Jurnal Riset Pendidikan Matematika 11 (1), 2024, 41-52

Exploration of students' mathematical communication abilities

Zahrin Nurun Na'im^{1,*,a}, Mohammad Mukhlis^{1,b}

¹Tadris Matematika, Universitas Islam Negeri Kiai Haji Achmad Siddiq Jember, Jember, Indonesia E-mail: ^azahrin060201@gmail.com, ^bmmukhlis166@gmail.com *Coresponding author

ARTICLE INFO

ABSTRACT

Article history

Received: 13 Oct 2023 Revised: 8 Nov 2023 Accepted: 31 May 2024

Kata kunci

kemampuan	komunikasi
matematis, bangu	ın ruang sisi
datar, kubus, balok	1

Keywords

mathematical communication skills, flat-sided spaces, cubes, beams It is of great importance for students to develop their mathematical communication skills. Mathematical communication skills play a pivotal role in mathematics education. Mathematical communication enables students to share ideas, clarify their understanding, and express their knowledge. The acquisition of proficient mathematical communication skills will facilitate a more profound comprehension of mathematical concepts. The objective of this study is to describe students' mathematical communication skills regarding flat-sided cubes and cuboids, with a focus on three indicators of mathematical communication skills as outlined by the National Council of Teachers of Mathematics (NCTM). The research design employed is that of descriptive research with a qualitative approach. The findings indicated that students exhibited a diverse range of mathematical communication skills. The students were divided into three categories according to their mathematical abilities. A more detailed examination of each category can be found in this article.

Sangat penting bagi siswa untuk mengembangkan kemampuan komunikasi matematis mereka. Kemampuan komunikasi matematis memainkan peran penting dalam pendidikan matematika. Komunikasi matematis memungkinkan siswa untuk berbagi ide, mengklarifikasi pemahaman mereka, dan mengekspresikan pengetahuan mereka. Perolehan kemampuan komunikasi matematis yang mahir akan memfasilitasi pemahaman yang lebih mendalam tentang konsep-konsep matematika. Tujuan dari penelitian ini adalah untuk mendeskripsikan kemampuan komunikasi matematis siswa pada materi bangun ruang sisi datar kubus dan balok dengan fokus pada tiga indikator kemampuan komunikasi matematis yang diuraikan oleh National Council of Teachers of Mathematics (NCTM). Desain penelitian yang digunakan adalah penelitian deskriptif dengan pendekatan kualitatif. Hasil penelitian menunjukkan bahwa siswa menunjukkan kemampuan komunikasi matematis yang beragam. Para siswa dibagi menjadi tiga kategori sesuai dengan kemampuan matematika mereka. Pembahasan lebih rinci dari setiap kategori dapat ditemukan dalam artikel ini.

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How to Cite: Na'im, Z., & Mukhlis, M. (2024). Exploration of students' mathematical communication abilities. *Jurnal Riset Pendidikan Matematika*, 11(1) 41-52. https://doi.org/10.21831/jrpm.v11i1.66639

INTRODUCTION

The National Council of Teacher Mathematics (NCTM) suggests that the learning objectives of mathematics include five basic abilities, namely problem solving, reasoning, connection, representation, and communication (Nurbayan & Basuki, 2022; Ruswanto et al., 2018; Wardhana & Lutfianto, 2019). Mathematical communication has an important role in mathematics or mathematics education (National Council of Teacher Mathematics, 2000; Elfareta & Murtiyasa, 2022). As argued by Rusdi et al. (2020) that mathematical communication is a standard that must be developed in mathematics education. Through mathematical communication, students can share ideas, clarify, and express what is understood (Rahmawati et al., 2023). Good mathematical communication skills will lead students to a deeper understanding of concepts. In line with the opinion of Uyen et al. (2021) in their article explained that students who communicate their mathematical ideas will be the students' process to strengthen their mathematical understanding. Students who communicate mathematical ideas will be the students' (Abidin, 2019; Arina & Nuraeni, 2022). Meanwhile, students who listen to or see their friends' explanations will be helped to construct and develop their own understanding (Utomo et al., 2015). This shows the importance of mathematical communication skills for students.

