of proof techniques such as direct proof, indirect proof like counter-position, or there is also proof through mathematical induction.

According to (Ashkenazi & Itzkovitch, 2014), mathematical induction is a technique developed in the complexion of a statement relating to discrete mathematical objects, such as number theory, graph theory, and combinatorics. Every student needs to recognize the true values of recursion inherent in the deductive proving process of inductive measures in order for students to understand mathematical induction as a valid technique (Dogan, 2016). In performing mathematical proofs using mathematical induction, procedures and proof steps must be fluent and systematic for the fluent procedure in mathematical proof is very important. Procedural fluency is one of the five mathematical skills that are interconnected and related and cannot be separated from each other. The five mathematical skills according to Kilpatrick and Swafford (2001) are 1) understanding concepts, 2) procedural fluency, 3) strategic competence, 4) adaptive reasoning, and 5) productive disposition (Bautista, 2013).

According to Watson and Sullivan (2008), smoothness involves the implementation of procedures flexibly, accurately, efficiently and precisely and has factual knowledge and concepts that come to mind easily. This definition combines the ability to readily perform mathematical mechanisms (procedures) and the mathematical understanding learned with regard to concepts and provides a more focused space on aspects of fluency (Cartwright, 2018). (NCTM, 2014) mentions that analyzing procedural fluency can reveal insights and errors and help plan the next steps in teaching. Thus, by analyzing student procedures can help to better understand students' abilities. The three indicators to measure procedural fluency are: 1) Choosing and utilizing the procedure; 2) apply the procedure appropriately; 3) modify the procedure.

Table 1. Creative Thinking Level

Level	Characteristic
Level 4 (Highly Creative)	Students are able to solve a problem with more than one solution and can represent another solution. One solution meets originality. It can also cause new problems. One problem has different solutions and different methods to solve them. Some of the problems built meet novelty, fluency and flexibility. He tends to say that building a problem is more difficult than solving a problem, because he has to have a certain way of making the solution.
Level 3 (Creative)	Students are able to solve a problem with more than one solution, but cannot represent another way of addressing it. One solution meets originality. Alternatively, he may represent another way to solve a problem, but he cannot create a new solution. On the other hand, he could also cause new problems. One problem has a different solution, but there are no different methods to solve it. He tends to say that building a problem is more difficult than solving a problem, because he has to have a certain way of making it a solution.
Level 2 (Quite Creative)	Students are able to solve problems with one original solution but do not meet fluency or not flexibility. Alternatively, he or she may represent another way to solve the problem; However, this is not new or not eloquent. Other traits, he (or she) can also cause new problems without fluency and flexibility. He tends to say that building a problem is more difficult than solving a problem, because he is unfamiliar with the task and difficult to estimate numbers, formulas or solutions.
Level 1 (Almost Creative)	Students are able to solve a problem with more than one solution but cannot represent it in any other way to solve it. The solution does not meet originality (novelty). It can also cause some problems. He tends to say that building a problem is quite difficult than solving a problem, because it depends on the complexity of the problem. He tends to understand that different methods or strategies for solving problems are another form of formula, even though both are the same.
Level 0 (Not Creative)	Students cannot solve problems with more than one solution and cannot represent in any other way to solve them. Solutions do not meet originality, fluency, and flexibility. He also couldn't cause any novelty and flexibility issues. All the problems built up don't meet novelty, fluency and flexibility. He tends to say that building a problem is easier than solving a problem, because he knows the solution.

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One of the courses that study and apply proof is the Number Theory which was studied in the first semester by Mathematics Education of STKIP Pamane Talino. After conducting observation on students who studied Number Theory, the result showed that students were still problematic in term of proving. When viewed from the procedural fluency of one of the students when he was doing proving, it was obtained that although it was a known the statement to be proven, the student had not been able to use the correct proof procedure and had not been able to produce proof correctly. Furthermore, the students were dealt with difficulty when manipulating or modifying procedures because they were used to solve problems according to examples. This state of comprehension of the students made it difficult to think creatively to complete the proof. Therefore, creative thinking is very necessary when doing the proof process.

The ability to think creatively in mathematics is necessary, and it is expected to present ideas that the students clearly had in mind. Therefore, the students realize that there are different opinions in the topic that they have learned, and with the different opinions comes cognitive conflict which is the encouragement for the students to make a change (Lince, 2016). Meissner (Švecová et al., 2014) puts emphasis on the idea that creative thinking can be developed through challenging questions. According to (Siswono, 2011) creative thinking can be divided from level 0 to level 4 based on fluency, flexibility, and novelty in solving mathematical problems. The creative thinking level is shown in Table 1.

Each level of creative thinking certainly has a different procedural fluency. Therefore, this study would analyze procedural fluency in proof based on the level of students' mathematical creative thinking. The result of this research showed the visible smoothness of the students in doing proving and their ability to solve proof problems. Thus, this finding can be used as a basis in developing further learning.

