Received: 12 June 2026
Accepted: 24 July 2026
Published: 31 July 2026

p-ISSN: 2716-1722
e-ISSN: 2715-8128

DIDAKTIKA

Jurnal Pendidikan Sekolah Dasar

Volume 9, Nomor 1, 64–78, 2026

Journal homepage: https://journal.uny.ac.id/index.php/didaktika

Uncovering Learning Obstacles in Elementary Students Fraction Understanding: A Multi-Dimensional Analysis for Didactical Design

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Abstract

Learning fractions in elementary school remains difficult for students because the concept is abstract and requires strong conceptual understanding. Although prior studies have examined epistemological, didactical, or ontogenic learning obstacles separately, few have triangulated multiple data sources to capture how these three dimensions interact, particularly within a Didactical Design Research (DDR) framework at the lower-grade elementary level, this gap in integrated, multi-source analysis is what the present study addresses. This study identifies and analyzes the three types of learning obstacle experienced by third-grade students in learning fractions at UPT SDN 060817 Medan and examines how these obstacles interact as a basis for designing DDR-based instruction. A qualitative descriptive approach was used with 15 third-grade students and one classroom teacher as subjects. Data was collected through a mathematical concept-understanding test, classroom observation, semi-structured interviews, and instructional-artifact analysis, then triangulated. Results show that only 33.33% of students reached mastery, with a class average of 53.2. Fifteen learning obstacles were identified: five epistemological, five didactical, and five ontogenic, and these interact with one another rather than occurring independently. Identifying these three obstacle categories carries direct practical significance for guiding teachers in designing more contextual, concrete-media- and technology-supported fraction instruction. The findings provide an empirical foundation for a Hypothetical Learning Trajectory (HLT) integrating concrete media, PhET Simulation, Problem-Based Learning, and local Medan contextualization.

Keywords: conceptual understanding, Didactical Design Research, fractions, learning obstacles.

Mengungkap Hambatan Belajar dalam Pemahaman Pecahan pada Siswa Sekolah Dasar: Analisis Multi-Dimensi untuk Desain Didaktis

Abstrak

Pembelajaran materi pecahan di sekolah dasar masih menjadi salah satu topik yang sulit dipahami siswa karena konsepnya bersifat abstrak dan memerlukan pemahaman konseptual yang baik. Penelitian terdahulu umumnya mengkaji hambatan epistemologis, didaktis, atau ontogenik secara terpisah, sementara penelitian yang mengintegrasikan ketiga dimensi learning obstacle tersebut secara simultan melalui triangulasi multi-instrumen pada jenjang kelas rendah sekolah dasar masih terbatas,learning obstacle yang dialami siswa kelas III pada materi pecahan di UPT SDN 060817 Medan, sekaligus mengkaji interaksi antar hambatan tersebut sebagai dasar penyusunan pembelajaran berbasis Didactical Design Research (DDR). Penelitian menggunakan pendekatan kualitatif deskriptif dengan subjek 15 siswa kelas III-A dan satu guru kelas. Data dikumpulkan melalui tes pemahaman konsep matematis, observasi pembelajaran, wawancara guru dan siswa, serta analisis buku ajar, kemudian dianalisis secara triangulasi. Hasil penelitian menunjukkan hanya 33,33% siswa mencapai ketuntasan belajar dengan nilai rata-rata kelas 53,2. Teridentifikasi 15 jenis learning obstacle yang terdiri atas lima hambatan epistemologis, lima hambatan didaktis, dan lima hambatan ontogenik, yang saling berinteraksi satu sama lain. Identifikasi ketiga kategori learning obstacle ini memiliki signifikansi praktis dalam memandu guru merancang pembelajaran pecahan yang lebih kontekstual serta didukung media konkret dan teknologi interaktif. Temuan ini menjadi dasar empiris bagi penyusunan Hypothetical Learning Trajectory (HLT) yang mengintegrasikan media konkret, PhET Simulation, Problem Based Learning dan kontekstualisasi lokal Medan.

Kata kunci: Didactical Design Research, learning obstacle, pecahan, pemahaman konsep matematis, sekolah dasar

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INTRODUCTION

Mathematics plays an important role in developing logical and systematic thinking skills, making its instruction in elementary school a crucial foundation for students' intellectual development (Yayuk, 2021). One topic considered both important and difficult for third-grade students to understand is fractions numbers that represent part of a whole and are written in the form $\frac{a}{b}$, where a is the numerator and b is the denominator (Ritawati et al., 2024). Their abstract nature makes this material difficult to understand without adequate instructional media support (Arsyad et al., 2024 & Yunita & Irawan, 2025)

International study results show that the mathematics ability of Indonesian students still needs improvement. Based on the Programme for International Student Assessment PISA (2022), Indonesia obtained an average mathematics score of 366, still below the OECD average. In addition, the results of the Trends in International Mathematics and Science Study (TIMSS) show that fractions are one of the topics most difficult for students to understand, particularly in solving word problems, comparing fractions, and performing fraction operations (Ina et al., 2019). In line with this, the results of the 2021–2025 Minimum Competency Assessment (AKM) for Numeracy Kemdikbudristek (2023), also show that the numeracy achievement of Indonesian students remains in a category requiring reinforcement, particularly in the ability to understand and apply mathematical concepts in various life contexts.

The mathematical understanding of Grade III students regarding fractions was still very low. The assessment involved 15 students who completed two diagnostic worksheets. The average score for Sheet 1 was 30.67, while the average score for Sheet 2 was even lower at 21.67, resulting in an overall average score of only 26.17. Individual scores also showed considerable variation, with several students obtaining 0 on Sheet 2 and only one student achieving a perfect score of 100, indicating substantial differences in students' prior knowledge. Overall, these findings reveal that most students had not yet mastered fundamental fraction concepts, including identifying fractions, interpreting the roles of the numerator and denominator, and solving fraction-related problems. Since the overall average was far below the school's Learning Objective Achievement Criteria (KKTP/KKM) of 75, the results indicate significant learning difficulties that require immediate instructional intervention through more meaningful, contextual, and student-centered learning approaches.

The low mathematics achievement reflected in PISA, TIMSS, and AKM is not only evident at the national level but is also apparent in the local context at UPT SDN 060817 Medan. This similarity in the pattern of problems indicates that learning obstacles in fraction material are a real issue also experienced by Grade III students at the research site. Therefore, identifying learning obstacles is important as a basis for designing instruction that is more contextual, appropriate to students' characteristics, and capable of meaningfully improving students' understanding of the fraction concept. *Learning obstacle* refers to a learning difficulty experienced by students of varying abilities, not only low-achieving students but also high-achieving ones (Fitriana & Maarif, 2022; Munir, 2023; Oviyanti et al., 2024). In line with this, Dasilvawati (2022) states that a learning obstacle is a situation in which students face difficulty understanding subject matter or solving problems. According to Brousseau (2002), learning obstacles are classified into three types based on their source: (1) ontogenic obstacle, (2) epistemological obstacle, and (3) didactical obstacle.

