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Written mathematical communication skills in solving set theory problems: a descriptive study of preservice elementary teachers

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Abstract
This study examined the written mathematical communication skills of preservice elementary teacher education students in solving problems related to set theory. A descriptive qualitative approach was employed with 28 first-semester students at a private university in Yogyakarta as subjects. Using a written essay test, with three set theory questions, data were gathered. These questions were designed to cover four aspects of mathematical communication: figuring out the problem, turning it into a mathematical model, working it out, and coming to a conclusion. The results showed clear variations in student performance. The majority of students were in the high-skill category (43%), followed by the moderate category (36%) and the low category (21%). Students in the high category fulfilled almost all indicators well, whereas those in the moderate category often made mistakes in mathematical modeling and conclusion writing. Meanwhile, students in the low category struggled from the initial problem comprehension stage through to formulating a conclusion. Overall, although a considerable number of students exhibited high-level skills, the findings suggest that written mathematical communication skills among preservice elementary teachers are not yet evenly developed. More than half of the students fell into the moderate and low categories, indicating a need for improvement, particularly in careful problem interpretation, accurate mathematical modeling, and the organization of solution steps into a clear and coherent written form.
Keywords: Written mathematical communication skills; set theory; problem solving; preservice elementary teachers; mathematics education

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INTRODUCTION

Mathematical communication is a foundational skill in mathematics learning, crucial for articulating ideas, explaining reasoning, and deepening conceptual understanding. Recognized as one of the five key process standards by the National Council of Teachers of Mathematics (NCTM) (Arabacı et al., 2023), it enables learners to translate abstract concepts into various forms such as diagrams, symbols, and language (Gatsmir & Palupi, 2023; Qohar & Fazira, 2022). This skill serves as a bridge between internal reasoning and external expression, supporting cognitive development through critical thinking and application of concepts (Annisak & Wandini, 2023). The ability to communicate mathematically is closely linked to overall mathematical proficiency. Students with a solid conceptual understanding tend to express their ideas more clearly, both orally and in writing, while those who struggle with concepts often face difficulties in organizing and conveying their problem-solving (Azizah et al., 2020; Hidayati & Djamaan, 2025; Purbaningrum et al., 2022). Mathematical communication manifests in two primary forms: oral communication, which involves explaining and discussing ideas verbally, often in collaborative settings, and written communication, which requires precise use of notation, representations, and structured explanations (Anzora et al., 2023; Teledahl et al., 2024).

Research indicates that this skill is influenced by a range of internal and external factors, including instructional approaches, feedback, cultural context, digital tools, and language ability (Anisa et al., 2023). While previous studies have extensively examined mathematical communication among school students in topics like algebra, geometry, and number patterns (Azizah et al., 2020; Hotimah & Lestari, 2023), there remains a notable gap concerning preservice elementary teachers (PGSD students). This is a significant omission, as strong written mathematical communication is essential for future educators, not only to enhance their own conceptual mastery but also to effectively teach mathematical ideas to young learners (Melissaa et al., 2023; Ristiana & Fauzi, 2024).

A key topic in teacher education curricula is set theory, which provides a foundational language for mathematics and logical reasoning (Dong, 2024). Despite its importance, little research has focused on how preservice teachers communicate mathematically in this domain. Addressing this gap, the present study aims to describe the variations in written mathematical communication skills among PGSD students when solving set theory problems. The findings are expected to inform teacher education programs in designing targeted interventions to strengthen these essential competencies.

METHODS

Research Design and Participants

This research was designed as a descriptive qualitative study, chosen to capture in detail the range of students' written mathematical communication abilities without the constraints of quantitative scoring alone. The focus was on describing how preservice teachers communicate their mathematical thought processes in writing and identifying common patterns or variations in their performance. The study was conducted at a private university in Yogyakarta, Indonesia, involving 28 first-semester students enrolled in the Elementary School Teacher Education program (PGSD) during the 2025/2026 academic year. These students were taking an introductory basic mathematics course at the time of the study. The first semester was chosen because it is when students typically take foundational mathematics courses (including set theory), and it provides an opportunity to assess their initial level of mathematical communication skills as they begin their teacher education. All participants gave informed consent to be involved in the study. To protect confidentiality, each student was assigned an anonymous code for data analysis.