METHOD

The type of the conducted research was qualitative research. Qualitative research is a research procedure that produces descriptive data in the form of written or spoken words from people and observable behavior (Moleong, 2018). In this study, procedural fluency in proof was being analyzed based on the level of students' mathematical creative thinking. In this paper, the proof test was given to 36 students of the 1st semester Mathematics Education Study Program of STKIP Pamane Talino. The test results were taken by analyzing 5 students, while interviews were conducted to see more deeply about students' procedural fluency in proving mathematical induction. The results of the students were also adjusted to the level of students' creative thinking which is divided into 5 levels, namely levels 0-4. The determination of the subject was done by using purposive sampling based on several criteria, including: having received the material specified in the number theory course, students who had creative thinking skills level 0 to level 4 based on the results of the creative thinking test, and producing evidence of complete answers that can be analyzed in order to obtain data in accordance with the research focus.

The data collection of this research were the test and interview method. The instrument in this study was the researcher himself with the assisted instrument in the form of an essay test as many as 2 questions of proof of mathematical induction with 3 steps of completion in sequence and interview guidelines. Test instrument used in this study was using an essay-shaped test consists of 2 questions, namely M1 and M2. The problem was designed to find out the procedural fluency of students in proving mathematical induction. The problems that become instruments of this research were solved using the induction step. Test instrument was first validated by experts in order to produce a good and valid instrument. For the validity of the data, triangulation method was used to compare the results of proof and interview tests so that valid and saturated data were obtained.

Data analysis using Milles Huberman technique (Sugiyono, 2010) with the following activities: 1) Data reduction, data which are obtained in the field are in large quantities, so it was necessary to reduce it. In this study, data reduction was done by categorizing the data according to procedural fluency indicators. 2) Presentation of data. The presentation of data aims to make it easier to understand what is produced and plan for future work. In this research, the presentation of data was done by using narrative text. 3) Conclusion withdrawal. In the activity of withdrawal, conclusions must certainly be supported by valid and consistent evidence. In this study, the conclusions resulted from triangulation methods by

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comparing between test results and interviews. Thus, procedural fluency was obtained and was able to do proving for each level of creative thinking.

RESULT AND DISCUSSION

Problems M1 and M2 was solved using a mathematical induction proof procedure. Procedural fluency can be seen in accordance with the following indicators: 1) Selecting and utilizing procedures, 2) implementing procedures appropriately, and 3) modifying procedures. As for the mathematical induction, proof process was used to analyze the results of student work in the following step: 1) Assumed that S(k) was correct for a certain integer selected arbitrarily k ≥ 1, and 2) assumed that S(k) correctly implied that S(k +1) was true (Utomo & Huda, 2020). The research results can be seen in the following explanation:

Creative Thinking Level 4 – Highly Creative (SL4)

1. Selecting and utilizing procedures

The subject level 4 (SL4) was correct in choosing the procedure, which used the mathematical induction proof procedure in solving the problem. When confirmed, the subject used proof induction of mathematics because the given problem could be solved by mathematical induction. The subject's answer in solving the M1 problem can be seen in Figure 1.

1. $P_n = \{n \in Z^+ | 1+2+3+\cdots+n = \frac{n(n+1)}{2}\}$ maka,

a. Adb P_n dengan $n = 1$ benar.

$$n = \frac{n(n+1)}{2}$$
$$1 = \frac{1(1+1)}{2}$$
$$1 = \frac{2}{2}$$
$$1 = 1$$

Sehingga, P_n dengan $n = 1$ benar.

b. Asumsikan P_n dengan $n = k$ benar. Maka, $1+2+3+\cdots+k = \frac{k(k+1)}{2}$ benar.

c. Adb P_n dengan $n = k+1$ benar

Maka $1+2+3+\cdots+(k+1) = \frac{(k+1)((k+1)+1)}{2}$

$1+2+3+\cdots+k = \frac{k(k+1)}{2}$ ……… *Pernyataan* b

$1+2+3+\cdots+k+(k+1) = \frac{k(k+1)}{2} + (k+1)$

$1+2+3+\cdots+(k+1) = \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$

$1+2+3+\cdots+(k+1) = \frac{k(k+1)+2(k+1)}{2}$

$1+2+3+\cdots+(k+1) = \frac{k^2+k+2k+2}{2}$

$1+2+3+\cdots+(k+1) = \frac{k^2+3k+2}{2}$

$1+2+3+\cdots+(k+1) = \frac{(k+1)(k+2)}{2}$

$1+2+3+\cdots+(k+1) = \frac{(k+1)((k+1)+1)}{2}$

Sehingga P_n dengan $n = k+1$ benar.

Jadi terbukti bahwa $1+2+3+\cdots+n = \frac{n(n+1)}{2}$, Untuk semua $n \geq 1$

Figure 1. Subject SL4's answer to M1 problem

According to Figure 1, it can be seen that the subject was working on the M1 problem by using mathematical induction proof, so the procedure the subject chose to solve the problem was correct.

2. Apply the procedure appropriately

The subject of SL4 could apply the proof procedure in a mathematical induction appropriately. Initially the subject defined the problem to be proven as Pn i.e. Pn = {n ∈ Z+|1+2+3+...+n = $\frac{n(n+1)}{2}$}. According to assumption that S(k) was correct for a particular integer selected arbitrarily k ≥ 1, the subject performed this step by describing that Pn with n = 1 was correct. The calculations the subject performed for the first step were correct so that it was proven for Pn with n = 1 correct. Second step, which was to assume that S(k) was true and implied that S(k+1) was true. At point b in Figure 1, the subject assumed that Pn with n = k was true so it was proven that Pn with n = k + 1 was true. The proof step of Pn with n = k+1 was correctly done by the subject on the M1 problem indicates that the subject was fluent in performing the proof procedure, because the subject was based on point b to prove point c. The subject SL4 has applied all of step in the proof procedure with mathematical induction correctly to solve the M1 problem. Therefore, the subject of SL4 had applied the second indicator of procedural fluency well i.e. could apply the procedure appropriately.

