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An Authentic Learning Approach to Assist the Computational Thinking in Mathematics Learning for Elementary School

Ratna Wardani[1], Wu-Yuin Hwang[2], Masduki Zakaria[3], Priyanto[4], Muhammad Irfan Luthfi[5], Irma Nuur Rochmah[6], Andy Ferry Rahman[7], Muhammad Trio Maulana Putra[8]
[1,3,4,5.6.7.8] Electronics and Informatics Engineering Education, Universitas Negeri Yogyakarta
[2] National Central University, Taiwan, Graduate Institute of Network Learning Technology
Email: ratna@uny.ac.id

ABSTRACT

Programme for International Student Assessment (PISA) 2018 report revealed that in mathematics the ability of Indonesian students to solve mathematical problems is still very low. The Trends in International Mathematics and Science Study (TIMSS) strengthened the PISA report by mentioning that in general, Indonesian students' ability in solving fraction problems is low. The study aims to improve student's ability in solving fractions by using computational thinking assisted by an authentic learning approach. A system was developed to help the students in learning fractions. Totally twenty-two fifth-grade students were assigned in this study. While they learned fractions using the system, they needed to capture an object around them to make a representation of the fraction problem. Moreover, by using the system, they also need to find the best and fastest way to solve a fraction problem. The result showed that their learning achievements improved especially in fraction representation, fraction operation, linguistics, and creativity. Multiple regression results showed that students' activity can predict the students' learning achievement. The more active the learn fraction using the system, the higher score they get. In the final, the result of this study contributes essential implications along with a conclusion and suggestion for future research.

Keywords*: Computational Thinking, Authentic Learning, Fraction Learning*

INTRODUCTION

Computational thinking (CT) was becoming a trend and this capability should be developed as early as possible in the 21st century [1]. Although still quite slow, in Indonesia, CT had been included in the Indonesian education curriculum in the regulation of the Minister of Education and Culture no. 37 of 2018 for junior high school and senior high school students.

Several studies were conducted to introduce ct to elementary school children through programming activities [2] [3] [4] and Science, Technology, Engineering, and Mathematics (STEM) learning activities [5] where mathematics had the strongest correlation with ct activity [6]. The application of mathematics learning in supporting CT in students had a positive impact [7], besides that the application of CT had a significant positive impact on student mathematics learning outcomes [8] [3].

One of the math subjects that were difficult for students to understand was fractions. Based on the results of the Trend In International Mathematics and Science Study (TIMSS) in 2015, Indonesia ranked 45 out of 48 in natural sciences and 45 out of 50 countries in mathematics. TIMSS put the fraction as one of the materials that measured in the domain of the number. Fractions in TIMSS covered the basic concepts of fractions and fraction operations [9]. Moreover, the basic concept of fractions was also related to students' ability to represent fractions using words, numbers, or models. In general, students' ability in solving fraction problems was low. In TIMSS 2015 the average percentage of Indonesian students' corrected answers for all fraction questions was 24.45%. This percentage was far below the international average, which was 46.98%. Furthermore, for TIMSS numeracy where all questions were only about the basic concept of fractions, the average percentage of Indonesian students' corrected answers was 42.67% compared to the international average of 47.33%. The TIMSS 2015 results confirmed the low ability of Indonesian students in the topic of fractions.

Four causes make fractions challenging to learn [10]: (1) rarely used in everyday life and difficult to explain with natural numbers; (2) the written forms of fractions were complicated; (3) the difficulty in putting fractions in a number line in sequence; and (4) fractional arithmetic had many rules and it was complicated. However, even the fractions were challenging to learn, the students need to understand them. If the students' did not understand the concept of the fraction, it could lead to math anxiety [11]. Above all, the fraction was fundamentally essential for more advanced mathematical and logical reasoning skills (e.g. Proportional, probabilistic, and algebraic thinking) [12].