Instrument and Data Collection

Data collection involved a written essay test based on set theory problems, adapted from previous work by Azizah et al. (2020). The test was translated into Indonesian and refined for local clarity. It comprised three essay questions aimed at gauging important problem-solving steps related to set theory, including union, intersection, and difference, framed as word problems. Each question required students to demonstrate four critical indicators of written mathematical communication: understanding the problem, translating it into a mathematical model, solving the problem, and drawing a conclusion (See Table 1).

Students had 40 minutes to complete the test under instructor supervision, with anonymity maintained by coding the papers. Scoring was based on the quality of each communication indicator, using a rubric similar to Azizah et al.'s. Scores ranged from 0 (not achieved) to 2 (fully achieved) for each indicator per question. Rather than summing scores, the analysis focused on the fulfillment patterns of indicators to identify performance trends and areas of weakness. Two researchers independently scored the tests to ensure consistency and objectivity in the evaluation of written communication.

Table 1. Written mathematical communication indicators

Stages of Solving the Problem	Written Mathematical Communication Indicators
Understanding the problem	Write what is known and what is asked about the problem using words or mathematical symbols completely, clearly, and correctly
Translate into a mathematical model	Use symbols, variables, or mathematical equations to model problems into mathematical sentences completely, clearly, and correctly
Addressing the problem	a. Write the procedure or steps to solve the problem completely, clearly, and correctly b. Use symbols, variables, or equations when writing problem-solving
Concluding	a. Change symbols, variables, ot mathematical equations to the problem situation to write a conclusion b. Write conclusions and reasons when solving problems in acomplete, clear, and correct way

Source: Azizah et al. (2020)

Data Analysis

Each student's written mathematical communication performance was analyzed and classified into three ability levels: high, medium, and low. The distribution of students across these categories was calculated in percentages and visualized using bar charts (Figure 2). In addition, bar charts for each test item were presented in Figure 1 to show the number of students who achieved each indicator on each problem. These visualizations helped identify stages of problem solving that were well mastered by students, as well as those that remained challenging.

To deepen the qualitative analysis, one representative student from each ability category was selected for further examination. These three students, labeled M1 (high category), M2 (medium category), and M3 (low category), represented the typical characteristics of their respective groups. The written solutions of M1, M2, and M3 were analyzed in detail by examining the attainment of each indicator, the patterns of reasoning demonstrated, and the types of errors observed. The qualitative findings from these cases were used to illustrate characteristic strengths and weaknesses at each ability level. Subsequently, the Results section presents the overall class-level findings, while the Discussion section provides a detailed description and interpretation of M1's, M2's, and M3's performances with reference to relevant literature.

RESULTS AND DISCUSSION

Results

Test data from 28 PGSD students showed that there were variations in written mathematical communication skills. Students' ability to understand problems and answer questions showed the most mastered indicators. Students who could not meet these two indicators were also unable to answer and conclude the solution to the problem correctly. Figure 1 below is a bar chart showing the results of the written mathematical communication test.

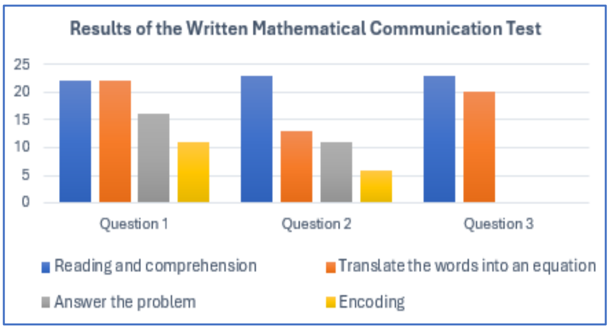


Figure 1. Student's test result

The diagram in Figure 1 shows that the subjects' ability to understand the research was the most commonly mastered indicator in each question. Then, the ability to translate into mathematical models began to show a decline in the number of subjects who mastered it. Only 13 of the 23 students were able to translate the questions into mathematical models in question 2. Meanwhile, there was a decline in the students' ability to answer problems at the conclusion stage. In fact, in question number 3, no subjects were found who could answer the problem and conclude it. When viewed based on the total score, the students' test results were grouped into three categories. The categories used were high, medium, and low, as shown in Figure 2 below.