According to Brousseau (2002), learning obstacles are classified as ontogenic obstacles (limitations in cognitive readiness), epistemological obstacles (knowledge limited to a specific context and therefore difficult to generalize), and didactical obstacles (arising from the teacher's methods, approaches, or teaching materials). According to Piaget's theory (Aini et al, 2025; Pitadjeng, 2021 & Rahman, 2025), elementary school students aged 6–12 are at the concrete operational stage and therefore require the manipulation of real objects to understand abstract concepts such as fractions.

Previous research has extensively examined learning difficulties with fractions, but most studies have highlighted a single dimension of obstacle in isolation. Hariyani et al., (2022) focused on epistemological obstacles without examining ontogenic obstacles in depth. Wally et.al., (2024), highlighted ontogenic obstacles in the form of whole-number scheme interference without linking them to didactical factors, while (Atiaturrahmaniah et al., 2021), emphasized didactical factors in the form of a lack of concrete media without involving an analysis of textbook artifacts. The three dimensions of learning obstacles according to Brousseau (2002), epistemological, didactical, and ontogenic, have thus rarely been analyzed simultaneously through multi-instrument triangulation, particularly among lower-grade elementary students and within a Didactical Design Research (DDR) framework that links obstacle findings to instructional design. This study fills that gap by analyzing the three types of learning obstacle in an integrated manner among Grade III students at UPT SDN 060817 Medan, while also examining how the three interact as a basis for designing didactical instruction. Based on the above, this study aims to identify and analyze the three types of learning obstacle among Grade III students at UPT SDN 060817 Medan in learning fractions, as an empirical basis for developing a DDR based Hypothetical Learning Trajectory (HLT).

METHODS

This study used a qualitative approach with a descriptive method. Descriptive qualitative research aims to explore and interpret a particular phenomenon in depth within its natural context Sugiyono (2021). In line with this, Arikunto (2019) states that descriptive research aims to describe a situation, event, or phenomenon as it is. This method was chosen because it aligns with the research objective, namely, to identify and describe the types of learning obstacle experienced by students in learning fraction material naturally and contextually.

This study was conducted at UPT SDN 060817 Medan, located on Jalan Sakti Lubis Gg Bengkel No. 19, Siti Rejo I, Medan City, North Sumatra, during the second semester of the 2025/2026 academic year (January–March 2026). The research subjects consisted of 15 Grade III-A students (9 female, 6 male) and one classroom teacher. All students were involved using a total sampling technique because the class population size was relatively small (fewer than 30 students), allowing data to be obtained from the entire population without the risk of selection bias (Sugiyono, 2021). For in-depth data collection, four students were selected through purposive sampling based on the results of the concept-understanding test, to represent four ability categories (very low, low, moderate, very good), considering that a comparison across ability levels was needed to ensure that the identified learning obstacles were not experienced only by low-achieving students but were more broadly shared.

Data collection was carried out using four complementary instruments: (1) a mathematical concept-understanding test consisting of 20 items (10 multiple-choice, 10 essay), developed based on six indicators synthesized from several experts. Based on a synthesis of indicators of mathematical concept understanding according to (Zulkarnaen et al., 2022 ; Rahayu et al., 2023 & Septia et al., 2022) these indicators consist of restating a concept, presenting various representations, classifying examples and non-examples, applying concepts to problem-solving, identifying necessary and sufficient conditions, and explaining a concept systematically; (2) teacher and student observation sheets; (3) semi-structured interviews with the teacher and students; and (4) artifact studies in the form of an analysis of the textbook and teaching module.

To obtain a comprehensive understanding of students' learning obstacles in fraction material, the researcher used a data triangulation technique. According to (Faustyna, 2023 & Nurfajriani et al., 2024), data triangulation is an approach used in research to increase the reliability and validity of findings by combining various data sources, methods, and theories.

Triangulation was carried out by comparing the patterns of findings from the four instruments; an obstacle was established as a learning obstacle only if it was supported by a consistent pattern across at least two different data sources. Classification followed Brousseau's (2002) criteria: an epistemological obstacle was established when the difficulty originated from the student's own limited conceptual understanding; a didactical obstacle was established when the difficulty originated from the learning process, media, or teaching materials; and an ontogenic obstacle was established when the difficulty was influenced by the student's cognitive, affective, or psychomotor readiness.

The concept-understanding test instrument used in this study. Before being used in the research, the mathematical concept-understanding test instrument was first validated through a content validity test. The validation process involved one validator, a lecturer from the Mathematics Education Study Program with expertise in mathematics instruction. Table 1 below presents the results of the validator's validation test.

Table 1. Results of the Validation Test of the Mathematical Concept-Understanding Test

No.	Indicator	Indicator Statement	Score (First)				Score (Second)			
			4	3	2	1	4	3	2	1
1	Alignment of items with the indicator	Item is aligned with the indicator of mathematical concept understanding			✓				✓	
2		Item measures the intended ability		✓				✓		

No.	Indicator	Indicator Statement	Score (First)				Score (Second)			
			4	3	2	1	4	3	2	1
3		Item is aligned with fraction material		✓				✓		
4	Clarity of item language	Item language is easily understood by students			✓				✓	
5		Item sentence does not create ambiguous meaning		✓				✓		
6		Mathematical terms are used appropriately		✓				✓		
7	Level of item difficulty	Difficulty level matches students' ability			✓				✓	
8		Item is neither too easy nor too difficult		✓				✓		
9		Item is appropriate to the grade level		✓				✓		
10		Item has a variation in difficulty level (easy, medium, difficult)		✓				✓		
Total Item Score			**27**				**37**			
Total Maximum Item Score			**40**							

Based on Table 1, the instrument validation was carried out in two stages by one expert lecturer in Mathematics Education using a content validity test, with assessment aspects covering the alignment of items with the indicators of mathematical concept understanding, the alignment of the material with the learning objectives, the accuracy of item construction, the use of language appropriate to the characteristics of elementary school students, and the clarity of the working instructions. In the first stage, the instrument obtained a validity percentage of 67.5%, categorized as valid, along with several suggestions for improvement, including clarifying the wording of items to avoid ambiguous interpretation, simplifying the language to match the cognitive developmental level of Grade III elementary students, clarifying the working instructions, and adjusting the illustrations and context of the items to students' everyday lives. After revisions based on this feedback, the instrument was resubmitted for a second stage of validation and obtained an increased validity percentage of 92.5%, categorized as very valid. This increase indicates that the instrument met the aspects of content, construction, and language, and was therefore deemed suitable for use as a research instrument to identify students' mathematical concept-understanding ability and to analyze learning obstacles in fraction material.