Some researchers conducted a study on fraction learning to solve this challenge. For example, a study conducted by [13]. They utilized a system installed on the multi-touch tabletop and tablet PC. In the study, they also applied a graphical and symbolic fraction representation to the system. The result of the study shows that the students' understanding of the fraction concept was improved. Before that, a study was also done by [14]. They applied virtual manipulatives that utilize graphic images and symbolic notations for fraction equivalence learning. and long before that, a study was also conducted by [15]. They applied graphical partitioning and authentic support for learning common fractions to improve the students' understanding of fraction concepts.

Some elementary school mathematics teachers also conduct studies to improve the quality of learning fractions. In [16] the studies applied computational thinking in adding and subtracting fractions. He stated that adding and subtracting was pretty tricky. So, He asked the students to construct flowcharts to solve the addition and subtraction problem. At the end of his studies, He finds out that most of his students could find the optimal way to solve the fraction addition and subtraction problem. Another study was conducted by [17], who applied a scratch-like tool in a laptop to facilitate the student in fraction learning. She also built a website called action fractions (https://www.canonlab.org/actionfractionslessons) that the content was integrated mathematics – computational thinking curriculum designed using everyday mathematics.

From the results of the studies above, we could know that some researchers and teachers tried to solve the fraction learning problem, for example, the use of multiple representations like graphical and symbolic representations, authentic support, and computational thinking. The use of graphical and symbolic representation will help the students to improve their knowledge of the fraction concept. Also, authentic support will improve the students' understanding of the fraction concept. Meanwhile, computational thinking encourages students to keep finding the optimal way of solving a fraction problem.

In this study, we utilized a system that could facilitate students in fraction learning. In this system, we will put forward authentic learning to assist computational thinking. The expected result from this study was, the student's understanding of fractions topic will be improved through graphical and symbolic representation, the students' could connect the fraction to their real life, and finally, the students' could find the optimal way to solve a fraction problem. Finally, we could investigate the effects of the systems on the students' performance in mathematics, especially in fraction learning.

A. Computational Thinking in Mathematics Learning

It had been explained before that CT had an important role in this 21st century. CT was described as thought processes involved in expressing solutions as computational steps or algorithms that could be carried out by a computer [18] [19] [20]. Reference [21] also stated that CT was a fundamental skill for everyone, to every child's analytical ability we should add CT. Moreover, the [22] described CT as a framework. A study conducted [20] [23] shows that CT could engage youth within these rich computational environments in three-stage: use, modify and create (Figure 1)

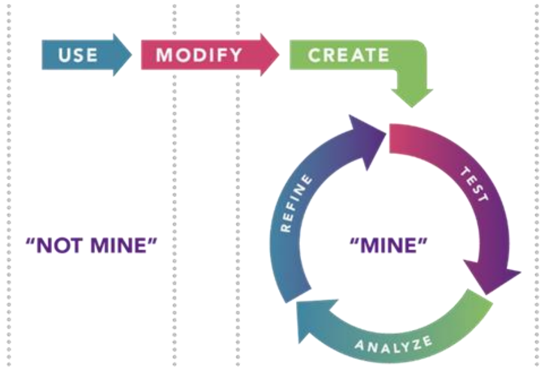

Figure 1. Use-Modify-Create [23]

In the use stage [23] the students first interact with CT artifacts. Then, they develop another CT skill by modifying someone else's finding in the modified stage. Finally, they could be encouraged to develop their ideas for CT in the creation stage.

Several studies were conducted to discover learning activities that could introduce CT skills to students. Reference [24] tried to find out which learning activities were suitable for providing CT experiences to elementary school students, whether it was through a digital device (plugged) or without a digital device (unplugged). And the results of the study revealed that the combination of plugged and unplugged activities could increase the motivation and ability of elementary school students' CT. A combination of CT learning activities with and without digital devices could provide a CT learning experience [25] [26]. Moreover, [27] found that to make the CT learning experience more meaningful, it was necessary to develop CT through analogic reasoning with unplugged activities.