Baroody (1947) explained two reasons for the urgency of mathematical communication, namely mathematics as a language and mathematics learning as a social activity. Mathematics as a language means that mathematics is not only a tool to help think and solve problems, but mathematics becomes a language to communicate various ideas clearly and precisely (Werdiningsih & Junaedi, 2019; Ikhsan et al., 2020). Meanwhile, mathematics learning as a social activity means that mathematics learning is a social activity in which there is interaction between students or between teachers and students. In this interaction, good mathematical communication skills are needed so that students can understand the material being studied. As Nuraina & Mursalin (2018) wrote in their article "In teaching and learning interactions, communication plays a significant role, because teaching and learning can take place well if there is mutual communication between teachers and students, and between students and students".

Good mathematical communication skills will help students express the difficulties they experience learning a material. This makes it easier for teachers to determine the right method in reexplaining parts that have not been understood by students (Samawati & Ekawati, 2021). While students lacking strong communication abilities may arrive at the right solution, their instructor remains unable to discern whether a misunderstanding of the underlying principles may have nevertheless led to the proper outcome, leaving conceptual gaps unaddressed. This will be detrimental to students. Students will assume the concept that has been understood is correct and will be used so on at the next level (Maulyda et al., 2020). Consequently, possessing mathematical communication abilities is tremendously significant for learners since effectively conveying mathematical ideas is crucial to comprehension and problem-solving. In accordance with the opinion of Yaniawati et al. (2019) in her article, "mathematical communication skills are very important for every student".

Based on the explanation above, it shows the importance of mathematical communication for students. Therefore, researchers are interested in exploring students' mathematical communication skills on the material of flat-sided cubes and beams. This can be used as a consideration or further research in determining the right learning strategy to improve students' mathematical communication skills.

Previous research related to mathematical communication skills was conducted by Rohid et al. (2019). The study analyzed students' mathematical communication skills on algebraic material. The results showed that students' mathematical communication skills still need to be improved. Consistent with the findings presented in the study led by Nada et al. (2022) which shows that the mathematical communication skills of junior high school students on triangle and quadrilateral materials are classified as moderate. Therefore, students' mathematical communication skills. As explained by Kusumah et al. (2020) in their article "Communication ability is needed to understand mathematical ideas appropriately. Weak communication ability will weaken other mathematical abilities.".

One of the materials in mathematics that needs attention is flat-sided space. In this material, students are expected to be able to determine the surface area and volume of flat-sided spaces (cubes,

beams, prisms, and pyramids) either from each flat-sided space or a combination of several flat-sided spaces (Putro & Setyadi, 2022). However, there are still many students who have difficulty in learning flat-sided space material. As the results of (Fahlevi & Zanthy, 2020)'s research state that students who have low, medium, and even high mathematical abilities have difficulty in learning flat-sided spaces. This difficulty is caused by students not understanding the concept. In addition, the abstractness of mathematics makes students unable to convey what is understood and what has not been understood in learning a material (Nisa et al., 2020; Wahyuni & Alfiana, 2022; Putri & Aini, 2023). This makes it difficult for teachers to find out the misconceptions experienced by students. In this case, students' mathematical communication skills play an important role in overcoming this.

To determine students' mathematical communication skills, there are several indicators to measure them. NCTM states that indicators of mathematical communication skills include 1) the ability to express mathematical ideas through oral, written, and visual demonstrations and depictions; 2) the ability to understand, interpret, and evaluate mathematical ideas both orally, in writing, and in other visual forms; and 3) the ability to use mathematical terms, notations and structures to present ideas and describe relationships with situation models (NCTM, 1989; Faradina et al., 2016).

METHODS

This study aims to describe students' mathematical communication skills. To find out the picture, a qualitative approach was taken by searching and analyzing data. The instruments in this study used researchers as the main instrument and mathematical communication ability tests as supporting instruments. The test is in the form of story problems on the material of flat-sided cubes and beams that have been validated by experts. Data collection techniques were carried out through tests, think aloud, and documentations.