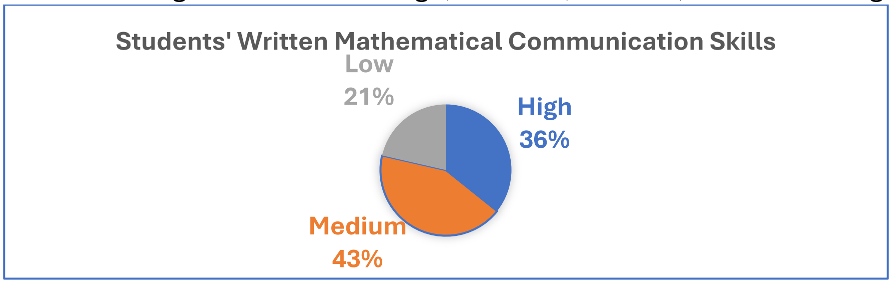


Figure 2. Test result category

Figure 2 shows that most students fall into the moderate category, namely 12 students with a percentage of 43%, and 10 students fall into the high category with a percentage of 36%. In addition, 6 students fall into the low category with a percentage of 21%. These results indicate that most students have moderate mathematical communication skills, characterized by an ability to understand problems but not yet able to answer them with written steps. Based on the student test results, the researcher selected 1 research subject in each category (high, medium, and low). Subject M1 was a student in the high category, subject M2 was a student in the medium category, and subject M3 was a student in the low category in the mathematical communication test.

Discussion

Overall, the results highlight substantial variability in the written mathematical communication skills of preservice elementary teachers, even when tackling the same set of problems. To understand the nature of these differences, we examine the work of three representative students (M1, M2, and M3), each exemplifying one of the performance categories (high, medium, and low, respectively). By analyzing their approaches and errors in detail, we can interpret the findings and connect them with broader educational insights and previous studies.

Understanding the Problem: The Foundation of Mathematical Communication

The analysis of students' written mathematical communication skills demonstrates that understanding the problem represents the most consistently mastered indicator across all ability categories. Figure 1 reveals that subject comprehension of problem statements was achieved by the majority of students in each of the three test questions, making it the strongest component of mathematical communication in this study. This finding aligns with the research by Hariyani et al. (2023), which states that initial understanding of the information in the question plays an important role in producing correct mathematical representations in the subsequent stages.

However, the capacity to understand problems varied significantly across students classified into different ability levels. Students in the high category, exemplified by M1, demonstrated complete and accurate comprehension by writing down what is known and what is asked using correct mathematical notation. As shown in Figures 3, M1 explicitly identified the members of sets A, B, and C using proper set notation with curly brackets, effectively translating the word problem into mathematical language from the outset.

Suppose:
- Set P = students who passed the first test.
- Set Q = students who passed the second test.

a. Write down the members of the P set and the Q set based on the table above!

Answer:

∈P = {Zainal, Fikria, Saiful, Merlina, Anisya, Nurul}
∈Q = {Ahmad, Zainal, Fikria, Aryani, Anisya, Rahma, Ratna}

b. Determine the students who included in:
- Class A
- Class B
- Class C

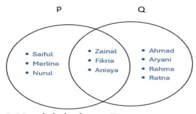
Explain why this is so!

Answer:

A = {Zainal, Fikria, Anisya}	because according to the provisions, students who pass both tests are placed in class A
B = {Saiful, Merlina, Nurul}	because according to the regulations, students who only pass the first test are placed in class B
C = {Ahmad, Aryani, Rahma, Ratna}	because according to the regulations, students who only pass the second test are placed in class C

c. Draw a Venn diagram! Explain the meaning of each area on the Venn diagram you draw!

Answer:



Ahmad, Aryani, Rahma, Ratna: Q

Saiful & Merlina & Nurul: is in the set P

Zainal, Fikria, & Anisya: located between sets P and Q

Figure 3. M1’s Work (English Version)

According to Rahmah et al. (2019), mathematical representation includes the use of symbols, images, or other forms as interpretations of thoughts that aid understanding and problem-solving. Thus, M1's enumerative presentation of sets is in accordance with the first indicator. In contrast, students in the moderate category (represented by M2) exhibited partial understanding. While M2 accurately identified the task requirements, for instance, determining the number of children who only like volleyball and only like soccer, as shown in Figure 4, this preliminary comprehension did not translate into accurate subsequent problem-solving steps.

Question 2

After collecting data from 50 children, it was found that 32 children liked volleyball, 40 children liked soccer, and 25 children liked both.

a. Determine the number of children who:
- Only like volleyball
- Only like soccer
- Like both sports
- Don't like either sport

Give your reasons!