In mathematics, CT began to be used by students in problem-solving in learning mathematics, especially in learning fractions. For example, research conducted by [28] applies CT in the teaching of mathematics. The results showed that the experimental group using CT outperformed the control group (which did not use CT). The study recommends integrating CT and mathematics in the classroom to improve students' problem-solving skills. Another study was conducted by [3], which facilitated the learning of mathematics using CT for the sixth-grade students. They divided the research into two phases, the first phase used to scratch to acquire basic CT concepts, and the second phase focused completely on math tasks. From this research, it can be concluded that the use of scratch helps to develop students' mathematical and CT abilities.

Some studies combine learning mathematics with CT skills because there is a close relationship between these abilities. Mathematics Thinking (MT) has similarities with CT because solving mathematical problems is a construction process that requires an analytical perspective that forms the basis for programmers or computer scientists [29]. In addition, [5] [6] revealed that STEM is correlated with CT expertise. Furthermore, CT can improve students' abstraction skills in learning mathematics [30].

Reference [30] found that there is a correlation between AI (Artificial Intelligence), CT (Computational Thinking), and ME (Mathematics Education) which can provide significant benefits. It is necessary to consider how to design mathematics learning to develop the three objects of knowledge. Furthermore, the use of mathematics learning has a positive impact on improving the CT skills of elementary school students [7] [31]. In addition, students' mathematics learning outcomes also increase when applying CT [8] [3].

B. Multiple Representation in Mathematics Learning

Several studies used various types of mathematics learning content to find the right type of content to support the improvement of

CT skills. Reference [32] revealed that operations and numbers, algebra, measurements & functions, and statistics & probabilities are types of content that are suitable for developing CT abilities. Some of these types of content require basic fractional skills to master. However, several studies from [33] [34] revealed that most elementary school students have difficulty understanding fractions because they are always confused with the basic concepts of fractions. This problem has arisen for a long time, Reference [35] stated that the complex relationship between different representations and basic operations in fractions causes difficulties in students' understanding of fractions.

This different representation problem can be solved by using multiple representations in studying fractions. The application of multiple representations in mathematics learning has been proven to help students achieve mathematical concepts in problem-solving more deeply [36] [37] [38]. In [39] also revealed that students find it easier to apply their knowledge and skills by receiving information in multimedia representations. In addition, the use of different representations such as numbers, number lines, explanatory texts, has been shown to improve student achievement in fractions learning [40].

C. Authentic Learning

To better understand fractions learning, teachers need to connect students' fractional knowledge with the real world. By connecting students' knowledge with the real world, students can get a more meaningful learning experience [41] [42]. The integration between knowledge and the real world is called authentic learning [43] [42]. With an authentic learning approach, students will feel that their knowledge has relevance to everyday life [44]. It can motivate students to continue learning to solve more everyday problems [45].

In this research, we will apply the authentic context to the system because it has been proven that applying authentic context in mathematics learning can encourage students' cognitive, operative, affective engagement and enjoyment of mathematics.

METHODS

The method used in this study was qualitative because the problems of the study need to be explored to gain a deeper understanding [46]. Furthermore, Creswell explained: this exploration was needed, in turn, because of a need to study a group of the population, identify variables that could then be measured, or heard voices silenced." [46].

A. Research Stages

The research stages of this study showed in Figure 2. There were five steps in this research stage: (a) System Development where we developed a system that could facilitate students to learn fractions with CT assisted by authentic context; (b) Activity Design: where we designed a learning activity. Students learned the fractions topic with computational thinking assisted by authentic context; (c) Study where the student started to learn fractions and to do practice using the system. In this stage also, we designed the pretest questions for the pretest and post-test based on the school curriculum and discussion with the mathematics teacher; (d) Data Analysis of the study where the students' activity toward the system will be collected and stored in an online database. After that, we extracted it and analyzed the data; and (e) Conclusion and Writing where We conclude and write the result of the study.