RESULTS AND DISCUSSION

Results

The research results are presented based on the findings from the four instruments used, covering teacher observation, the mathematical concept-understanding test, student observation, teacher interviews, student interviews, and analysis of the textbook and the teacher's teaching materials. These findings were then analyzed to identify the three types of learning obstacle.

Teacher Observation Results

Observation of the teacher was conducted over two sessions, on 13 and 24 January 2026, in Class III-A of UPT SDN 060817 Medan. Based on the results of the two observation sessions, it can be concluded that instruction on fractions in Class III-A was dominated by the lecture method without the support of concrete media or interactive technology. The absence of group discussion activities, the non-use of student worksheets, and weak scaffolding in explaining the concept of the numerator and denominator represent instructional conditions with strong potential to create and reinforce learning obstacles. These observation findings directly identify the presence of didactical learning obstacles, particularly LOD-1 (dominance of lecturing), LOD-2 (absence of concrete media and worksheets), and LOD-5 (lack of variation in representation).

Student Observation Results

Observation of the 15 Grade III-A students was conducted alongside the teacher observation from 13–31 January 2026, focusing on students' activity, responses, and engagement during instruction and while working on the fraction concept-understanding test, using an instrument with 10 indicators covering aspects of concept delivery, representation, fraction comparison, arithmetic operations, and learning responses. The observation results show very large variation in achievement among students, ranging from student SH with the highest achievement (80%), five students in the 60–70% range, to four students with the lowest achievement (10%), indicating heterogeneity in conceptual understanding within a single class. Despite this variation, three learning obstacles were identified that were consistently experienced by nearly all students: difficulty comparing two fractions using the symbols >, <, or = (including by the highest-achieving student); difficulty performing addition and subtraction operations on fractions, particularly in word problems, with an error pattern of adding or subtracting the numerators and denominators simultaneously without understanding the structure of a fraction; and the inability of 93.33% of students (14 of 15) to represent fractions in various forms, reflecting a limited mental schema resulting from the lack of variation in representation during instruction. Overall, these findings confirm a significant and widespread learning obstacle across nearly all levels of student ability, indicating that the root of the problem does not lie solely in students' individual capacity but in an instructional design that has not yet succeeded in building meaningful conceptual understanding.

Results of the Mathematical Concept-Understanding Test

The concept-understanding test was administered to 15 Grade III students. Based on the test results, only 5 students (33.33%) achieved scores above the minimum mastery criterion (KKM ≥ 65), while 10 students (66.67%) had not yet reached the KKM. The average score was 53.2. The distribution of test result categories is presented in Table 2.

Table 2. Recapitulation of Students' Mathematical Concept-Understanding Categories

No.	Category	Score Range	Number of Students	Percentage
1	Very Good	85–100	1	6.67%
2	Good	70–84	2	13.33%
3	Fair	55–69	5	33.33%
4	Poor	40–54	3	20.00%
5	Very Poor	< 40	4	26.67%
Total			**15**	**100%**

Source: Research Data Analysis Results

The data in Table 2 show that most students (46.67%) fall into the Poor and Very Poor categories. This condition indicates a significant learning obstacle in mastering the fraction concept. Students experienced the most difficulty in presenting fractions in various pictorial representations, comparing fractions, and applying the concept to solving everyday problems.

Teacher Interview Results

A semi-structured interview with the homeroom teacher of Class III-A was conducted on Saturday, 31 January 2026, in the prayer room (Mushola) of UPT SDN 060817 Medan for approximately 10 minutes, covering six main aspects: students' level of understanding, the most difficult part of the material, error patterns, teaching methods and media, obstacles faced by the teacher, and recommendations for improvement. The interview results revealed that students' understanding of fraction material was good but uneven.

> *"Students' level of understanding of fraction material could be said to be fairly good, but it is not even, because there are some students..." (Classroom Teacher)*

This is in line with the concept-understanding test data showing that only 5 of 15 students (33.33%) reached the KKM, with a class average score of 53.2. The teacher identified that students' main difficulty lay in conceptual understanding of the numerator and denominator, rather than merely computational procedure, so that students tended to memorize solution steps without understanding the underlying logic.

> *"Then they often make mistakes in determining the numerator and denominator... students also often make mistakes in interpreting the meaning of word problems." (Classroom Teacher)*

The most common errors were found in interpreting word problems, which require adequate literacy skills to understand the narrative context before determining the appropriate fraction operation, while the still-developing literacy skills of Grade III students posed a particular challenge in this process. Regarding the instructional approach, the teacher acknowledged that teaching of fraction material had not yet involved concrete media or special teaching aids and still relied on the lecture method, which the teacher considered less effective because it made it harder for students to imagine the abstract concept of fractions.

> *"I also haven't used any special media in teaching fractions. Without using media, I think the method I use is still not very effective... especially without the media help, students also find it harder to imagine the concepts of fractions. The use of concrete media needs to be increased concrete media or technology so that students' interest can also increase." (Classroom Teacher)*

This finding is consistent with the researcher's observation results, in which the teacher's own acknowledgment also identified two main obstacles in instruction: the gap in ability among students, which made it difficult to provide explanations evenly, and the limited instructional time allocation (approximately 30 minutes per session), considered insufficient to ensure understanding among all students. As a recommendation, the teacher emphasized the need to increase the use of concrete media and instructional technology to increase students' interest and active participation.