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3. Modifying procedures

The subject of SL4 could perform calculations on the proof process by modifying the form of the statement in such a way that the statement to be proven was obtained. The M2 problem was a matter of proof by mathematical induction for the problem of diversity as follows: Prove that a(a+1) was divisible by 2. The subject's answer SL4 on the M2 problem can be seen in Figure 2.

Buktikan bahwa $a(a+1)$ habis dibagi oleh 2.
$P_a = \{a \in Z | a(a+1)$ habis dibagi oleh $2\}$ maka
1. Adb P_a dengan $a = 1$ benar

   $2|a(a+1)$ maka dapat ditulis $a(a+1) = 2.b$ dengan b merupakan bilangan bulat

$$a(a+1) = 2.b$$
$$1(1+1) = 2.b$$
$$1(2) = 2.b$$
$$2 = 2.b \ ada \ b = 1 \in Z$$

   Sehingga P_a dengan a = 1 benar
2. Asumsikan P_a dengan $a = k$ benar maka, $k(k+1)$ habis di bagi 2 atau $2|k(k+1)$ benar.
3. Adb P_a dengan $a = k+1$ maka $2|(k+1)((k+1)+1)$

   $2|k(k+1)$ maka dapat ditulis $k(k+1) = 2.b$ dengan b merupakan bilangan bulat

$$k(k+1) = 2.b$$
$$k(k+1) + (2k+2) = 2.b + (2k+2)$$
$$k^2 + k + 2k + 2 = 2b + 2k + 2$$
$$k^2 + 3k + 2 = 2(b+k+1)$$
$$(k+1)(k+2) = 2(b+k+1)$$
$$(k+1)\big((k+1)+1\big) = 2.c \text{ misal } c = b+k+1 \text{ anggota bilangan bulat}$$
$$2|(k+1)((k+1)+1)$$

   Sehingga P_a dengan a = k + 1 benar

Jadi terbukti bahwa $a(a+1)$ habis dibagi oleh 2, untuk setiap a anggota bilangan bulat.

Figure 2. Answer of subject SL4 to M2 problem

From Figure 2, it was obtained that the subject of SL4 defined the statement to be proven as Pa. Then after that entered at the initial step of mathematical induction or the basic step and it was proven that 2 = 2.b with b = 1 ∈ Z. Further entering the induction assumption step, here it was known for the modifications made by the subject SL4 in order to form a statement in such a way that it become a proven statement. From the statement in step 2, namely Pa with a = k true then, k(k+1) was divisible 2 or 2|k(k+1) was true, the subject SL4 described the form of 2|k(k+1) to k(k+1) = 2*b*. Furthermore the subject of SL4 added both segments with (2k+2) so that $k^2+3k+2 = 2(b+k+1)$ of the statement was generated (k+1)(k+2) = 2(b+k+1) to (k+1)((k+1)+1) = 2c or if it was written in a section form of 2|(k +1)((k+1)+1) which was the statement to be proven.

The subject of SL4 could modify the procedure to form the initial statement into a statement to be proven. When confirmed, the subject replied that the subject defined in advance the statement to be proven and broke it down in order to be a form that corresponds to the statement in step 2 so that the settlement step as written. It can be concluded that the subject could modify the procedure with correct modifications and not deviate from any theory or nature.

Creative Thinking Level 3 – Creative (SL3)

1. Selecting and utilizing procedures

The subject SL3 solved the problem that by using a procedure by mathematical induction to solve it, then it was correct. When confirmed, the subject gave the reason that the mathematical induction material that had just been studied, in addition to the example of mathematical induction, was similar to the given problem so that the subject also worked on the problem by way of mathematical induction. The answer of the subject SL3 in solving the M1 problem can be seen in Figure 3.

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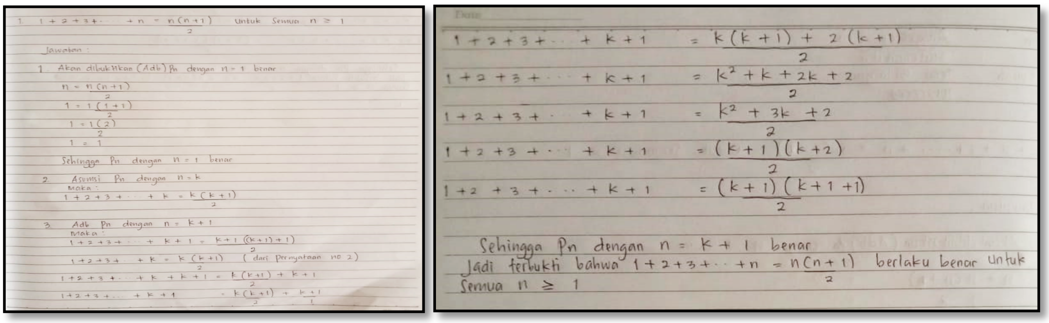

Figure 3. Subject SL3's answer to M1 problem

According to Figure 3, it can be seen that the subject SL3 worked on the M1 problem by using mathematical induction proof, so that the procedure that the subject chose to solve the problem was correct.