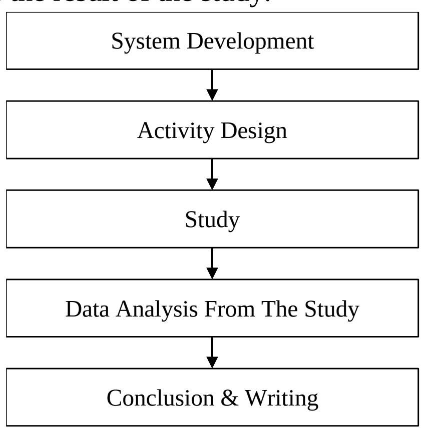

Figure 2. Research Stages

B. Research Variables

This study uses three types of variables: (a) Control variables included learning time and learning materials. Related to this variable, every student spent four days finishing three kinds of fraction learning activities (fraction representation, simplification, and calculation) by applying computational thinking in an authentic context. All students used the same learning materials which was the fraction app; (b) Independent variables included the student learning activity that consisted of the number of their exercises using the app, the corrected answered during their exercise, and final score of their exercise; (c) Dependent variables included the students learning achievement that consisted of post-test score in the aspect of representation, post-test score in the aspect of simplification, post-test score in the aspect of calculation, post-test score in the aspect of linguistics, post-test score in the aspect of creativity, and the final score of the post-test.

C. Research Subjects

Twenty-two of fifth-grade students has participated in this study. They divided into three experimental groups which consisted of six to seven students. A pretest was applied for all experimental groups to measure the students' prior knowledge in fraction learning. After the pretest, the activity continues with the fraction learning using The Fraction App in 4 days. In the last week of the study, we conducted a post-test for all experimental groups to measure the students' achievements.

D. Research Tools and Analysis

In this study, a pre-test was conducted to evaluate the students' prior knowledge and a post-test to measure the students' achievement after the experiment. Both tests were designed by mathematics teachers based on the related material in this study. Pre-test and post-test included fourteen items and were divided into three aspects: fraction representation, fraction simplification, and fraction calculation. Both post-test and pre-test had the same difficulty level of questions. The system recorded data related to students' usage of the application. The recorded data then will be extracted and analyzed to examine the student's performance during learning activities.

An independent sample t-test was employed in analyzing stage to test whether there was a significant difference between the pretest and post-test of the experimental group. Pearson product-moment correlation and stepwise multiple regression Pearson correlation analysis were also employed to test whether the variables correlated. Moreover, we also conducted multiple Stepwise Regression to predict the strongest factor of all variables.

RESULTS AND DISCUSSION

This study applied CT assisted with authentic learning to support fraction learning for the students. In this part, we also presented the learning achievement of the experimental group, followed by the analysis of the correlation between the students' learning activity to the students' learning achievement. We also analyze the students' learning activity more deeply to find out which factors contributed the most to the students' learning achievement.

A. Authentic Learning Model

The learning model developed in this study uses an authentic learning approach for learning mathematics for elementary school students, especially in learning fractions. The learning steps in this model are explained based on Figure 3.

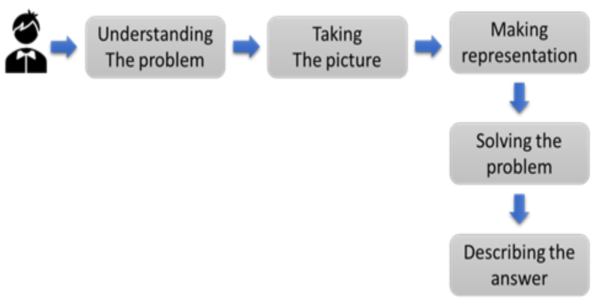

Figure 3. Authentic Learning Model

In the first stage of the learning process, students are asked to understand the given problem. Understanding the problem becomes the most important part of the learning process so that students have insight into the solution of the problem. To get a good understanding, students are asked to take pictures/objects related to the problem using the application. They can take pictures of a rectangle or circular shape objects around them. Students use the picture to make a representation of the problem to be solved (Figure 4a). The activity of making presentations is used to train students so that they can express problem abstractions. Students were also asked to describe the environment in which they took pictures (Figure 4b).