This research was conducted in junior high school consisting of 29 students. Determination of research subjects was carried out by giving a test of mathematical communication skills to students. Furthermore, the results of student work were analyzed based on three indicators of mathematical communication skills according to NCTM. For each indicator, two students were taken as research subjects. The research subjects were asked to retake the same test while speaking aloud whatever was in their mind during the process of understanding, solving problems, and answering questions on the test (think aloud) (Charters, 2003). Then, the test results and think aloud were analyzed to obtain data on mathematical communication skills. If there is a lack of data, researchers take data through interviews with subjects. In addition to tests and think aloud, data collection was also obtained through documentation. The documentation used in this research is in the form of photos and videos. The documentation was taken as an alternative way to record student activities. Researchers also documented the test results of research subjects. Documentation is used as supporting evidence of authenticity and complementary data in conducting research.

The data analysis process used the Miles and Huberman model, namely data reduction, data presentation, and conclusion drawing (Sugiyono, 2017). Data analysis was validated using triangulation of techniques and sources. Researchers used a variety of data collection techniques, namely tests, think aloud, and interviews. In addition, data were also obtained from two research subjects for each indicator of mathematical communication ability. Through these various techniques and sources, researchers obtained various points of view that could be integrated to produce complete and accurate information about students' mathematical communication skills.

RESULT AND DISCUSSION

Based on the data obtained from the research subjects, a description of mathematical communication ability is obtained according to the three indicators according to NCTM. S1 and S2 will be analyzed based on the first mathematical communication indicator. S3 and S4 will be analyzed based on the second indicator. S5 and S6 will be analyzed based on the third indicator.

First indicator: the ability to express mathematical ideas through oral, written, and visual demonstration and representation.



Figure 1. Answer of S1

S1 drew two block-shaped aquariums. The first aquarium is the aquarium before being filled with decorative stones. The second aquarium is the aquarium after being filled with decorative stones. In the second aquarium, the water level in the aquarium has increased. After drawing, S1 can understand the points asked in the problem. At the think aloud stage, S1 concluded that what was asked in the question was the difference in water level in the aquarium before and after being filled with decorative stones. According to S1, the water level in the aquarium will increase after the aquarium is filled with decorative stones. Furthermore, S1 also wrote the known things in the problem including the length and width of the aquarium.

S1 can visualize the situation in the problem by drawing two conditions of the aquarium before and after being filled with decorative stones. S1 expresses mathematical ideas by writing the known things in the problem, namely p and l. p symbolizes the length of the aquarium while l symbolizes the width of the aquarium. S1 has understood the point asked in the problem correctly. However, S1 can only complete the answer at that stage. S1 did not write down all the known things in the problem. S1 only wrote the length and width of the aquarium.

The following is an excerpt of SI's think aloud results

(S1 drew two aquariums)

"So I drew the aquarium first. The aquarium is a block. Initially the aquarium contains only water which fills half the aquarium (S1 draws an aquarium that contains only water). Then the aquarium was filled with decorative stones like this, so the water level in the aquarium increased (S1 drew an aquarium containing decorative stones). What is asked in this question is the difference in the water level in the aquarium before and after being filled with decorative stones. After filling the decorative stone, the water level in the aquarium will increase." (S1 wrote the known things in the problem)

"The known things in the problem are the length of the aquarium p equal to 100 cm and the width of the aquarium l equal to 40 cm. Then I don't know how to find the height."

Based on the results of think aloud, S1 has understood the point asked in the problem correctly. S1 has shown the first indicator of mathematical communication ability, namely being able to express mathematical ideas through oral, written, and visual representations. However, S1 can only complete the answer at that stage. S1 did not write all the known things in the problem. S1 only wrote the length and width of the aquarium. S1 was confused to continue his answer. He did not understand the other things known in the problem and the next step to solve the problem.



Figure 2. Answer of S2

S2 drew a block and its size. However, the placement of the measurements in the drawing was incorrect. S2 put the size of 40 cm on the rib of the beam that is longer than the rib of the beam that he gave the size of 100 cm. S2 also wrote the volume of water in the aquarium on the rib that was parallel to the 100 cm rib. Next, S2 wrote down several things that were known and asked in the problem. S2 continued his answer by finding t using the beam volume formula. However, S2 could not find the result.

S2 has shown the first indicator of mathematical communication ability. S2 visualized the aquarium told in the problem by drawing the block and its size. However, there was an error in placing the size.