2. Apply the procedure appropriately

The subject of SL3 started the procedure with the basic step, which was to assume that Pn with n = 1 was correct. This step is correct both in terms of assumptions and calculations so it was proven that Pn with n = 1 was correct. For the next step that was the assumption of induction, the subject SL3 assumed Pn with n = k then 1+2+3+...+k = (k(k+1))/2 the assumption was done correct and only the subject did not write that the assumption on point b was considered true, whereas this assumption needed to be declared correct in order to be the basis for proving the next point. When confirmed, the subject said that the subject forgot to write down the truth of the statement, but the subject understood that the truth value of the statement should need to be written down. The correct subject answer should be as follows: assumed Pn with n = k then 1+2+3+...+k = (k(k+1))/2 was considered correct. Then in the next step, the subject SL3 proved Pn with n = k+1 correct.

The subject of SL3 performed proof for the third step departing from the statement in the second step. For mathematical calculations, it was carried out on the proof of the third step and had been done correctly so that the results obtained that proved for Pn with n = k+1 were correct. Not forgetting the subject of SL3 also wrote the conclusion of the proof process with mathematical induction that he had done correctly. Therefore, the subject of SL3 was quite complete to write the proof procedure with mathematical induction, although there was a slight deficiency in writing the truth of the assumption of the second step statement. The subject of SL3 was considered to be able to apply the proof procedure by mathematical induction correctly.

3. Modifying procedures

The subject of SL3 could prove by modifying the form of the statement in such a way that the statement was obtained to be proven. Actually, the proof step carried out by the subject was not in accordance with the correct mathematical induction step but the modification of the procedure carried out by the subject SL3 was interesting enough so that the results could be proven or obtained a statement to be proven. The subject's answer to the M2 problem can be seen in Figure 4.

According to Figure 4, the subject SL3 still used the mathematical induction step correctly, by changing the value a in the statement to the number 1. From the results of the initial step, it was obtained that 2 = 2b there was b = 1 ∈ Z so it was proven to be true 2|a(a+1). Entering the second step, subject SL3 assumed the statement by replacing the correct a = k was true so 2|k(k+1) was true. When entering the third step, the induction assumption step, usually started from the statement in the second step, the statement was assumed to be true for a = k, but the subject of SL3 did not do so. The subject of SL3 went directly into the statement to be proven to be k+1((k+1)+1) and described it into the form k+1(k+2) on the right side. After that the subject SL3 processed the calculation in the right segment that was k+1(k+2) became k(k+1)+2(k+1). Next step, the subject used the assumption of the statement in the second step, namely k(k+1)+2(k+1) to 2b+2(k+1). Next the form was converted into a distributive form of multiplication against the sum to 2(b+(k+1)) so that it was proven to be 2|k+1((k+1)+1). The subject could make modifications to the procedure because the procedure used

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by the subject was not the proper mathematical induction procedure according to the steps described in the study.

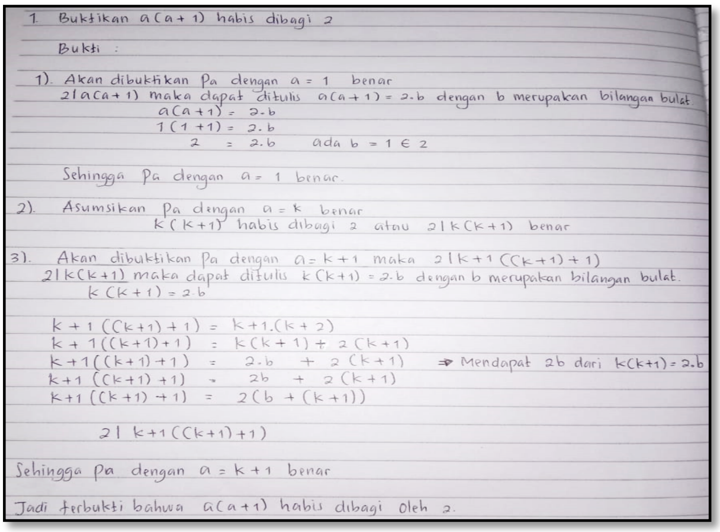

Figure 4. Subject SL3's answer to M2 problem

Creative Thinking Level 2 – Quite Creative (SL2)

1. **Selecting and utilizing procedures**

The subject SL2 solved the problem and it was correct because the subject used a proofing procedure with mathematical induction. When confirmed, the subject SL2 conveyed that the subject only followed what was exemplified at the time of material delivery, so the subject used this mathematical induction. The subject's answer to solve the M1 problem can be seen in Figure 5.

