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Students' Onto-Semiotic Approach in Solving Mathematics Problems

Teguh Wibowo [1,a] *, Elena Susianti [2,b], Puji Nugraheni [3,c], Riawan Yudi Purwoko [4,d]

[1,2,3,4] Program Studi Pendidikan Matematika, Universitas Muhammadiyah Purworejo
Jl. K.H. Ahmad Dahlan No. 3 & 6, Purworejo 54151, Indonesia
E-mail: [a] twibowo@umpwr.ac.id, [b] susiantielena@gmail.com, [c] pujinugraheni@umpwr.ac.id, [d]riawanyudi@umpwr.ac.id
* Corresponding Author

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ABSTRACT

Penelitian ini bertujuan untuk mengetahui dan menganalisis tahapan pendekatan onto-semiotik siswa dalam menyelesaikan masalah matematika. Penelitian ini berdasarkan pada objek matematika pendekatan onto-semiotik yaitu bahasa, konsep, proposisi, prosedur, situasi, dan argumen. Penelitian ini merupakan penelitian kualitatif dengan jenis fenomenologi. Subjek penelitian adalah siswa SMA kelas XI. Teknik pengambilan subjek menggunakan teknik sampel bertujuan. Instrumen penelitian mengunakan tes berupa soal uraian program linear. Hasil penelitian menunjukkan dalam pendekatan onto-semiotik memuat tahapan bahasa meliputi mengubah objek kontekstual ke dalam bentuk variabel, menuliskan simbol matematika, menuliskan model matematika, dan yang lain; tahapan konsep meliputi siswa membuat dan mengisi tabel dengan mengelompokkan objek-objek yang ada pada soal untuk mempermudah menyelesaikan masalah matematika; tahapan proposisi meliputi alasan siswa tidak dapat mengganti variabel; tahapan prosedur meliputi menyederahanakan koefisien dan konstanta, cara mendapat nilai dari masing-masing variabel, dan yang lain; tahapan situasi meliputi menuliskan model matematika yang akan digunakan untuk mencari nilai masing-masing variabel, mencari titik potong garis, dan yang lain.

This study aims to identify and analyze the stages of the onto-semiotic approach of students in solving mathematical problems. This research is based on the mathematical object of the onto-semiotic approach, namely language, concepts, propositions, procedures, situations, and arguments. This research is qualitative research with a phenomenological type. The research subjects were high school students of eleventh grades. The technique of taking the subject is using a purposeful sampling technique. The research instrument used a test in the form of a linear program description question. The results of the study show that the onto-semiotic approach contains language stages including changing contextual objects into variables, writing mathematical symbols, writing mathematical models, and others; the concept stage includes students creating and filling in tables by grouping the objects in the problem to make it easier to solve mathematical problems; the proposition stage includes the reasons students cannot replace variables; the procedure stage include simplifying coefficients and constants, how to get the value of each variable, and others; the situation stage includes writing a mathematical model that will be used to find the value of each variable, find the point of intersection of the line, and others.

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INTRODUCTION

The 2013 Curriculum is a curriculum that prioritizes problem-solving skills as one of the most important aspects in learning mathematics, this is written in the basic competencies contained in the Regulation of the Minister of Education and Culture of the Republic of Indonesia Number 64 of 2013 concerning Basic and Secondary Education Content Standards. The basic competencies state that students are expected to be able to demonstrate logical, careful and meticulous attitudes, honesty, responsibility, and not easily give up in solving mathematical problems. At every school level, problem solving must be the focus of NCTM mathematics learning (Rahayu & Afriansyah, 2015, p. 30). So that every student must have the ability to solve mathematical problems.

One approach to solving mathematical problems is the onto-semiotic approach. The onto-semiotic approach is a person's perspective in describing mathematical objects in solving mathematical problems (Hasanah & Budiarto, 2021, p. 35). The objects of mathematics in solving problems with the onto-semiotic approach are very broad. Font et al., (2015, p. 4) states that there are 6 main objects of mathematical activity, namely situations, procedures, language, concepts, propositions, and arguments. Godino et al., (2007, p. 132) states the difference between the cognitive approach and the onto-semiotic approach in mathematics education. The cognitive approach is said to be a mental process because understanding is only seen as a thinking process. The onto-semiotic approach is said to be a competency because students' skills in describing mathematical objects are highly considered. Skills in describing mathematical objects will later become a benchmark for how well students understand solving mathematical problems.