Excerpts of researcher interview with S2

P: Why did you put the measurements 100 cm, 40 cm, and 120 l on this part of the beam?
S2: Because the problem explains that the length of the beam is 100 cm, the width of the beam is 40 cm, and the volume of water in the beam is 120 l.
P: Is the placement of these measurements correct?

S2 : I don't know ma'am.

The answer of S2 also shows the indicator of expressing mathematical ideas by writing down the known and questioned information in the problem. S2 explained that p symbolizes the length of the aquarium, l symbolizes the width of the aquarium, v symbolizes the volume of water in the aquarium before it is filled with decorative stones, and the surface area in question is the surface area of the cube-shaped decorative stone. S2 understood that what was asked was the difference in the height of the water in the aquarium before and after it was filled with decorative stones. Next, S2 looked for t by using the beam volume formula. The formula used is correct, namely $v = p \times l \times t$. The mistake S2 made at this stage was that S2 immediately entered the known value in the problem without equalizing the unit first. In the process of working, S2 was also not careful in writing the sign of the arithmetic operation equal to (=) into a times sign (×). So that S2 was confused himself to continue his answer.

The explanation above shows that S1 and S2 have the first mathematical communication ability only and even then it is still not perfect. Like the results of Imanisa and Effendi's research that there are students who are able to master one of the indicators of mathematical communication skills even though not perfectly (Imanisa & Effendi, 2023). S1 and S2 visualize the conditions in the problem by describing the aquarium in block form. S1 drew the condition of the aquarium before and after it was filled with decorative stones, while S2 focused more on the shape of the aquarium and its size. In this case, S2 is not right in putting the size on the block. This is relevant to the results of Pujianti, et al.'s research that students have difficulty in making pictures of the problems they do because they have not been able to visualize the images appropriately (Pujianti et al., 2023). S1 and S2 also expressed mathematical ideas by writing the known things in the problem using symbols. S1 and S2 only wrote down some of the things known in the problem. The lack of information caused S1 and S2 to be confused and did not understand the next step to answer the question.

The mistakes made by S1 and S2 were because they did not understand the mathematical concepts needed to solve the problem, they were unable to capture all the known information in the problem, they did not equate units, and they lacked accuracy in the process of working. This shows low mathematical communication skills, so students are unable to solve the problem. As the results of research by Pardosi, et al. showed that the low mathematical communication skills of students will also lead to low problem solving skills of these students (Pardosi et al., 2023). As the results of Laia and Harefa's research show that there is a positive relationship between mathematical communication skills and problem solving skills. Mathematical communication skills help students produce mathematical models needed in problem solving (Laia & Harefa, 2021).

Second indicator: the ability to understand, interpret, and evaluate mathematical ideas both orally, in writing, and in visual form.



Figure 3. Answer of S3

S3 answered the question by detailing what was known and asked in the question first. S3 mentions the known includes the length of the beam, the width of the beam, and the volume of water. S3 did not write about the decorative stone cubes that have a known amount and surface area. Then S3 wrote that what was asked was t. After being clarified, S3 explained that t meant the difference in water height before and after being filled with decorative stones. S3 continued his answer by looking for the height of the water in the aquarium before being filled with stones using the beam volume formula. The formula used by S3 is correct. However, S3 could not find the answer.

The following are excerpts of S3's think aloud results (S3 writes things that are known and asked in the problem) "It is known that the length of the beam is 100 cm, the width of the beam is 40 cm, the volume is 120 L. Asked t the difference in water level before and after being filled with decorative stones. Because the water level in the aquarium will increase a lot if it is filled with decorative stones." (S3 looking for the water level before being filled with decorative stones) "First find the water level before it is filled with decorative stones $v=p \times l \times t$ $120=100 \times 40 \times t$ $120=400 \times t$ 120=400I can't find the t."