Student Interview Results

Student interviews were conducted semi-structurally on 31 January 2026 with four students. The purposive selection of four students based on the concept-understanding test results aimed to obtain a comprehensive picture of the learning experiences and obstacles faced across the full spectrum of student ability in the class. Student AI (score 18, very poor category) showed the most fundamental confusion, namely an inability to distinguish fractions from whole numbers, as she stated, "I don't know what a fraction is, Miss. I thought it was just an ordinary number. I've never learned to draw fraction pictures like that before," and met only 1 of the 10 observation indicators (10%). Student AS (score 48, poor category) showed a misconception in reading fraction symbols (reading 2/5 as "two-five") and experienced significant difficulty with word problems, as she stated, "All the questions are difficult, Miss. The hardest are the word problems I don't know where to start," indicating a mathematical literacy obstacle in identifying contextual information and converting it into a mathematical operation. Student SANA (score 66, fair category) was already able to read and write fraction symbols but still struggled with visual representation and word problems, "The word problems are hard, Miss. The numbers make me confused. I don't know what to work on first," while also expressing a hope for learning media based on animated pictures or computer games. Student SH (score 86, very good category) had mastered addition of fractions with the same denominator but still struggled with word problems and comparing fractions, and suggested the use of concrete objects in instruction, "Learning about fractions would be easier if the teacher gave examples using everyday objects like cake, pizza, or martabak that are divided into pieces."

Overall, the interview results with the four students confirm the observation and concept-understanding test findings that the main learning obstacles lie in the ability to visualize fractions, interpret word problems, and connect the fraction concept with real-life contexts. A significant finding is that these obstacles are systemic and found across the entire spectrum of ability, from the lowest-achieving student (AI) to the highest-achieving (SH), confirming that this issue is not caused solely by low individual ability. The shared desire among all students to learn through more concrete, visual, and interactive media strengthens the argument for the need for a technology-based instructional design, such as PhET Simulation, as the solution proposed in this study.

Analysis of the Textbook and Learning Materials

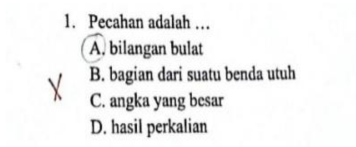
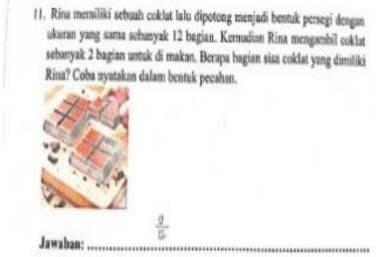
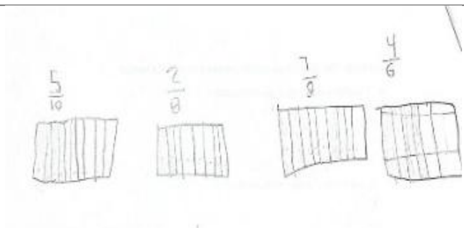
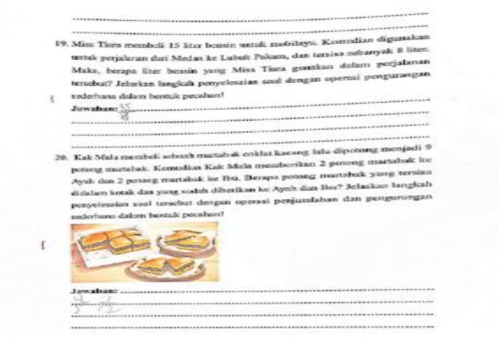
Analysis of the textbook (Fractions chapter, adapted from the Gakko Tosho curriculum) and the teacher's teaching module revealed three main weaknesses. First, the visual representation of fractions throughout the material relies only on the area model and the number line, without circle models, sets of objects, or paper folding, which would be more intuitive for children. Second, the context of the textbook does not match the everyday life of Medan students, for example the use of the deciliter unit and single character names that do not reflect local diversity. Third, the sequence of concepts moves too quickly toward decimal numbers, appearing on the seventh of ten pages of material, before understanding of common fractions is reinforced. The teacher's teaching module had designed concrete activities such as paper folding, but in practice the teacher relied more on the lecture method, showing a gap between planning and instructional practice.

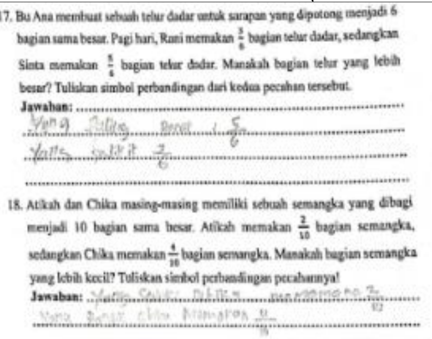
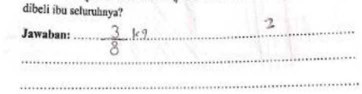
Methods should be clearly describing research design and providing clear procedures that were performed. In other word, methods should be answers of question "How did you do it?". The first paragraph is usually used to describe the study design and procedures in detail. List of methods in the same order when they will appear in the results, in a logical order in which you did the research.

Summary of Learning Obstacles

Based on the triangulation of data from the four research instruments described above, 15 types of learning obstacle were identified in the instruction of fraction material in Class III-A of UPT SDN 060817 Medan. These fifteen obstacles are distributed into three categories based on their source according to Brousseau (2002) epistemological, didactical, and ontogenic, each consisting of five learning obstacles. These are summarized by the researcher in Table 3.

Table 3. Summary of Learning Obstacles

Code	Category	Type of Obstacle	Brief Evidence & Data Source
LOE-1	Epistemological	Misconception of the fraction concept (regarded as a whole number / two separate numbers)	 1. Pecahan adalah ... A. bilangan bulat B. bagian dari suatu benda utuh C. angka yang besar D. hasil perkalian Question no. 1: most students answered "whole numbers." Interview: students did not know the difference between fractions and whole numbers. *"Because I learned about integers, I thought fractions were integers" (Student AS)*
LOE-2	Epistemological	Confusion between the function of numerator and denominator	 11. Rina memiliki sebuah coklat lalu dipotong menjadi bentuk persegi dengan ukuran yang sama sebanyak 12 bagian. Kemudian Rina mengambil coklat sebanyak 2 bagian untuk di makan. Berapa bagian sisa coklat yang dimiliki Rina? Coba nyatakan dalam bentuk pecahan. Jawaban: ... Question no. 11: many students reversed the numerator and denominator. Observation: some students wrote fractions incorrectly. *"Because there are two numbers, '5 and 2'. So I read it as five two" (Student AI)*
LOE-3	Epistemological	Limited visual representation (only squares/circles)	 93.33% of students could not represent fractions in question no. 14. Interview: students admitted they only knew how to draw circles and squares. *"Because it's easier, Miss, and I only learn circles and squares." (Student AS)* *"Draw squares and circles" (Student AI)*
LOE-4	Epistemological	Difficulty interpreting word problems	 19. Miss Tiara membeli 15 liter bensin untuk mobilnya. Kemudian digunakan untuk perjalanan dari Medan ke Lubuk Pakam, dan tersisa sebanyak 8 liter. Maka, berapa liter bensin yang Miss Tiara gunakan dalam perjalanan tersebut? Jelaskan langkah penyelesaian soal dengan operasi pengurangan sederhana dalam bentuk pecahan! Jawaban: 15/8 20. Kak Mala membeli sebuah martabak coklat kacang lalu dipotong menjadi 9 potong martabak. Kemudian Kak Mala memberikan 2 potong martabak ke Ayah dan 2 potong martabak ke Ibu. Berapa potong martabak yang tersisa didalam kotak dan yang sudah diberikan ke Ayah dan Ibu? Jelaskan langkah penyelesaian soal tersebut dengan operasi penjumlahan dan pengurangan sederhana dalam bentuk pecahan! Jawaban: ...