Afifah (2019, p. 235) said that the stages of students in the onto-semiotic approach are language, concepts, procedures, situations, propositions, and arguments. Based on the research results of Fónt, et al. (2007) and Afifah (2019) there are differences in the order of the stages of mathematical objects in the onto-semiotic approach.. Font et al.,(2015, p.3) studied the stages of mathematical objects based on derived material in 17-year-old students while Afifah (2019, p. 237) studied the stages of mathematical objects based on training in implementing the onto-semiotic approach in learning for junior high school students.

Before discussing further about the onto-semiotic approach, first discuss the ontology and semiotics which are the basis for the birth of the onto-semiotic approach. Thabrani (2015, p. 107) states that ontology etymologically comes from Greek, namely *ontos* which means something tangible and *logos* which means science. Gaifman (2012, p. 48) states that philosophers understand ontology as a question about things that exist. Nasution (2018, p. 1) states that ontology is automatically related to abstraction, modeling, and simulation. Ernest (Larvor, 2016, p. 196) says that ontological objectivism there are objects and classes of beings that exist independently of the point of view, beliefs, or conceptual schemes of a particular person and objects are objective in the sense that they exist automatically for people who know. So what is meant by ontology in this study is the study of an object that is used, what the form of the object is, what the object means, and how the bond between the object and the absorption of student knowledge.

In addition to ontology, the theory that underlies the birth of the onto-semiotic approach is semiotics. Sobur (2003, p. 16) states that semiotics or semiology etymologically comes from Greek, namely *semeion* which means sign or *seme* which means interpretation of signs. Santosa (2013, p. 3) defines semiotics as the science of signs. Sobur (2003, p. 16) says that with signs someone will try to find order and will get a little grip. So what is meant by semiotics in this research is a statement about objects that are displayed in the form of tables, symbols, images, graphs, etc.

Font et al., (2015, p. 3) states that semiotics and ontology in mathematical cognition form a relationship that can overcome problems of understanding and representing knowledge by describing mathematical objects. So Font et al.,(2015) calls this an onto-semiotic approach. Hasanah & Budiarto (2019, p. 35) states that a person's perspective in describing mathematical objects used to solve mathematical problems is called an onto-semiotic approach. Burgos et al. (2021, p. 36) states that the onto-semiotic approach is an approach that can connect two complementary perspectives on mathematics, namely as a problem-solving activity and as a system of objects and processes that regulate and appear in mathematical activities. Based on several opinions, the researcher concludes that the onto-semiotic approach is a student's perspective in solving mathematical problems by describing mathematical objects.

In the onto-semiotic approach there are mathematical objects. These mathematical objects will later become references in analyzing the stages of the onto-semiotic approach. In the research conducted by (Godino et al., 2007, p. 132), the mathematical objects in question consist of *language* including terms, mathematical expressions, mathematical notation, graphs, and others that are used as tools for action; *definitions* are obtained through concepts or descriptions in the form of numbers, points, straight lines, means, functions, or others; *propositions* include theorems, properties, or others that have the same function, namely connecting between concepts; *arguments* function to validate and explain propositions and procedures both inductively and deductively; *procedures* include algorithms, mathematical operations, and techniques in solving mathematical problems; and *situations* include problems, practice questions, extra-mathematical or intra-mathematical applications, and so on. The objects of mathematics in the onto-semiotic approach according to Hasanah & Budiarto (2019, p. 36) are language, concepts, procedures, propositions, and arguments. While the context does not appear in these objects. And according to Afifah (2020, p. 382) are language, concepts, procedures, computations, propositions, and arguments. These mathematical objects are the basis for the stages of the onto-semiotic approach to students. So it can be concluded that there are 6 stages of the onto-semiotic approach.