Based on the test results and think aloud, S3 has shown indicators of the second mathematical communication ability. S3 is able to interpret and evaluate mathematical ideas. S3 understands that the water level in the aquarium will be higher after being filled with decorative stones. To find the difference in water level in the aquarium before and after being filled with decorative stones, S3 chose the right step by finding the initial water level (before being filled with decorative stones). The formula used by S3 is correct, namely $v = p \times l \times t$. However, the mistake made by S3 was to directly enter the volume of water, the length of the beam, and the width of the beam in the formula without equalizing the units first. Another error occurred in the calculation process $120 = 100 \times 40 \times t$. S3 wrote the result is $120 = 400 \times t$. Furthermore, the calculation process stopped. S3 said that he was confused so he did not continue his calculations.



Figure 4. Answer of S4

Not much different from S3, S4 also wrote the same thing. S4 wrote down the known things in the problem even though it was not explicit, namely only the length of the beam, the width of the beam, and the volume of water. Furthermore, S4 looked for the height of the water using the beam volume formula. The steps and formula used were correct. However, the results obtained were not correct.

The following is an excerpt of S4's think aloud results (S4 writes what is known in the problem)

"It is known that the length is 100 cm, the width is 40 cm, the volume of the beam is 120 liters. Then find the water level, the difference between the water level before and after being filled with decorative stones. After filling with decorative stones, the water level in the aquarium will increase."

(S4 looking for the water level before being filled with decorative stones)

"First find the water level before it is filled with decorative stones using the formula for the volume of a block.

 $v=p \times l \times t$ $120=100 \times 40 \times t$ $120=4000 \times t$ t=33,3

We have found the water level in the aquarium before it was filled with decorative stones. As for the water level after the decorative stone, I don't know how to find it."

The second indicator of mathematical communication ability was seen in S4. S4 can interpret and evaluate mathematical ideas. S4 has understood that the water level in the aquarium will be higher after it is filled with decorative stones. S4 decided to find the water level in the aquarium before it was filled with decorative stones. S4 used the known information in the problem to find the water height using the beam volume formula. However, an error occurred because S4 did not equalize the units first, so the final result obtained was wrong. S4 calculated that the water height was 33.3. S4's answer stopped at this stage. S4 said that he did not know the next step because he did not understand the material of cubes and beams.

S3 and S4 were correct in interpreting that the water level in the aquarium would increase after the aquarium was filled with decorative stones. They decided the first step and the right reason was to find the height of the water in the aquarium at first. They also use the right formula based on what is known in the problem. At this stage, the mistakes made by S3 and S4 were not equalizing the units first. As Listiana's observation shows that students do not change units first (Listiana, 2022).

Another mistake made by S3 occurred in the calculation process of 100×40 and 120 = 400. This makes S3 confused and unable to continue the answer. Students cannot find the value of t they are looking for. This is in line with the explanation of Suwarno, et al. in his article that one of the mistakes students often make when solving story problems is wrong in the calculation process (Suwarno et al., 2023). In contrast to S4 who could successfully calculate and find the size of the water level in the aquarium at first. But still the final result is not correct because the numbers entered in the formula do not have the same units. The mistake was due to lack of accuracy and lack of practice in working on problems. This is relevant to the results of Dewanti, et al's research which

explains that solving problems in the form of stories requires higher accuracy. If you are not careful, it will be difficult during the process of solving the problem (Dewanti & Muna, 2023).

The third indicator: the ability to use mathematical terms, notations to present ideas and describe relationships and models of situations.



Figure 5. Answer of S5

The first step that S5 did was to first list the known and asked in the problem. S5 wrote p, l, v, and lp, p symbolizes the length of the water in the aquarium before it is filled with decorative stones, l symbolizes the width of the water in the aquarium before it is filled with decorative stones, v as a symbol of the volume of water in the aquarium before it is filled with decorative stones, while *lp* is the surface area of the cube-shaped decorative stone. S5 changed the volume unit from liter to cm^3 . Furthermore, S5 wrote that what was asked in the question was t. S5 understood that what was asked in the question was the difference in the height of the water in the acuarium before and after it was filled with decorative stones. To answer this, S5 looked for the height of the water before being filled with decorative stones first using the beam volume formula. However, S5 symbolized the height of the water before being filled with decorative stones as the same as the symbol for the difference in the height of the water before and after being filled with decorative stones. S5 wrote the formula v = $p \times l \times t$. Then S5 entered the numbers in the formula correctly. Next, S5 looks for the length of the ribs of the decorative stone cube using the formula $lp = 6s^2$. After finding the ribs of the cube, S5 looks for the volume of each decorative cube using the formula v = $s \times s \times s$.