Answer:

- children who like volleyball = 7

  calculation = 32 - 25 = 7
- children who like soccer = 15

  calculation = 40 - 25 = 15
- both = 25
- neither = 25

  calculation = 50 - 25 = 25

  (50 are total number of children)

Figure 4. M2’s Work (English Version)

Students in the low category, exemplified by M3, demonstrated fundamental difficulties from the problem comprehension stage itself. As illustrated in Figure 5, M3 failed to correctly identify set members, made errors in distinguishing odd and even numbers, and did not employ proper mathematical notation (such as curly brackets for sets). These conceptual errors at the foundational stage directly impeded all subsequent problem-solving phases.

Question 3

Given:
A = {natural numbers less than 20}
B = {positive even numbers less than 15}
C = {positive odd numbers less than 10}
D = {natural numbers greater than 7 and less than 15}

a. Determine the members of sets A, B, C, and D!

Answer:

A = 1, 2, 3, 4, 5 19
B = 1, 3, 5, 7, 11, 13, 15
C = 2, 4, 6, 8, 9
D = 8, 9, 10, 11, 12, 13, 14

b. Determine the elements of $B \cap C$, $B \cap D$, and $C \cap D$ and provide the reason why these elements are included in the intersection!

Jawab:

B ∩ C: member		$B \cap D$	$C \cap D$
1 3 5	2 4 6	1, 2, 5 : 11	2 4 6 8 9
7 11 13	8 9	7, 11, 13 13	
		15	
		8, 9, 10, 11, 12, 13, 14	8, 9, 10, 11, 12, 13, 14
The intersection is not in the set			Intersection: 8 9

c. Present the data in a Venn diagram!

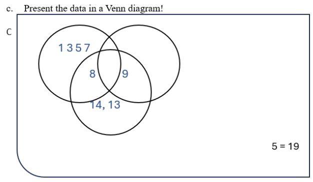


Figure 5. M3's Work (English Version)

The consistent mastery of the problem comprehension indicator suggests that preservice elementary teachers generally possess basic reading and comprehension skills necessary for mathematics. However, the subsequent challenges in translating this understanding into mathematical models and solutions indicate that the ability to understand problems alone is insufficient for effective mathematical communication.

Translating Into Mathematical Models: Where Communication Breaks Down

The analysis of students' performance on the second indicator reveals a notable decline in achievement compared to problem comprehension. Figure 1 demonstrates that the number of students successfully achieving this indicator decreased substantially across the three test questions. Particularly in question 2, only 13 of the 23 students who understood the problem were able to correctly translate the information into mathematical models. This significant drop indicates that translating problem scenarios into formal mathematical representations presents considerable difficulty for a substantial portion of preservice teachers.

Subject M1, representing students in the high category, successfully met the indicator of translating problems into mathematical models by employing mathematical symbols and variables appropriately. As demonstrated in Figure 3, M1 used set notation correctly and identified the specific mathematical operations needed (intersection, union, complement) to address the problem. M1's approach demonstrates awareness of how to convert word problems into formal mathematical language. According to Schoenherr et al. (2024), the use of external visualizations (e.g., diagrams) significantly supports mathematics learning. In the third indicator, M1 uses Venn diagrams as visual representations to solve problems, which are suitable for modeling relationships between sets. The successful achievement of this indicator in the high category reflects the power of appropriate mathematical representation in supporting communication and problem-solving.

Subject M2's performance on mathematical modeling reveals a pattern of procedural and conceptual errors that had cascading effects on the solution process. As shown in Figure 4, M2 attempted to determine the number of children who did not like both sports by directly subtracting the total number of students from the number of students who liked both sports, without considering the complete set structure. This approach demonstrates a fundamental misunderstanding of set union and intersection operations, core concepts required for accurate mathematical modeling in set theory problems.

More specifically, when working with the Venn diagram representation (as shown in Figure 6), M2 wrote down the number of members of sets P and Q without first subtracting the number of members of set R (the intersection), resulting in errors in both the calculation and the mathematical model. Such mistakes are common among students or university students when understanding the concepts of intersection and union, which often cause the calculation process to be incorrect and affect the completion of the next stage. M2 continued the procedural error into the visual representation and conclusion-drawing stages, leading to an inappropriate final result. This finding is consistent with the research by Sari et al. (Sari et al., 2025), which shows that errors in the early stages of mathematical modeling (e.g., incorrectly writing set members or misunderstanding data) will directly affect the quality of conclusions in problem solving.