Based on the three studies above, the researcher suspects that the stages of the onto-semiotic approach in this study are: (a) the language stage functions to describe the use of mathematical terms, mathematical images or mathematical symbols; (b) the concept stage includes the use of definitions or descriptions of a design used by students in solving mathematical problems; (c) the proposition stage functions to provide statements about the answers written where the statement contains the relationship between concepts and descriptions of the steps that students have used in solving mathematical problems; (d) the procedural stage functions as a strategy or problem-solving technique by explaining the steps of the answers that have been written; (e) the situation or context stage is the use of written mathematical formulas or models in solving a problem for students; (f) the argument stage functions to explain each answer that has been written for students in solving mathematical problems.

To examine the stages of students' onto-semiotic approaches, one of which is through solving mathematical problems. Kurniawan et al. (2019, p. 164) stated that problem solving is not only limited to the use of formulas, but rather more directed at the part of understanding the problem, determining the solution, and implementing the solution plan. This means that in solving a problem, students need to master a sufficient concept as a basis.

Based on the background of the problem and the theoretical studies that have been put forward, the purpose of the researcher in this study is to find out and analyze the stages of students' onto-semiotic approaches in solving high school students' mathematical problems. The findings of the study are expected to be a reference in efforts to improve the quality of education, especially at the high school level.

METHOD

This study uses qualitative research because the data to be obtained cannot be measured by numbers and requires in-depth analysis and interpretation from the researcher. This study uses a phenomenological approach. According to (2016, p. 14) the term phenomenology is used to refer to the researcher's experience of various types and types of subjects encountered. This research was conducted at State Senior High School 1 Pejagoan, Kebumen Regency, Central Java Province. The subjects of this study were students of class XI State Senior High School 1 Pejagoan in the Odd Semester of the 2021/2022 Academic Year. The selection of research subjects used the purposive sampling technique or purposive sampling method. Sugiyono (2015, p. 216) stated that purposive sampling is a data collection technique by considering something. Researchers take subjects based on the test results given. The test questions used for taking subjects with research will be different. Students selected as research subjects are students who can solve the first test questions correctly. The material to be tested is a linear program. The number of subjects in this study depends on how many students successfully answer the test given correctly.

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The data collection techniques used in this study were tests, interviews, and documentation. In addition to being the main instrument, the researcher created a test instrument in the form of linear programming questions. The researcher will create two different questions, where the first test question is used to take research subjects and the second test question is used to obtain the data needed by the researcher. The test questions are made by adopting existing questions. In order for the instrument to function optimally, the questions are first tested for validity. The validity test is carried out by reviewing the questions by a validator who is an expert in mathematics.

The technique for checking the validity of data in qualitative research generally uses triangulation techniques. The triangulation technique used in this study is technical triangulation. Sugiyono (2015, p. 274) states that technical triangulation is a test of data credibility which is carried out by checking data from the same source but with different techniques. Data obtained from the results of the test are then checked by interview. In addition to using triangulation, researchers also conduct confirmability tests. According to Mekarisce (2020, p. 150) in qualitative research, confirmability is defined as a form of researcher availability in disclosing to the public about their research which then provides an opportunity for other parties to assess the results of their findings and obtain approval from those parties. Researchers conduct confirmability by reflecting on the findings in related journals and consulting with experts, namely fellow lecturers.

The data analysis used in this study is data reduction, data presentation, and conclusions and verification. Data reduction means that the data obtained from each data collection technique will be reduced (summarized), then each result will be reduced again to further narrow down the research results and one conclusion. The presentation of data in this study is in the form of a narrative in the form of sentences that show the stages of students' onto-semiotic approaches in solving mathematical problems. Conclusions in qualitative research may be able to answer the formulation of the problem that has been formulated because as has been conveyed, the problems and formulation of problems in qualitative research are still temporary and will develop after the researcher is in the field.

RESULTS AND DISCUSSION

Research Results

The subjects of this study were students who could correctly answer the subject selection test given by the researcher. Students who had been selected were then given linear programming questions to determine the stages of the onto-semiotic approach in solving mathematical problems. The results of the answers to these questions were then analyzed to determine the stages of the students' onto-semiotic approach in solving mathematical problems. Furthermore, the researcher also conducted interviews with the subjects by looking at the results of the subjects' answers. In this study, the results of the students' linear programming description test answers were analyzed and described based on mathematical objects in the onto-semiotic approach. In the research test questions, it was asked how much maximum profit a trader would get from selling bicycles. The following are the stages of students in solving the questions.