S5 was able to present mathematical ideas and connect them using mathematical models. S5 was able to find the height of the water in the aquarium before it was filled with decorative stones. S5 could also find the length of the ribs and the volume of the cube-shaped decorative stone. This shows the third indicator of mathematical communication ability, namely being able to present ideas and describe relationships with situation models. However, S5's answer ended at that stage only. He did not reach the stage of finding the difference in water level in the aquarium before and after it was filled with decorative stones. S5 admitted that he could not answer the next step. This was because S5 only focused on the individual cubes and blocks. He was confused to relate the relationship between the two flat-sided shapes in the problem.

Jurnal Riset Pendidikan Matematika, 11 (1), 2024 - 49 Zahrin Nurun Na'im, Mohammad Mukhlis



Figure 6. Answer of S6

S6 started her answer by first listing what was known and asked in the problem. S6 looked for the height of the water before it was filled with decorative stones. At this stage, the steps taken by S6 were correct. He had equalized the units first and calculated correctly so that the height of the water in the aquarium was 30 *cm*. Next, S6 looked for the length of the ribs of the cube-shaped decorative stone, and then looked for its volume. After that, S6 added the previous volume of water to the volume of the cube. The mistake S6 made at this stage was that S6 did not multiply the volume of the cube by the number of decorative cubes. The decorative cube-shaped stones that were put into the aquarium were 200 pieces. However, S6 only added the volume of water with the volume of one cube-shaped decorative stone. S6 continued by finding the new water height using the beam volume formula. The volume of the beam used at this stage is the volume of water plus the volume of one cube. After S6 found the new height, S6 looked for the difference in water height before and after being filled with decorative stones. Actually, the steps and decisions taken by S6 were correct. However, because S6 only added one cube-shaped decorative stone, the next calculation was wrong. So that the final result in the form of the difference in water height before and after being filled with decorative stones is also wrong.

Based on the test results and think aloud, S6 showed the third indicator of mathematical communication ability. S6 was able to use mathematical terms, notations and structures to present ideas and describe relationships with models of situations. The steps and calculations performed by S6 were also correct. However, in the process of working, there were errors caused by S6's inaccuracy. This made the final result of the calculation also wrong.

S5 and S6 were able to present mathematical ideas and connect them using mathematical models to answer the questions in the problem. This is in accordance with the third indicator of mathematical communication ability according to NCTM. However, S5 was not able to show this indicator perfectly. S5 was only able to find the relationship for each flat-sided space. So that S5 could not find the final result of the point asked in the question. Meanwhile, S6 was able to search until the final stage to answer the question. The formula used was also correct. However, the final result obtained by S6 was not correct. This is because there was one step that was missed, S6 did not multiply the volume of the cube by the number of decorative stones. To overcome these various errors, S5 and S6 need more practice problems so that they are trained to be more thorough and can find relationships from various kinds of problems presented in the problem. As the results of research by (Nada et al., 2022) stated that one of the causes of students not being able to solve the answer correctly is student inaccuracy in working on the problem.

CONCLUSION

Students' mathematical communication skills on the material of flat-sided cubes and beams need to be improved. Students have mathematical communication skills in certain indicators but are still low so that students cannot solve and answer questions in the problem correctly. For the first indicator, students can express mathematical ideas by writing some of the things known in the problem using symbols and describing the problems in the problem visually. For the second indicator, students can interpret and evaluate mathematical ideas but each error occurs in the calculation. As for the third indicator, students can present mathematical ideas and connect them using mathematical models but there are still errors due to lack of accuracy and practice problems. Errors and difficulties experienced by students are caused because students do not understand the concept of flat-sided spaces. Students are also not careful so they make mistakes in calculations and do not pay attention to units. To improve students' mathematical communication skills, teachers need to provide practice problems in the form of story problems about contextual problems or PISA problems. In addition, teachers also need to choose learning strategies that involve students actively. So that the teacher can know the level of students' mathematical communication skills and the level of students' understanding of the material taught.

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