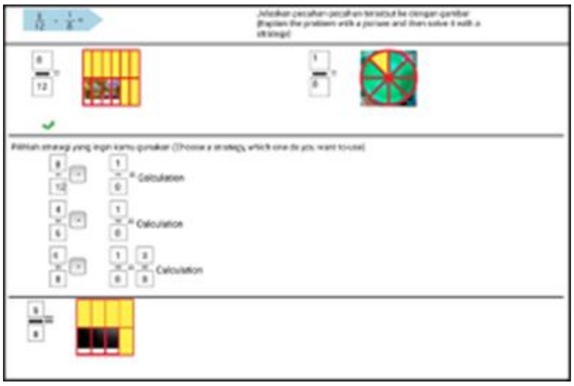

Figure 4a. A fraction representation from a captured object

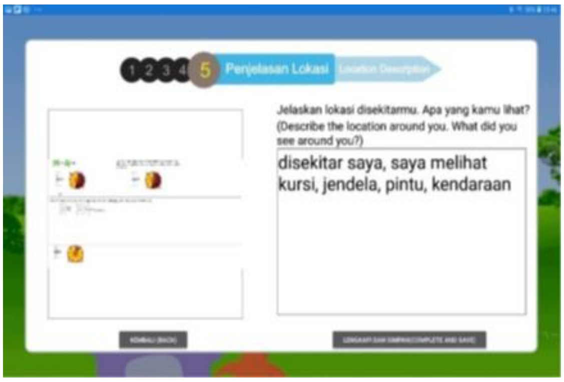

Figure 4b. A fraction representation from a captured object

In problem-solving, we direct students to solve fraction problems in the best and fastest way. For this purpose, the application is equipped with a problem-solver board (Figure 5). Through this problem-solver board, students will be motivated and challenged to find the best and fastest solution. By the Problem-Solver board, students can solve the fraction problem by doing simplification and calculation. This method can train students in the ability of Computational Thinking.

Figure 5. Problem-Solving Board

B. Students Activity Design

Due to the COVID-19 situation, the study was carried out with a special design. Experimental, implementation, and testing of applications in schools are carried out by implementing health protocols as regulated by schools and the government.

The experimental design in this study was carried out by dividing students into 3 experimental groups. Each group consisted of six to seven students. Students in each group utilize the application in fractions learning activities for one week. Learning activities are carried out at students' homes due to the COVID-19 situation. After one group has finished learning fractions, the next group will continue with the same activity. Thus, this research experiment was completed in three weeks, with an additional two weeks of carrying out the pre-test and post-test.

We divided the learning activities in this experiment into four sessions: (1) On the first day, the devices were handed over to students. Students accompanied by their parents come to school to receive a set of devices consisting of a) A bag; b) a sanitizer; c) a tablet; d) a smartphone e) a charger set; f) a guide book; (2) Training session which was also conducted on the first day. After students receive a set of devices, they will be trained to use the Fractions Application.

The training is carried out to ensure students understand how to use the Fractions Application so that there are no problems during fractions learning activities at home; (3) Fractions learning session where students use the Fractions App to study at home. Through an authentic learning approach, students need to capture objects from their environment to solve problems in Fractions Applications. Furthermore, the students have to do their homework with parental assistance. This activity is designed to involve parents in student learning. Because students have been studying at home since the beginning of the Covid-19 pandemic, parents play a major role in the student learning process; (4) After one week, students return the device to school accompanied by their parents. In this experiment, students learn fractions by using the Fractions Application from day 2 to day 6. Next, we rank students' activities in learning.

C. *Students' Learning Achievement*

Analysis of student achievement was carried out by separating the pre-test and post-test into five parts: Representation, Simplification, Operations, Linguistics, and Creativity. We analyzed the results using the T-Test to find out the difference between pretest and post-test.