Code	Category	Type of Obstacle	Brief Evidence & Data Source
			66.67–73.33% of students scored 0 on questions no. 17–20. Interview: students stated they did not understand the meaning of the problems. *"All the questions are difficult, Miss. The most difficult one is the story question, I don't know where to start" (Student AS)*
LOE-5	Epistemological	Difficulty comparing fractions with different denominators	 Questions no. 17 and 18 were often unanswered. Observation: students appeared confused in determining which fraction is larger.
LOD-1	Didactical	Instruction dominated by the lecture method (teacher-centered)	Two observation sessions: the entire process was dominated by teacher lectures; no group activities or independent student exploration.
LOD-2	Didactical	Lack of concrete media, technology, and worksheets (LKPD)	Observation: only the board was used. Teacher interview: has never used technology-based media. *"I haven't used any special media to teach fractions. I usually explain the material directly to the students in class III-A... I don't think my method is very effective without media... especially since without the help of media, it's more difficult for students to understand the concepts of fractions." (Class Teacher)*
LOD-3	Didactical	Absence of local Medan contextualization in the textbook	Textbook analysis: use of deciliters, a 1-meter ribbon context, and character names that do not reflect the local Medan context.
LOD-4	Didactical	Concept sequence moves too quickly toward decimals	Textbook analysis: transition to decimals begins on page 7 of 10 pages without strengthening fraction representation.
LOD-5	Didactical	Lack of variation in representation in the textbook and teaching	Observation and textbook analysis: the same representation is repeated without variation, leading students to develop only one mental schema of fractions.
LOO-1	Ontogenic	Limited capacity for abstraction (concrete operational stage)	Observation: many students did not understand abstract explanations. Interview: students stated they needed to "see directly." *"Hmmm... using real media, Miss." (Student SH)*
LOO-2	Ontogenic	Procedural error in adding fractions	 Test result question no. 12: some students answered 1/4 + 2/4 = 3/8 (both numerator and denominator were added).
LOO-3	Ontogenic	Whole-number scheme interference in fractions	Test results and observation: several students made reversed errors when comparing fractions with different denominators.
LOO-4	Ontogenic	Low confidence in asking questions or expressing opinions	Two observation sessions: the teacher gave opportunities to ask questions, but no students asked. Interview: students admitted feeling afraid or shy.
LOO-5	Ontogenic	Limited literacy in understanding word problems	Essay questions no. 11–13 and 17–20 received the lowest scores. Interview: students did not understand "what is being asked" in the problems.

Note: Code LOE = Epistemological Learning Obstacle; Code LOD = Didactical Learning Obstacle; Code LOO = Ontogenic Learning Obstacle.

These three categories of learning obstacles do not stand alone but interact with and reinforce one another. Didactical learning obstacles, in the form of a lack of variation in representation and use of concrete media, directly contribute to the formation of epistemological learning obstacles in the form of limited visual representation. At the same time, students' lack of cognitive developmental readiness, which constitutes an ontogenic learning obstacle, makes an abstract instructional approach increasingly ineffective.

Discussion

The findings reveal that learning obstacles related to fraction material among third-grade students are multidimensional in nature, encompassing three principal dimensions: epistemological, didactical, and ontogenic obstacles.

Epistemological Learning Obstacles

The most prominent epistemological learning obstacle identified was students' misconception of the meaning of fractions coupled with limited visual representation ability. This finding is consistent with Hariyani et al. (2022), who reported that elementary school students tend to understand fractions merely as "a part of one object" without grasping the concepts of equivalence and comparison. According to Laird (as cited in Saputri, 2025), difficulty comparing fractions with unlike denominators and linking fractions to visual representations constitute common epistemological obstacles among elementary students. This is further corroborated by Van Hoof et al. (2022), who found that many elementary students perceive a fraction with a larger denominator as having a greater value, a misconception rooted in limited understanding of fraction visual representation.

Within this epistemological obstacle, most students were found to be confused in distinguishing the functions of the numerator and denominator. This condition arises because students' understanding of fractions remains limited to a symbol consisting of two stacked numbers, rather than as a representation of a part of a whole. During instruction, students tended to memorize the positions of the numerator and denominator rather than understand the meaning of each component through concrete experience. Consequently, when presented with images or situations that differed from the examples given by the teacher, students frequently confused the roles of the numerator and denominator.

The misconception regarding the numerator and denominator is not merely a failure to remember mathematical terminology but reflects students' incomplete conceptualization of fractions as multiplicative relationships rather than isolated symbols. When instruction emphasizes symbolic notation before students experience part-whole relationships through concrete representations (Afridayanti et al., 2022; Rakhmawati et al., 2023), students tend to construct procedural rather than conceptual knowledge. Consequently, the numerator and denominator are perceived as two independent whole numbers instead of interdependent quantities describing a single fractional unit.

Didactical Learning Obstacles

The didactical learning obstacles identified centered on a teacher-centered instructional approach and minimal use of learning media. This condition aligns with Atiaturrahmaniah et al. (2021), who reported that many elementary students struggle to understand fractions because teachers make insufficient use of supporting media. Setyaningrum et al. (2023) similarly emphasized that fraction instruction relying solely on lecture-based methods without interactive media leaves students unable to visualize fraction concepts. The absence of local contextualization in the textbooks examined in this study reinforces Brousseau's (2002) argument that instructional designs misaligned with students' context can become a significant source of learning obstacles. Consistent with this, Herawati et al. (2026) found that didactical obstacles in fraction instruction are closely linked to students' low learning readiness and limited conceptual understanding, rather than instructional quality alone.

Classroom observations further indicated that instruction remained dominated by lecture-based delivery and abstract explanation on the whiteboard, without the use of concrete or technology-based media. This condition made it difficult for students to connect fraction symbols with their actual meaning as a representation of part of a whole. The findings indicate that the didactical obstacles in fraction learning were primarily caused by instructional practices that emphasized procedural teaching rather than conceptual understanding. Classroom observations showed that instruction was dominated by teacher explanations and routine exercises, with limited opportunities for students to explore fraction concepts through concrete materials, visual representations, or collaborative discussion.