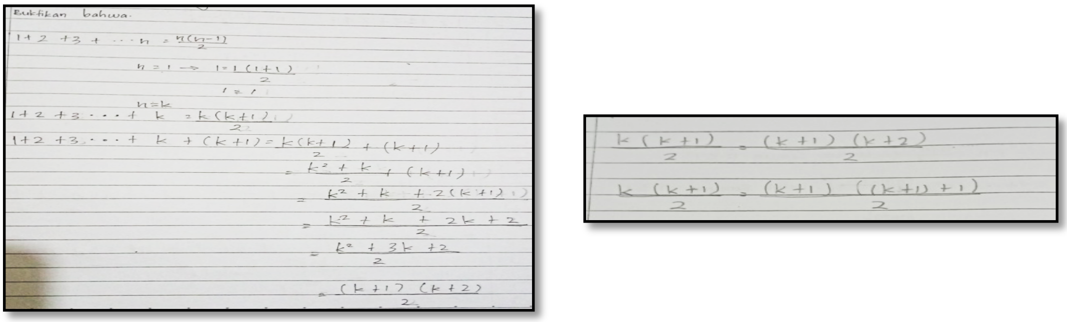

Figure 5. Subject SL2's answer to M1 problem

According to Figure 5, it can be seen that the subject SL2 did the M1 problem by using proof induction mathematics, so that the procedure chosen was correct but the mathematical induction step written by the subject was not given information, so it became difficult to understand even though the calculation process was classified as correct. In addition, the mathematical induction step was also not given conclusions so that the answer seemed unfinished.

2. **Apply the procedure appropriately**

The subject SL2 changed the value of n to 1 so that it was obtained $1 = \frac{1\,(1+1)}{2}$ and produced a value of 1 on the left and right segments. The results of the subject SL2 were correct. Next the subject replaced the value n with k so that the statement changed to 1+2+3+...+k = $\frac{k\,(k+1)}{2}$. The subject directly proved for the statement n replaced with k+1 departing from the statement n = k. In the induction assumption step, the proof of the subject SL2 was proven and produced a statement to be proven that was 1+2+3+...(k+1) = $\frac{(k+1)((k+1)+1)}{2}$. The process of proving this statement was

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classified as true but because the evidentiary step was not given an explanatory statement so that the proof process became less understandable. In general, the mathematical induction procedure performed by the subject SL2 was correct because it met two steps of mathematical induction.

3. Modifying procedures

The subject of SL2 could prove by modifying the form of the statement in such a way that the statement to be proven was obtained. The answer's SL2 to prove the M2 problem can be seen in Figure 6.

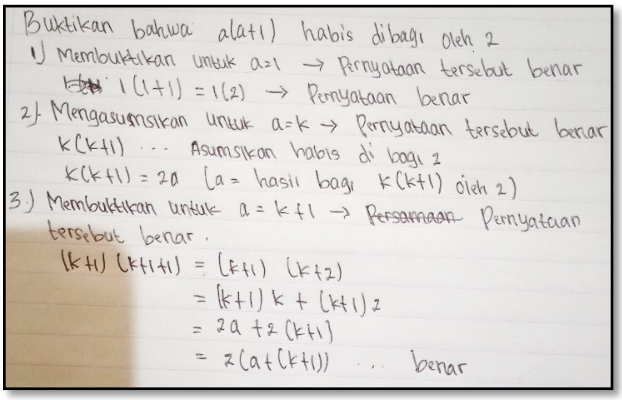

Figure 6. Subject SL2's answer to M2 problem

The answer of the subject SL2 was almost in the right direction but the explanation of the step was less clear so the proof process became less complete. In the basic step, the subject SL2 replaced the value of a with 1 statement was true, but what the statement was meant to be was not written so that the subject sentence SL2 became biased. Furthermore obtained 1(1+1) = 1(2) only mentioned true statement but not given a reason why the statement was true. Second step, subject SL2 assumed a true statement for a = k so that it became k(k+1) = 2a. Then the third step proved to a = k+1 and the statement was true, there was no mention of what the statement was meant to be so that the sentence became biased. Next the subject multiplied the right segment to (k+1)k+(k+1)2. For (k+1)k on the right side was changed to 2a according to the assumption of the statement in the second step so that the shape in the right side became 2a+2(k+1)=2(a+(k+1)). The final form did show that the left segment (k+1)(k+1+1) was halved, but the subject of SL2 did not write the statement so that the procedure of proof of the subject of SL2 became incomplete. The subject of SL2 could try to modify the proof procedure carried out only that the proof done by the subject had not been completed.

Creative Thinking Level 1 – Almost Creative (SL1)

1. Selecting and utilizing procedures

The subject of SL1 chose a mathematical induction method to prove the problem. When asked about the subject, the subject replied that due to the way the mathematical induction had just been taught so as to solve this problem the subject SL1 also used that method, but he did not really understand about mathematical induction. The procedure chosen was correct, using mathematical induction, only the procedure of proof with mathematical induction carried out by the subject was still not appropriate. The answer of the subject SL1 in solving the M1 problem can be seen in Figure 7.

According to Figure 7, it can be seen that the subject SL1 worked on the M1 problem by using mathematical induction proof, so that the procedure the subject chose to solve the problem was correct even though the process and the result were wrong.