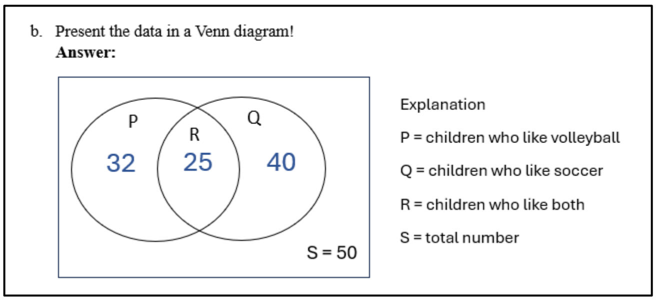


Figure 6. M2’s Work (English Version)

Subject M3's inability to translate problems into mathematical models stemmed from more fundamental misconceptions than those observed in M2. As shown in Figure 5, M3 wrote sets without proper notation (missing curly brackets) and made errors in identifying odd and even numbers, conceptual errors that represent basic mathematical misconceptions. Misconceptions about the concept of numbers or set operations cause students to fail to classify elements correctly, thus leading the mathematical representation process in the wrong direction. Because M3 could not correctly identify the members of the set intersection, the subsequent mathematical modeling process was fundamentally compromised from the outset.

The translation indicator serves as a critical threshold in the problem-solving process. The substantial decline in student achievement on this indicator, compared to the preceding comprehension stage, suggests that while preservice elementary teachers can read and understand problem statements, many lack the conceptual depth and procedural fluency necessary to convert problem scenarios into formal mathematical language. This finding has important implications for teacher education programs: instruction must move beyond surface-level problem comprehension to develop a robust understanding of mathematical concepts and their symbolic representations.

Solving the Problem: Execution and Visual Representation

The third indicator of written mathematical communication comprises two interconnected components: (a) writing the procedure or steps to solve the problem completely, clearly, and correctly, and (b) using symbols, variables, or equations when writing problem-solving. The achievement patterns on this indicator reveal important insights about students' ability to execute mathematical procedures and maintain appropriate mathematical notation throughout the solution process. Figure 1 demonstrates a sharp decline in the number of students successfully achieving both components of this indicator across all three questions. By question 3, the decline becomes particularly pronounced, indicating that sustaining accurate problem-solving procedures and consistent use of mathematical notation becomes progressively more difficult as problem complexity increases.

Subject M1 exemplifies successful problem-solving at the high proficiency level. As shown in Figure 3, M1 employed a Venn diagram as a visual representation to solve the set problem. According to Schoenherr et al. (2024), the use of external visualizations (e.g., diagrams) significantly supports mathematics learning. M1's approach was both procedurally systematic and visually clear, writing steps in an organized manner that conveyed substantial mathematical information. The integration of visual representation within the problem-solving procedure distinguishes high-category students from those in lower categories. M1 did not merely list computational steps; rather, the Venn diagram functioned as a mathematical tool that clarified the relationships between sets and guided the solution process transparently.

Subject M2's work on the problem-solving indicator reveals partial competence marred by conceptual confusion. While M2 attempted to construct a Venn diagram (as shown in Figure 6), the diagram did not correctly represent the set structure due to the earlier modeling error. Specifically, M2's failure to properly account for the intersection when filling in the diagram resulted in an inaccurate visual representation that could not support correct problem-solving. This finding demonstrates a critical relationship between the mathematical modeling indicator and the problem-solving indicator: errors in modeling directly propagate into the problem-solving phase. Previous research shows that subjects who have difficulty creating Venn diagrams tend to have obstacles in drawing accurate conclusions about set problems (Irawan et al., 2025).

Subject M3 was unable to construct visual solutions through Venn diagrams effectively. Because M3 could not correctly identify the members of the set intersection (a manifestation of earlier conceptual errors), the construction of an accurate Venn diagram was fundamentally impossible. When visual representations do not match the correct set structure, students tend to fail to connect concepts with the required solutions. This finding is in line with the research by Agostini et al. (2022), which found that students' errors in understanding the properties of numbers and the relationships between sets significantly affect their ability to construct accurate visual representations, thereby impacting their overall mathematical reasoning.