From the perspective of the Theory of Didactical Situations, such learning environments restrict students' opportunities to construct mathematical meaning through active reasoning and cognitive conflict, resulting in knowledge that is memorized rather than understood (Sidik et al., 2021). This condition was further exacerbated by the premature introduction of symbolic fraction notation without sufficient concrete and pictorial experiences, contrary to Bruner's progression from inactive to iconic and symbolic representations. Consequently, students struggled to develop meaningful fraction concepts and instead relied on procedural rules and whole-number reasoning, highlighting that the didactical obstacle stemmed not only from the absence of instructional media but also from an instructional design that was insufficiently aligned with students' cognitive development and conceptual learning needs.

Ontogenic Learning Obstacles

The most dominant ontogenic learning obstacle was students' limited capacity for abstraction combined with interference from whole-number schemas. This finding aligns with Piaget's theory (Aini et al., 2025; Pitadjeng, 2021 & Rahman, 2025), which posits that third-grade students remain at the concrete operational stage, meaning their understanding of abstract concepts such as fractions requires the support of concrete objects. According to Vessonen et al. (2021), children aged 7–8 years are in the intuitive fraction understanding phase and still require manipulation of concrete objects to build fraction comprehension. The whole-number schema interference identified in this study is also consistent with Wally et al. (2024), who demonstrated that inappropriate knowledge transfer from whole numbers to fractions is a common source of conceptual error. Research by Siegler et al. (as cited in Vamvakoussi et al., 2012) likewise indicates that students' tendency to apply whole-number logic to fractions constitutes a persistent ontogenic obstacle that only declines significantly between grades 3 and 4 of elementary school (Ni & Zhou, 2025).

These difficulties are further shaped by the cognitive developmental characteristics of third-grade students, who remain at Piaget's concrete operational stage (Aini et al., 2025 & Pitadjeng, 2021). At this stage, students grasp concepts more readily when presented through concrete objects, manipulatives, or visual representations. In addition, this study identified interference from the whole-number schema, namely students' tendency to apply whole-number reasoning when solving fraction problems. For instance, students judged $\frac{1}{4}$ to be greater than $\frac{1}{2}$ because 4 is numerically larger than 2. This error stems from students' prior learning experiences, which focused predominantly on whole-number concepts, causing that schema to be carried over into their learning of fractions. Students had not yet grasped that, in fractions, a larger denominator corresponds to a smaller unit size when the numerator remains constant. Consequently, students' pre-existing understanding of whole numbers actively impeded the formation of a correct fraction concept.

Difficulties in solving word problems were also attributable to students' limited ability to connect everyday contexts with mathematical representations. Interview data revealed that some students failed to grasp key information in the problem, making it difficult for them to determine the appropriate fraction operation. This finding suggests that the obstacles students experienced were not solely rooted in mathematical concepts but were also influenced by their still-developing reading literacy skills. One of the most important findings of this study is that epistemological, didactical, and ontogenic learning obstacles are not isolated phenomena but interact dynamically throughout students' fraction learning.

The findings suggest that didactical obstacles, characterized by teacher-centered instruction and limited use of concrete representations, restricted students' opportunities to construct meaningful conceptual understanding. As a result, students developed epistemological obstacles, including misconceptions about fractions and limited visual representation skills, which were further strengthened by ontogenic obstacles, particularly their tendency to apply whole-number reasoning to fraction concepts. These findings indicate that weaknesses in one dimension can trigger difficulties in others, creating a cycle that hinders conceptual development. This interaction extends Brousseau's Learning Obstacles framework by demonstrating that learning obstacles should be understood as interconnected rather than independent dimensions. Therefore, improving students' understanding of fractions requires instructional interventions that simultaneously address instructional design, conceptual knowledge construction, and students' cognitive developmental readiness, rather than focusing on a single type of obstacle.

Interaction Among the Three Types of Learning Obstacles

Overall, the three types of learning obstacles identified interact with and reinforce one another. Didactical obstacles, manifested as a lack of representational variety and minimal use of concrete media, contribute to the emergence of epistemological obstacles in the form of students limited visual representation ability. At the same time, students' cognitive developmental unreadiness (ontogenic obstacle) renders abstract instructional approaches increasingly ineffective. These findings underscore the need for instructional interventions that simultaneously address all three dimensions of learning obstacles. Taken together, the findings demonstrate that learning obstacles in fraction material are multidimensional, shaped by the interaction of epistemological, didactical, and ontogenic factors. Consequently, these obstacles cannot be resolved simply by providing more practice exercises; rather, they require a redesign of instruction that aligns with students' cognitive developmental characteristics.

Recommendations for the Application of Didactical Design Research (DDR)

Based on the identified learning obstacles, the application of Didactical Design Research (DDR) should be directed toward instructional designs that specifically target each obstacle identified. To address didactical obstacles, DDR-based instructional design needs to shift instruction from a teacher-centered approach to a more student-centered approach involving exploration, group discussion, and contextual problem-solving. The results of this obstacle identification directly inform the selection of the following DDR instructional strategies, with explicit links to the obstacle codes in Table 4.

Table 4. Linkage between Strategy and Learning Obstacle

Strategy	Learning Obstacle Addressed	Mechanism of Linkage
PhET Simulation	LOE-1, LOE-2, LOE-3, LOD-2, LOO-1	LOE-1: Students manipulate virtual shapes to directly understand the part-whole relationship, correcting the misconception that a fraction is a whole number or two unrelated digits. LOE-2: Real-time visualization between the shaded part and the total number of parts helps students understand the distinct functions of the numerator and denominator. LOE-3: The simulation provides various representations (bars, number lines, objects) beyond circles/squares, enriching students' visual understanding of fractions. LOD-2: The PhET Simulation serves as an interactive technological medium that fills the gap in concrete media, technology, and worksheets in learning. LOO-1: The visual and manipulative nature of the simulation aligns with the concrete-operational thinking stage of Grade III students, allowing them to "see and touch" fractions before moving on to abstract symbols.
Problem-Based Learning (PBL)	LOD-1, LOE-4, LOO-4, LOO-5	LOD-1: PBL restructures instruction around small-group investigation of authentic problems, shifting the teacher's role from lecturer to facilitator and fostering student centered learning. LOD-1: PBL restructures instruction around small-group investigation of authentic problems, shifting the teacher's role from lecturer to facilitator and fostering student-centered learning. LOE-4: Problems are collaboratively unpacked through guiding questions (what is known, what is asked, which strategy applies), providing students with a systematic routine for interpreting word problems. LOO-4: Small group work offers a lower-stakes environment than whole-class response, enabling students to voice ideas and pose questions among peers before presenting to the class, thereby building their confidence. LOO-5: Structured PBL scaffolding including guiding questions and teacher modeling of think-aloud reading guides students step by step in decoding problem text, directly developing the literacy skills needed to comprehend word-problem demands.