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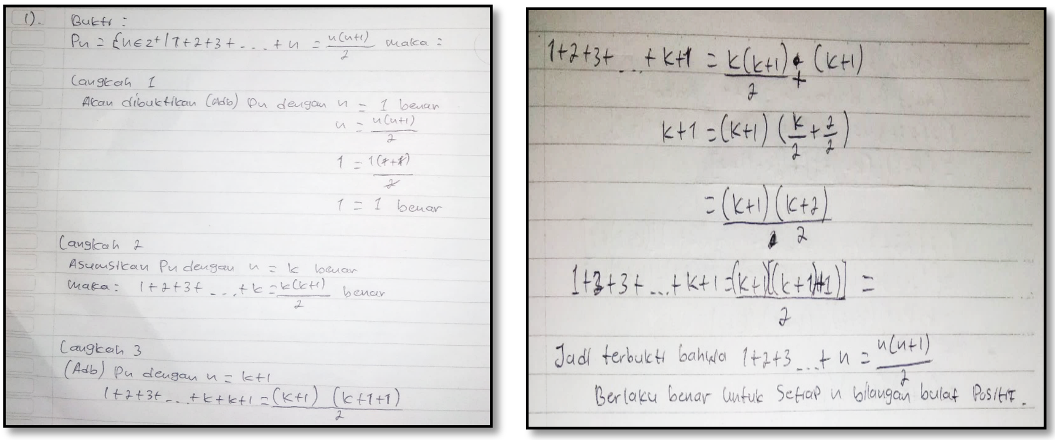

Figure 7. Subject SL1's answer to M1 problem

2. Apply the procedure appropriately

In proving mathematical induction, the initial procedure performed by the subject SL1 was to define the statement to be proved as Pn so that it became Pn = {n ∈ Z+|1+2+3+...+n = (n(n +1))/2 }. The subject of SL1 entered the first step which was the basic step of proof by mathematical induction. In this step, the subject of SL1 could describe the procedure of the first step of induction proofing appropriately. This was evident in proving Pn with n = 1 was true. The mathematical calculations performed by the subject SL1 were correct so it was proven that Pn with n = 1 was correct. Second step, the subject SL1 assumed Pn with n = k was correct then 1+2+3+...+k = (k(k+1))/2 was true. Third step, the subject SL1 would prove Pn with n = k+1 which was the induction assumption step. At the end of the procedure, the subject wrote on the conclusion that the statement was proven, the conclusion of the subject SL1 was not appropriate because the proof process did not get results. Therefore, SL1 applied the proof procedure by mathematical induction with the correct step complete with explanatory information but could not provide the right proof result because the process was wrong so that the statement became unproven.

3. Modifying procedures

The subject of SL1 attempted to modify the proof procedure by mathematical induction method. But the modifications made by the subject SL1 were not precise so that the proof process carried out by the subject SL1 was wrong. The subject's answer to M2 problem could be seen in Figure 8.

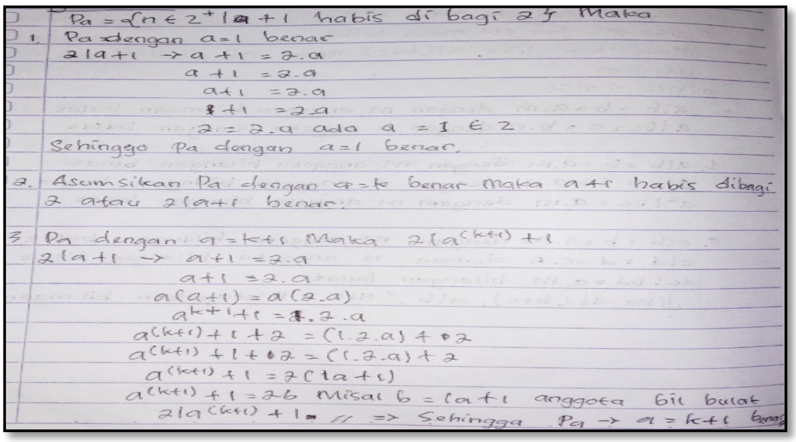

Figure 8. Subject SL1's answer to M2 problem

In Figure 8, the basic step that the subject SL1 did on proof with the mathematical induction method was correct, i.e. the subject SL1 proved the statement to be proven (Pa) by a = 1 was true. Next to the second step, subject SL1 assumed that statement Pa with a = k was correct. While the third step, the step to see the modification of the procedure carried out by the subject, the subject

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made an error in writing the statement to be proven. The subject of writing would prove the statement $2|a^{(k+1)}+1$ when the statement to be proven should be $2|(k+1)((k+1)+1)$. Therefore, the process of calculation and modification made by the subject was wrong. In the third step, the subject of SL1 did not depart from the assumption of the statement in the second step, so the process of proving the third step was certainly wrong and the modifications made by the subject SL1 were not appropriate.

Creative Thinking Level 0 – Not Creative (SL0)

1. Selecting and utilizing procedures

The subject SL0 solved the problem and it was correct, because the subject used a proofing procedure by mathematical induction. The subject SL0 explained that the subject used mathematical induction because of the way the mathematical induction proved the teaching. The procedure chosen by the subject SL0 was correct, namely using proof by mathematical induction, but the proof process written by the subject was incomplete. Here is the answer of the subject SL0 in solving the M1 and problem can be seen in Figure 9.