The analysis of the problem-solving indicator reveals that successful mathematical communication at this stage requires both procedural accuracy and effective use of visual or symbolic representations. The sequential deterioration in achievement across ability levels suggests that while high-category students integrate these components seamlessly, moderate-category students encounter difficulties in maintaining conceptual accuracy within visual representations, and low-category students cannot execute visual representations correctly due to foundational conceptual gaps.

Drawing Conclusions

The fourth indicator presents the most severe challenge to student performance. Figure 1 demonstrates a progressive decline in achievement across the three test questions, culminating in question 3, where no students were found who could fully meet both components of this indicator: (a) changing symbols, variables, or mathematical equations back to the problem situation to write a conclusion, and (b) writing conclusions and reasons when solving problems in a complete, clear, and correct way. This dramatic decline in the conclusion-drawing indicator is particularly significant because drawing conclusions represents the culmination of the entire problem-solving and communication process. The difficulty that students encounter in this final stage suggests that challenges in earlier indicators may be compounded by the time students reach the conclusion stage, or that students lack a specific understanding of what constitutes a mathematically complete and communicatively clear conclusion.

Even subject M1, classified in the high category with strong overall performance, exhibited a deficiency in the conclusion-drawing indicator. As noted in the analysis of Figure 3, M1's conclusion written by M1 does not mention that students in the intersection of sets also belong to each of these sets. Ningtyas and Ekawati (2021) state that written mathematical communication requires precision in expressing ideas because writing externalizes thoughts more accurately. Writing that is not systematic or omits important details can cause readers to misunderstand. This is in line with the findings of Hidayati and Djamaan (2025) that low-ability subjects often omit important elements (e.g., pictures/sketches), resulting in suboptimal written communication. Thus, M1's failure to mention the intersection of sets in the conclusion makes the solution incomplete and potentially leads to misinterpretation.

Subject M2's inability to draw appropriate conclusions represents a direct consequence of the compounding errors that originated in the mathematical modeling stage. Because M2's Venn diagram and calculations were inaccurate (as shown in Figure 6), the conclusion drawn by M2 could not be mathematically valid. However, beyond the issue of mathematical correctness, M2 was unable to connect the available information, namely the total number of children of 50, with the sum of members who like volleyball only, like soccer only, like both, and who do not like either. The lack of connection between this information had a direct impact on the conclusions drawn by M2.

Subject M3 did not achieve the conclusion-drawing indicator at all. With fundamental conceptual errors in place from the problem comprehension stage, and without accurate mathematical modeling or problem-solving procedures, M3 was unable to construct any valid conclusion. M3's difficulty in interpreting and organizing information indicates a lack of deep mathematical thinking in the problem-solving process. The result is in line with the findings of Kholid et al. (2021) that subjects with low mathematical abilities are unable to solve problems correctly, including writing mathematical conclusions.

The dramatic difficulty students encounter in drawing conclusions stems from the cognitive demand of this indicator. Drawing conclusions requires students to simultaneously: 1) Verify that the mathematical solution is complete and mathematically consistent; 2) Translate abstract mathematical results back into the concrete context of the original problem; 3) Express this back-translation in clear, grammatically correct language that maintains mathematical precision; 4) Ensure that the conclusion addresses all aspects of the original question.

This multi-faceted requirement explains why even high-category students may have difficulty achieving full proficiency. The pervasive difficulty in drawing conclusions among preservice elementary teachers raises particular concerns. These future educators must not only solve mathematical problems themselves but must also teach students how to express mathematical conclusions clearly. If preservice teachers struggle with this indicator, they may inadvertently model incomplete or imprecise conclusion-drawing practices for their future students, potentially perpetuating communication deficiencies across educational generations.

Integration of Indicators: The Sequential Dependency Model

The analysis across all four indicators reveals a fundamental principle: written mathematical communication in problem-solving follows a hierarchical, sequential structure in which earlier indicators establish prerequisites for later ones. Figure 1's data demonstrates this principle vividly: the percentage of students successfully achieving each indicator decreases progressively from the problem comprehension stage through the conclusion-drawing stage across all three questions.

This pattern is not coincidental but reflects the mathematical reality that each stage builds upon the preceding stage. As Ningtyas and Ekawati (2021) noted, previous research has demonstrated the interrelatedness of problem-solving stages, from the initial step to the final one. This sequential dependency principle has important implications: a student who misunderstands the problem (Indicator 1) cannot accurately model it (Indicator 2); a student who incorrectly models the problem cannot solve it correctly (Indicator 3); and a student who does not solve the problem correctly cannot draw a valid conclusion (Indicator 4).