Strategy	Learning Obstacle Addressed	Mechanism of Linkage
Local Medan Contextualization	LOD-3, LOE-4	**LOD-3:** Replacing generic contexts in the textbook (e.g., deciliters, unfamiliar names and settings) with familiar Medan references directly resolves the textbook's absence of local contextualization. **LOE-4:** When a word problem uses objects, places, and unit's students already recognize from daily life, less effort is spent decoding an unfamiliar scenario, so students can focus on the underlying fraction relationship rather than the setting.
Concrete-Representational-Abstract (CRA) Sequence	LOD-4, LOO-2, LOO-3	**LOD-4:** The CRA sequence deliberately extends and reinforces the concrete and pictorial stages of fraction understanding before symbolic/decimal notation is introduced, preventing the premature jump to decimals identified in the textbook. **LOO-2:** Moving from concrete manipulation to representational drawing before abstract symbols helps students see why, when parts share a common whole, only the numerators combine, correcting the mechanical error of adding both numerator and denominator. **LOO-3:** Anchoring instruction in physical and pictorial part-whole models helps students distinguish fraction reasoning from whole-number counting, reducing the intrusion of whole-number schemes into fraction comparison and computation tasks.

These learning obstacle findings serve as the primary foundation for designing a Hypothetical Learning Trajectory (HLT) and a didactical design that integrates PhET Simulation with the Problem-Based Learning (PBL) model, local Medan contextualization, and the principle of progressing from concrete to abstract instruction. This concrete-to-abstract principle is supported by Zhang et al., (2022), who demonstrated that the Concrete-Representational-Abstract (CRA) approach significantly improves elementary students' fraction comprehension, particularly among students experiencing mathematics learning obstacles. Accordingly, the didactical design developed in this study functions not only to identify learning obstacles but also as a systematically designed instructional solution based on the characteristics of the obstacles found among students. The effectiveness of PhET simulations in elementary instruction is further confirmed by Perkins et al., (2024), who found that students learning with PhET simulations achieved significantly higher conceptual understanding scores compared to control groups and were better able to articulate their reasoning in problem-solving tasks.

The findings of this study contribute to the theoretical development of Brousseau's Learning Obstacles framework by demonstrating that epistemological, didactical, and ontogenic obstacles should not be interpreted as isolated constructs. Instead, the present findings suggest that these dimensions interact recursively, creating cumulative barriers to conceptual learning. While previous studies have typically focused on one type of obstacle in isolation, this study shows that misconceptions about fractions originate from the simultaneous influence of instructional design, students' cognitive development, and conceptual knowledge. This integrated perspective extends the Learning Obstacles framework by emphasizing that meaningful conceptual change requires coordinated interventions across all three dimensions rather than isolated remediation of students' conceptual errors.

From a practical perspective, the findings indicate that improving fraction learning requires more than increasing the number of practice exercises or procedural explanations. Instruction should prioritize conceptual understanding through multiple representations, contextualized learning experiences, and active student participation. Learning environments should incorporate concrete manipulatives, interactive technologies such as PhET simulations, and locally meaningful contexts to facilitate conceptual construction before introducing symbolic procedures. Furthermore, instructional sequences should consistently follow the Concrete–Representational–Abstract (CRA) progression, enabling students to gradually develop robust fraction schemas that reduce whole-number interference and support long-term mathematical understanding. These findings provide a strong empirical foundation for the development of the DDR-based instructional design proposed in this study.

CONCLUSION

This study revealed that learning obstacles in fraction learning among third-grade students at UPT SDN 060817 Medan are multidimensional, comprising 15 types of obstacles: five epistemological, five didactical, and five ontogenetic obstacles that interact and reinforce one another. *e*pistemological learning obstacles (fraction misconceptions, limited visual representation, and difficulty interpreting word problems), didactical learning obstacles (dominance of lecturing, limited media variety, and lack of local contextualization), and ontogenic learning obstacles (limited abstraction ability, whole-number schema interference, and low student confidence). These findings extend Brousseau's (2002) Learning Obstacles framework by demonstrating that the three dimensions operate as an interconnected system rather than as isolated factors in elementary fraction learning. Furthermore, the findings provide an empirical basis for developing a Hypothetical Learning Trajectory (HLT) and a Didactical Design Research (DDR)-based instructional design integrating concrete manipulatives, PhET simulations, and Problem Based Learning (PBL) to support students' conceptual understanding of fractions.

From a practical perspective, the findings suggest that addressing students' learning obstacles in fractions requires instructional designs that simultaneously consider conceptual understanding, teaching practices, and students' cognitive readiness, rather than relying solely on additional practice exercises. Although this study was conducted in a single elementary school, its findings may inform fraction instruction and learning design in other primary schools with similar instructional characteristics, particularly those facing comparable challenges in students' conceptual understanding and the use of learning media.

REFERENCES

Afridayanti, D., Rakhmawati, Y., & Afridayanti, D. (2022). Improving the Understanding of Addition and Subtraction of Fractions Through Cooperative Model of Rotating Trio Exchange (RTE) Type for Students Elementary School. *Journal of Learning Improvement and Lesson Study*, *2*(2), 27–32. https://doi.org/10.24036/jlils.v2i2.33

Aini, I. Q., Alim, R. F., & Firmansyah, Y. (2025). Implementation of Jean Piaget's cognitive Development theory in Science Learning in Elementary Schools and its Relevance to SDGs 4. *Journal on Smart Learning Technologies.*, *1*(2).

Alzanatul Umam M., Zulkarnaen, R. (2022). *Kemampuan Konsep Matematis*. Penerbit UNMA.

Arikunto, S. (2019). *Prosedur Penelitian: Suatu Pendekatan Praktik (Revisi)*. Rineka Cipta.

Arsyad, A., Rahman, M., & Putri, D. (2024). The Efficiency of Using Visual Learning Media in Improving The Understanding of Science Concepts in Elementary School Students. *Journal of Science Education*, *12*(45–58).