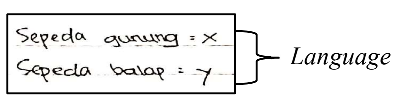

Figure 1. Subjects write down examples

Figure 1 shows that the subject initially wrote the analogy by changing the contextual object into the form of variables x and y. This is reinforced by the interview results which will be shown in Figure 2 below (R: Researcher, S1: Subject 1).

R : When solving the problem, what is the first thing you do?
S1 : Suppose a mountain bike with x and a racing bike with y.
R : Why do you suppose x and y.
S1 : Because you have often used it and the teacher also taught it like that.
R : If you suppose not to use x and y, is it still possible?
S1 : No
R : What is the reason?
S1 : Later there will be a step to draw a Cartesian diagram, in the diagram there are x and y axes. So you can't change the example other than x and y.

Figure 2. Transcript of the researcher's interview with the subject

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From Figure 1 and Figure 2, it shows that the subject meets the language stages. The language stages are proven by the subject associating a mountain bike into the variable x and a racing bike into the variable y. However, based on the interview results, it shows that the subject meets the argument stage and the proposition stage. The argument stage is shown by the interview results where the subject said the reason for writing the analogy with x and y was "because I often use it and the teacher taught it like that". The proposition stage is shown in the interview results where the subject said the reason why he could not replace the variables x and y with other variables, namely "later there will be a step where we draw in the Cartesian diagram, in the Cartesian diagram there are 2 axes, namely the x axis and the y axis, so he cannot change the analogy to other than x and y".

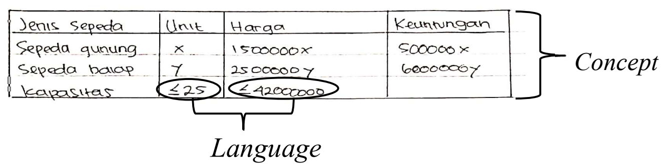

Figure 3. The subject makes a table

After changing the contextual object into a variable form, the subject then creates a table that can be seen in Figure 3. It can be seen that the subject creates a table and fills the table with the type of bicycle, unit, price, and profit in the table column and writes mountain bikes, racing bikes, and capacity in the table row.

R	:	After assuming, what is the next step?
S1	:	Create a table, the table is like this (while showing the table that was created)
R	:	Why create a table first?
S1	:	To make it easier to write the inequality system.
R	:	After creating a table, what is the next step?
S1	:	Filling in the table.
R	:	What are the contents?
S1	:	The contents are the type of bicycle, unit, price, profit, and capacity.
R	:	Why do you use the ≤ sign in the unit capacity and price cells?
S1	:	The question clearly states that the trader only wants to buy 25 bicycles and does not want to spend more than IDR. 42,000,000.

Figure 4. Transcript of the researcher's interview with the subject

From Figure 3 and Figure 4, it shows that the subject fulfills the concept stage and language stage. The concept stage is proven by the subject creating a table and filling the table with the type of bicycle, unit, price, and profit in the table column and writing mountain bikes, racing bikes, and capacity in the table row and completing the contents of the table. The language stage is proven by the subject using the ≤ sign in the unit capacity and price capacity cells which are strengthened by the subject's answer in the interview results which said "in the question it is clearly written that the trader only wants to buy 25 bicycles and does not want to spend more than 42,000,000".