The case studies of M1, M2, and M3 demonstrate how errors in earlier stages propagate through subsequent stages. M2's error in mathematical modeling, failure to properly account for set intersection, resulted not only in an incorrect problem-solving procedure but also in an invalid conclusion and flawed visual representation. Conversely, M1's accurate work through all stages demonstrates how correct execution at each stage enables success in the subsequent stage. Sari et al. (2025) documented this phenomenon explicitly: errors in the early stages of mathematical modeling will directly affect the quality of conclusions in problem solving. This sequential dependency means that interventions aimed at improving mathematical communication cannot focus exclusively on any single indicator; instead, they must address the foundational understanding necessary for all indicators to function effectively.

An important nuance emerges from the data: students in the moderate category demonstrated the ability to understand problems (Indicator 1) without necessarily being able to model or solve them correctly (Indicators 2-3). This dissociation suggests that problem comprehension, while necessary, is not sufficient for overall mathematical communication competence. Conversely, students in the high category who achieved both understanding and modeling often still showed minor deficiencies in the conclusion stage, suggesting that even when cognitive processes function effectively through the problem-solving stage, the final communicative act of translating results back into the problem context requires particular attention.

The distribution of students across three ability categories (high: 43%, moderate: 36%, low: 21%) indicates that written mathematical communication skills among preservice elementary teachers are not evenly distributed. The concentration of students in the moderate and low categories (57% combined) suggests that more than half of the preservice teachers studied demonstrated performance levels below the high category, indicating considerable need for improvement in mathematical communication skills. The concentration in the moderate category, specifically characterized by the ability to understand problems but not to model or solve them with written steps, suggests a particular profile of students who recognize problem requirements but struggle with the conceptual depth necessary for accurate mathematical modeling and systematic problem-solving communication.

Collectively, these patterns suggest that many preservice elementary teachers possess foundational reading and comprehension skills sufficient to understand mathematical problem statements but lack the conceptual understanding, symbolic fluency, or communicative precision necessary to express solutions effectively in writing. This profile has particular implications for teacher education: instructional interventions must move beyond surface-level problem comprehension to develop robust mathematical conceptualization and explicit communication instruction.

Synthesis and Implications

Across all ability categories, students who successfully employed visual representations (such as Venn diagrams) demonstrated stronger problem-solving performance and more complete mathematical communication. This finding aligns with Schoenherr et al. (2024), whose meta-analysis demonstrated that learning with visualizations helps significantly in mathematics education. The implication is that teacher education programs should explicitly teach preservice teachers not merely to understand visual representations but to construct and interpret them as communication tools within the problem-solving process.

The consistent pattern of conceptual errors at multiple stages, particularly among moderate and low-category students, indicates that a deficiency in mathematical communication stems fundamentally from incomplete conceptual understanding of set theory and related mathematical operations. Procedural instruction alone, without deep conceptual development, cannot adequately address these deficiencies. Given the pervasive difficulty students encounter in drawing conclusions, preservice teacher education programs should explicitly emphasize the importance of precision and completeness in mathematical written communication. Students should be taught to verify that their conclusions address the entirety of the original problem and to articulate clearly the relationships between mathematical results and problem context. Because mathematical communication indicators are sequentially interdependent, instructional strategies should build systematically from problem comprehension through conclusion-drawing, ensuring that students develop proficiency at each stage before advancing. Interventions should address conceptual misunderstandings that emerge at early stages, as these errors will otherwise propagate through all subsequent indicators.

CONCLUSION

Based on the results of the analysis, this study concludes that students' written mathematical communication skills are still in the moderate category, with the main weaknesses being in the aspects of drawing conclusions and accurately representing information in visual form. Errors in reading and understanding the information in the questions have a direct impact on the mathematical modeling and solution steps written by the subjects. These findings confirm that PGSD students, as prospective elementary school teachers, need practice and guidance in writing down the solution process in a sequential, clear, and representative manner. This study contributes to mapping indicators of students' written mathematical communication and provides a basis for developing learning strategies that emphasize mathematical communication exercises, particularly in visual representation and drawing conclusions. In the future, it is recommended that a learning model or lecture intervention be specifically designed to improve the written mathematical communication skills of PGSD students.

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