Atiaturrahmaniah, Kudsiah, M., & Ulfa, E. M. (2021). Analisis Faktor Penyebab Kesulitan Belajar Matematika Materi Pecahan Siswa Kelas IV SDN Sukaraja. *Jurnal DIDIKA: Wahana Ilmiah Pendidikan Dasar*, *7*(2), 268–278. https://doi.org/10.29408/didika.v7i2.4657

Brousseau. (2002). *Theory of Didactical Situations in Mathematics*. Springer. https://link.springer.com/book/10.1007/0-306-47211-2

Dasilvawati, P. (2022). *Desain Didaktis Bangun Ruang Sisi Datar Berbasis Teori Van Hiele Dengan Media Geogebra*. Universitas Pendidikan Indonesia.

Faustyna. (2023). *Metode Penelitian Kualitatif Komunikasi (Teori dan Praktek)*. umsu press.

Fitriana, A., & Maarif, S. (2022). Analysis of Learning Obstacle to Numeracy Ability Based on Mathematical Resilience of Elementary Students. *(Jiml) Journal of Innovative Mathematics Learning*, *5*(2), 75–84. https://doi.org/10.22460/jiml.v5i2.11202

Hariyani, M., Herman, T., Suryadi, D., & Prabawanto, S. (2022). Mengembangkan Desain Didaktis Berdasarkan Hambatan Belajar dan Learning Trajectory Siswa pada Konsep Dasar Pecahan di Sekolah Dasar. *Jurnal Riset Pedagogik*, *6*(2), 416–425.

Ina V.S , Mulis , Michael. O, Martin , Pierre Foy, Dana .L, Kelly, B. F. (2019). *Lynch School of Education and Human Development Boston College*. TIMSS & PIRLS International Study Center.

Kemdikbudristek. (2023). *PISA 2022 dan Pemulihan Pembelajaran Di Indonesia*. Laporan PISA Kemdikbudristek. https://lpmpdki.kemdikbud.go.id/pisa-2022-dan pemulihan-pembelajaran-di-Indonesia

Munir. (2023). *Kajian Pedagogik Pendidikan Ilmu Komputer*. Penerbit Indonesia Emas Group.

Ni, Y., & Zhou, Y. D. (2025). Teaching and Learning Fraction and Rational Numbers: The Origins and Implications of Whole Number Bias. *Educational Psychologist*, *40*(1), 27–52. https://doi.org/10.1207/s15326985ep4001_3

Nurfajriani, W. V., Ilhami, M. W., Mahendra, A., Afgani, M. W., & Sirodj, R. A. (2024). Triangulasi Data dalam Analisis Data Kualitatif. *Jurnal Ilmiah Wahana Pendidikan*, *10*(17), 826–833. https://doi.org/10.5281/zenodo.13929272

Oviyanti, F., Maryamah, M., Sartika, A., Sanjaya, C. R., & Supriyadi, S. (2024). Analysis of Student Learning Difficulties in Islamic Education Subject at Elementary School. *EDUTEC : Journal of Education And Technology*, *7*(3), 1–5. https://doi.org/10.29062/edu.v7i3.851

Perkins, K., McKagan, S., & Moore, E. (2024). Transforming Science Education in Elementary Schools: The Power of PhET Simulations in Enhancing Student Learning. *Multimodal Technologies and Interaction*, *8*(11), 105. https://doi.org/10.3390/mti8110105

PISA. (2022). *The State Of Learning And Equity In Education*. OECD Publishing Paris. https://doi.org/10.1787/53f23881-en

Pitadjeng. (2021). *Pembelajaran Matematika Yang Menyenangkan*. Graha Ilmu.

Rakhmawati, Y., Retnawati, H., Sugiman, S., Kassymova, G. K., & Kudyarova, G. (2023). *Exploring pre-service teacher's knowledge of content and teaching (KCT) and computational thinking: The case of addition and subtraction*. 151–159. https://doi.org/10.31643/2023.18

Rahayu, Y., Pujiastuti, H. (2023). *Kemampuan Pemahaman Matematis*. Penerbit UNPAS.

Rahman, S. M. R. (2025). From Concrete to Abstract: Piaget's theory for Cognitive Development. *BRAC University Research Journal.*, 1–15.

Ritawati, Bernadeta; Liliana, Sepriana; Tupulu, N. (2024). *Materi Pecahan*. PT Nasya Expanding Management.

Septiadi, R., Wahidah, N. (2022). *Pemahaman Konsep*. Penerbit UNPAS.

Setyaningrum, Ermawati, R. (2023). Analisis Kesulitan Belajar Dalam Memahami Konsep Pecahan Pada Siswa Kelas V SD Negeri Sidomulyo. *Pendas : Jurnal Ilmiah Pendidikan Dasar*, *2*(1), 349–365. https://doi.org/10.23969/jp.v8i1.8270

Sidik, G. S., Suryadi, D., & Turmudi. (2021). Learning Obstacle of Addition Operation Whole Number in Elementary Schools. *Journal of Physics: Conference Series*, *1842*(1), 12070. https://doi.org/10.1088/1742-6596/1842/1/012070

Sugiyono. (2021). *Metode Penelitian Kuantitatif, Kualitatif, dan R&D*. Alfabeta.

Suryadi, D. (2019). *Landasan Filosofis Penelitian Desain Didaktis (DDR)*. Gapura Press.

Van Hoof, J., Janssen, R., Verschaffel, L., & Van Dooren, W. (2022). Incorrect Ways of Thinking About the Size of Fractions. *International Journal of Science and Mathematics Education*, *21*, 1481–1502. https://doi.org/10.1007/s10763-022-10338-7

Wally dkk, N. (2024). Analisis Hambatan Belajar Matematika Bagi Siswa Sd Pada Materi Pecahan. *Jurnal Pendidikan Guru Matematika*, 190–195. https://doi.org/10.33387/jpgm.v4i3.8246

Yayuk, E. (2021). *Pembelajaran Matematika Sekolah Dasar*. Universitas Muhammadiyah Malang Press.

Yunita, Y., & Irawan, T. A. (2025). Effectiveness of Interactive Media in Enhancing Students' Science Concept Understanding. *Educational Journal of History and Humanities*, *8*(4), 6687–6697.

Zhang, S., Yu, S., Xiao, J., Liu, Y., & Jiang, T. (2022). The Effects of Concrete-Representational-Abstract Sequence Instruction on Fractions for Chinese Elementary Students with Mathematics Learning Disabilities. *International Journal of Science and Mathematics Education*, *20*(1481–1498). https://doi.org/10.1007/s10763-021-10215-9