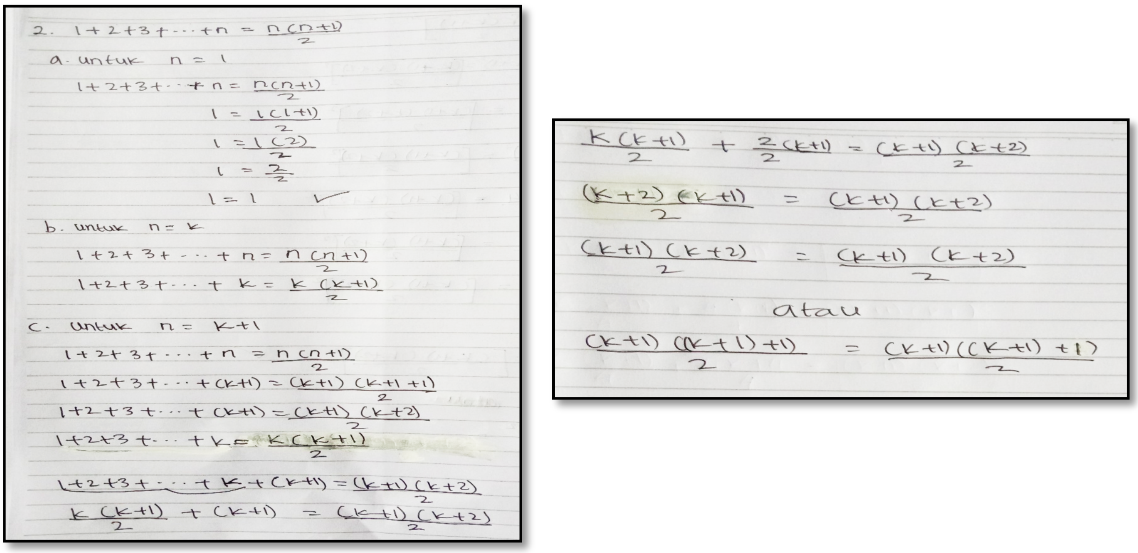

Figure 9. Subject SL0's answer to M1 problem

According to Figure 9, it can be seen that the subject SL0 worked on the M1 problem by using proof induction mathematics, so that the procedure chosen by the subject to solve the problem was correct even though the step of mathematical induction was incomplete because it was not accompanied by information and the proof process was not yet appropriate.

2. Apply the procedure appropriately

The subject of SL0 was incomplete in writing a description or explanation of the mathematical induction proof step. At the beginning of the procedure, the subject of SL0 immediately wrote down the statement to be proven which was $1+2+3+...+n = (n(n+1))/2$. Furthermore, prove for $n = 1$ against the statement to be proven which was the first step or the basic step of the mathematical induction procedure. The basic steps carried out by the subject SL0 produced the correct calculation process, so that the results of the left and right segments were equally large but not inferred by the truth of this step. For the next step, the subject SL0 substituted the value $n = k$ on the statement to be proven. This step was also without being given a clear explanation, so that the process carried out became less understandable. Furthermore, the subject substituted the value $n = k+1$ on the statement to be proven, nor was it added with a clear description. For the calculation process carried out by the subject there was also some errors. So, for the proof procedure by mathematical induction, it did not produce the correct answer. In addition, the subject also did not write the conclusion of the proof process that he did. Therefore, the subject of SL0 did not apply the proof procedure with proper mathematical induction, because the procedure performed was not equipped with explanatory information and the results which were obtained were also wrong.

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3. Modifying procedures

The subject of SL0 could not modify the proof procedure by mathematical induction method. This was because from the answer of the subject SL0, the subject did not perform the step of induction assumption so there was no modification process made by the subject SL0 in answering the M2 question. Subject SL0's answers to M2 problem can be seen in Figure 10.

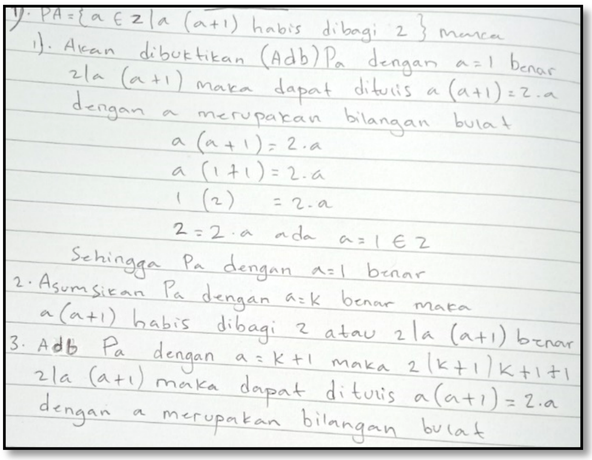

Figure 10. Subject SL0's answer to M2 problem

According to Figure 10, it can be seen that the subject performed the mathematical induction that was in the first step. The subject would prove the to be proven statement defined by the subject as Pa with a = 1 was true. The basic step process of proof by mathematical induction carried out by the subject was correct and proven. Next the subject entered the induction assumption step, in the second step the subject SL0 assumed the statement Pa with a = k true. Third step, the subject would prove the Pa statement with a = k+1 true. But the SL0's answer stopped to the statement and no proof procedure was performed. Therefore, the subject of SL0 did not modify the procedure to the proof process by mathematical induction.

From the analysis that has been spelled out, the obtained results showed that several students had developed creative thinking level 4. On the first indicator of choosing and utilizing the procedure, the chosen procedure to solve the problem was correct. In term of applying the indicators and procedures appropriately, the students had applied the first step (basic step) and second (assumption of induction) in the procedure of proof with mathematical induction correctly to complete the procedure problem. Meanwhile, the third indicator that was modifying the procedure, students could modify the procedure with correct modifications and did not deviate from any theory or nature. Students with a level 4 creative thinking level that had the characteristics of solving problems with novelty, fluency and flexibility were very in accordance with the skill criteria needed in procedural fluency. Procedural fluency refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently (Graven & Stott, 2012; Kilpatrick et al., 2001). Highly creative students who have good procedural fluency certainly produce satisfactory achievements or learning outcomes such as the opinion that procedural fluency in solving problems has a great impact in achieving learning outcomes that must be mastered (Bagay et al., 2021).