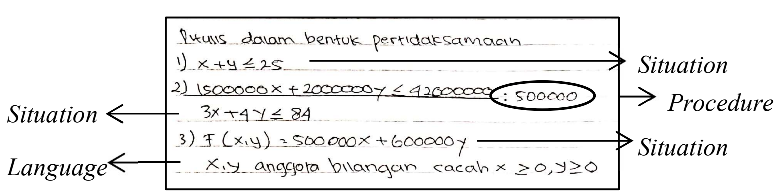

Figure 5. The subject writes the form of inequality

After making a table, the subject writes the form of inequality shown in Figure 5. This is also supported by the results of the researcher's interview with the subject shown in Figure 6. Based on Figures 5 and 6, it shows that the subject meets the situation stage, procedure stage, and language stage. The situation stage is proven by the subject writing all the mathematical models that will be used to solve the problem, namely "1) $x + y \leq 25$, 2) $3x + 4y \leq 84$, 3) $f(x, y) = 500{,}000x + 600{,}000y$". The procedure stage is proven by the subject simplifying $1{,}500{,}000x + 2{,}000{,}000y \leq 42{,}000{,}000$ by dividing it by $500{,}000$ to produce $3x + 4y \leq 84$. The language stage is proven by the subject writing "x, y are members of whole numbers, $x \geq 0, y \geq 0$".

P : After creating a table, what do you do?
S1 : Create a system of inequalities obtained from the table and determine the objective function.
P : It is written 3x + 4y ≤ 84, where is it obtained?
S1 : From simplifying 1,500,000x + 2,000,000y ≤ 42,000,000 divided by 500,000.
P : Why in the process, did you write 3x + 4y ≤ 84?
S1 : So that the numbers are not too large.
P : You also wrote f(x, y), what is this for?
S1 : f(x, y) is the objective function, used to find Mr. John profit.
P : What is the objective function in this question?
S1 : f(x, y) = 500,000x + 600,000y
P : You wrote x, y as members of the whole number. What is a whole number?
S1 : Whole numbers are positive integers starting from zero.

Figure 6. Transcript of researcher interview with subject

After the subject wrote down the form of inequality, the subject looked for the value of x, y from each inequality which can be seen in Figure 7. The subject assumed x = 0 to get the value of y and assumed y = 0 to get the value of x.

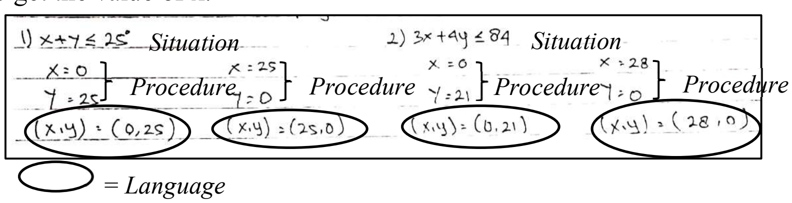

Figure 7. Subject looks for the value of x, y from each inequality

The results of the researcher's interview with the subject that strengthen what the researcher has explained in Figure 7 are presented in Figure 8 below.

P : After determining the mathematical model in the form of inequality, what is the next step?
S1 : Find the value of x and y from each inequality.
P : How to do it?
S1 : For x + y ≤ 25 by assuming x = 0 we get y = 25, so (x, y) = (0, 25). Then y = 0 we get x = 25, so (x, y)=(25, 0)
P : Why do you assume x and y to be 0?
S1 : If x is assumed to be 0, to determine the value of y. If y is assumed to be 0, to determine the value of x.

Figure 8. Transcript of the researcher's interview with the subject

Based on the results of the work and the results of the interview, it shows that the subject meets the stages of the situation, the stages of the procedure, and the stages of language. The stages of the situation are proven by the subject writing the inequalities that will be used to find the value of x, y from each inequality, namely $x + y \leq 25$ and $3x + 4y \leq 84$. The stages of the procedure are proven by the subject assuming x with 0 to determine the value of y, and assuming y with 0 to determine the value of x, as written by the subject, namely for $x + y \leq 25,\ x = 0, y = 25$; $y = 0, x = 25$, for $3x + 4y \leq 84$, $x = 0,\ y = 21$; $y = 0$, $x = 28$. The language stage is proven by the subject writing down the answer results from comparing x with 0 to determine the value of y, and comparing y with 0 to determine the value of x, namely for $x + y \leq 25$ then $(x, y) = (0, 25)$ and $(x, y) = (25, 0)$ and for $3x + 4y \leq 84$ then $(x, y) = (25, 0)$ and (x, y) = (28, 0).