Table 1. Comparison Data of Pretest and Post-test of the Experimental Group

Variables	Mean Difference	Std. Error	Sig
Representation	-.400	.169	.028
Simplification	-.200	.117	.104
Operation	-.750	.260	.010
Linguistic	-.550	.153	.002
Creativity	-1.800	.583	.006
Total Score	-3.700	.808	.000

Based on Table 2, the result show that there is a significant difference between pre-test and post-test of the experimental group ($p = .000 < .05$). Moreover, the difference also can be found in the creativity ($p = .006 < .05$), linguistics ($p = .002\ldots < .05$), operation ($p = .010 < .05$) and representation ($p = .028 < .05$). These results prove that authentic learning that is applied in learning fractions through the Fraction App can improve students' CT abilities. Through learning design in the Fraction App, students' abilities in representation or abstraction as well as creativity in problem solving are increasing.

The Problem-Solver Board on Fractions Application also improves students' skills in fractional operations. The features developed encourage students to find the best and fastest solution when solving fractional problems. In addition, students' linguistic skills also improve with features that allow students to describe problem solutions. That is, students' understanding of fraction problems is getting better. In other words, the faster and easier students understand a problem, the faster they can solve the problem.

D. *Correlation Between The Students' Activity to The Students' Learning Achievement*

We divided student activity into three variables: frequency of students practicing (Activity: total practice), correct answers (Activity: correct answer in practice), and activity score (Activity: final score). The three variables were analyzed to determine their correlation with student achievement.

Table 2. The Pearson Correlation between Students' Activity and Students' Learning Achievement

	1	2	3	4	5	6	7	8	9
1	1								
2	.843**	1							
3	.737**	.927**	1						
4	.391	.425	.439	1					
5	-.048	-.008	-.034	.110	1				
6	.347	.402	.337	.044	.025	1			
7	.130	.255	.231	.117	.134	.452*	1		
8	.564**	.725**	.681**	.594**	.074	.342	.181	1	
9	.569**	.721**	.675**	.687	.193	.538*	.427	.929**	1

1. Activity: Exercise Total
2. Activity: Exercise Correct Answer
3. Activity: Final Score
4. Post-test: Representation
5. Post-test: Simplification
6. Post-test: Operation
7. Post-test: Linguistic
8. Post-test: Creativity
9. Post-test: Total Score

***Correlation is significant at the 0.01 level (2-tailed)*
**Correlation is significant at the 0.05 level (2-tailed)*

Table 2 shows that there is a relationship between student activity and student achievement. Especially in students' creativity and learning achievement. It can be seen that the number of exercises that students do has a correlation with students' creativity ($r = .564$, $p < .001$) and student achievement ($r = .569$, $p < .001$). Based on this correlation, it can be concluded that the amount of students' exercise affects creativity and students' learning achievement.

E. *The Most Contributed Factor to The Students' Learning Achievement*

This study also explores what variables can be used to predict student learning achievement. We used multiple linear regression to test the existing variables. The results (Table 3 and Table 4) showed that all variables in student activities could predict student achievement.

Table 3. Multiple Regression model summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.725[a]	.526	.437	3.490

a. Predictors: (Constant), Activity: Final Score, Activity: Exercise Total, Activity: Exercise Correct Answer

Table 4. Multiple Regression Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig
		B	Std. Error	Beta		
1	(Constant)	5.574	3.514		1.568	.132
	Activity: Exercise Total	-.240	.578	-.137	-.416	.683
	Activity: Exercise Correct Answer	1.181	.838	.833	1.410	.178
	Activity: Final Score	.009	.968	.004	.010	.992

CONCLUSION

In general, this study has found that the student's learning achievement is improved after using the Fraction App. The number of students' exercises, the number of the correct answers from the exercises, and the final score of the students' activity are correlated to their learning achievement. Moreover, it can predict the students' learning achievement.

To improve the results of this research, we suggest conducting a replication study that is carried out in face-to-face classes at schools and in the school environment. The results of research conducted during the COVID-19 pandemic were then compared with the results of research during the new normal.

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