To think creatively level 3, on the first indicator of choosing and utilizing the procedure, students worked on the problem by using proof induction mathematics, so that the procedure chosen by the subject to solve the problem was correct. For the second indicator, with applying the procedure appropriately, students could apply the proof procedure by mathematical induction correctly, whereas for the third indicator that used the modification of the procedure, students made modifications to procedures from mathematical induction and could prove the statements to be proven so that students modified the procedure properly. In accordance with the opinion that people who think creatively can produce new ideas (Maharani, 2014), as well as modifications made by students to the solution of the problem of proof with this mathematical induction.

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To think creatively level 2, on the first indicator of choosing and utilizing the procedure, students worked on the problem by using proof induction mathematics, so that the procedure chosen by the subject was correct. For the second indicator, it was the apply the procedure appropriately rule, in general the mathematical induction procedure was carried out correctly because it met the basic step and assumption of induction and could produce a realization that was applicable. However, the mathematical induction step written by the subject was not given any information, so it became difficult to understand even though the calculation process was classified as correct. In addition, the mathematical induction step was also not given a conclusion so that the answer seemed unfinished. For the third indicator to modify the procedure, students could try to modify the proof procedure that was done only by the subject and had not been completed. Therefore, students with a level 2 creative thinking level had good procedural abilities because they could determine the right procedures yet had not been able to complete them appropriately. Though procedural fluency can not only determine the procedure but also can use the procedure well, because procedural fluency can be categorized as good if you know when to use mathematical procedures, know-how mathematical knowledge, where learners' abilities are remembered quickly and perform procedures correctly (Kusuma Dewi et al., 2020; Zakaria & Zaini, 2009).

To think creatively level 1, on the first indicator of choosing and utilizing the procedure, students worked on the problem by using proof induction mathematics, so that the procedure chosen to solve the problem was correct. For the second indicator that by applying the procedure appropriately, students applied the proof procedure with mathematical induction with the correct steps complete but could not provide the correct proof results because the calculation process was wrong so that the statement became unproven. For the third indicator of modifying the procedure, the student tried to modify the procedure but not appropriately. Even so, there were attempts to modify the procedure even if the results were wrong. This could be because students whose creative thinking skills are less developed are less able to construct ideas and understanding of mathematical concepts (Yayuk et al., 2020) whereas procedural fluency relates to students' comprehension of mathematical ideas and problems (Inayah et al., 2020).

To think creative level 0, on the first indicator of choosing and utilizing the procedure, by using proof induction mathematics, the procedure chosen by the subject to solve the problem was correct. For the second indicator that by applying the procedure appropriately, students did not apply the proof procedure with proper mathematical induction, because the procedure performed that was not equipped with explanatory information obtained was also wrong. For the third indicator to modify the procedure, students did not modify the procedure in the proof process by mathematical induction. Procedural fluency that is very lacking can be due to lack of understanding (Wladis, 2019), for example not understanding the correct mathematical induction step or lack of understanding in the correct calculation process. Therefore, conceptual understanding should be equipped with procedural fluency in order to become an expert problem solver (Kusuma & Retnowati, 2021).

CONCLUSION

Based on the results and discussions in this research, it can be concluded that mathematics education students with each level of creative thinking skills have different procedural fluency. For students who were very creative, (creative thinking level 4) they had a fluent procedural proof with excellent mathematical induction because they could use the proof procedure induction mathematics correctly and could modify the procedure in the correct rules. For students who were creative (creative thinking level 3), they had a fluent procedural proof with good mathematical induction. It was because students could use the procedure of proving mathematical induction methods correctly and could modify mathematical induction procedures creatively. For students who were quite creative (creative thinking level 2), they had a fairly good procedural fluency because those learners could choose to use mathematical induction methods to complete the proof. However, the mathematical induction process was rather incomplete, besides that it could modify the procedure even though the process had not been completed. For students who were almost creative (creative thinking level 1), they had less procedural fluency, because they had not been able to determine with certainty when the mathematical induction procedure was used in addition to the proof procedure that was being carried out. Furthermore, they did not produce the right results although in terms of modification procedures they began to try to modify but the results were still wrong. For students who were not creative (creative thinking level 0), they had a procedural fluency that was very lacking because they were less able to understand in what condition

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they would use the mathematical induction proof. They also could not apply the procedure of proving mathematical induction properly and there was no effort to modify the procedure to solve the problem. Therefore, students should be able to improve their procedural fluency by choosing and applying procedures correctly, conducting examinations or proofs of a procedure that has been carefully selected using symbolic methods, and developing or modifying procedures to decipher factors and solve mathematical problems appropriately.

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SHORT PROFILE

Pradipta Annurwanda was born in Magetan on May 7, 1990. The master of education degree was obtained at Sebelas Maret University in the mathematics education study program in 2014. Since 2014 until now, he is a lecturer in the mathematics education of STKIP Pamane Talino, West Kalimantan. Courses that have been mastered are Linear Programing, Statistics, and Research Methodology.

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