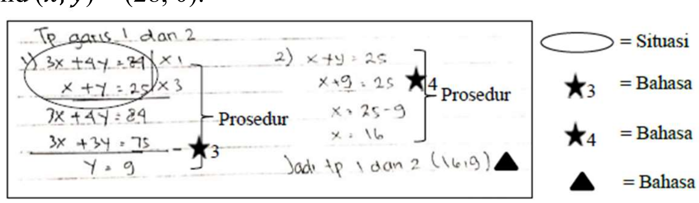

Figure 9. Subject looking for the intersection point of lines 1 and 2

Next, after the subject finds the value of x, y from each inequality, the subject looks for the intersection of inequality lines 1 and 2 which can be seen in Figure 9. It can be seen that the subject eliminates the variable x to get the value of the variable y. Then the subject substitutes the value of the variable y to get the value of the variable x. This is also supported by the results of the researcher's interview with the subject in Figure 10.

Based on Figure 9 and Figure 10, it shows that the subject meets the situation stage, procedure stage, language stage, and proposition stage. The situation stage is proven by the subject writing $3x + 4y = 84$ and $x + y = 25$. The procedure stage is proven by the subject writing $3x + 4y = 84$ multiplied by 1 then $x + y = 25$ multiplied by 3 resulting in $3x + 4y = 84$ and $3x + 3y = 75$ then subtracted to produce $y = 9$. The language stage is shown in the results of the subject's answers and interviews which say "eliminated" in the steps of the subject to find the value of y so as to get $y = 9$. The procedure stage is proven by the subject writing $x + y = 25$ then entering the value of $y = 9$ to become $x + 9 = 25$ then writing $x = 25-9$ resulting in $x = 16$. The language stage is shown by the subject who says "y = 9 is substituted" in the steps of the subject to find the value of x so that the value $x = 16$ is obtained. The language stage is proven by S1 writing down the results of finding the intersection point of lines 1 and 2 and writing it "So the intersection point 1 and 2 (16, 9)".

P : After finding the value of x and y for each inequality, what is the next step?
S1 : Find the point of intersection of the lines.
P : How to do it?
S1 : Eliminate 3x + 8y = 84 and x + y = 25 to produce y = 9.
P : After getting y = 9, the next step?
S1 : y = 9 is substituted into x + y = 25, we get x = 16.
P : How many points of intersection are found?
S1 : The point of intersection is (16, 9)

Figure 10. Transcript of researcher interview with subject

After finding the intersection point of inequality lines 1 and 2, the subject draws a graph on the Cartesian diagram shown in Figure 11. Then determine the area of the solution set and give a name to each corner point in the area of the solution set. This is also supported by the results of the researcher's interview with the subject in Figure 12.

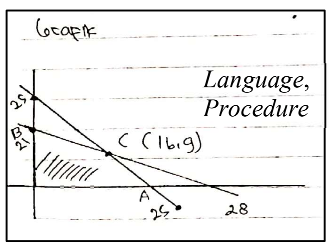

Figure 11. Subject draws a graph

P : After finding the point of intersection, what is the next step?
S1 : Drawing a graph on a Cartesian diagram.
P : How do you do it?
S1 : Earlier, you found the x and y values of each inequality, then drew them on a Cartesian diagram.
P : In the graph that is drawn, there is a hatching, what is this hatching?
S1 : That is the solution set area.
P : How do you determine the solution set area?
S1 : The solution set area is below the line.
P : After the solution set area is determined, what is the next step?
S1 : Mark the corner points with A, B, and C.
P : What are the coordinates of points A, B, and C?
S1 : Point A (25, 0), B (0, 21), and C (16, 9).

Figure 12. Transcript of the researcher's interview with the subject

Based on Figures 11 and 12, the subject fulfills the language stage, procedure stage, and argument stage. The language stage is proven by S1 drawing a graph on the Cartesian diagram which is a representation of the x, y values of each inequality and the intersection point that the subject has searched

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for. The stages of the procedure are proven by after drawing the graph, the subject determines the area of the solution set, and determines the corner points in the area of the solution set. The stages of the argument are shown in the results of the researcher's interview with the subject who said "earlier in the inequality the sign was ≤, the Area of the Solution Set is below the line".

After drawing the graph on the diagram, the subject tested points A, B, and C against f(x, y) shown in Figure 13 and the interview results in Figure 14.

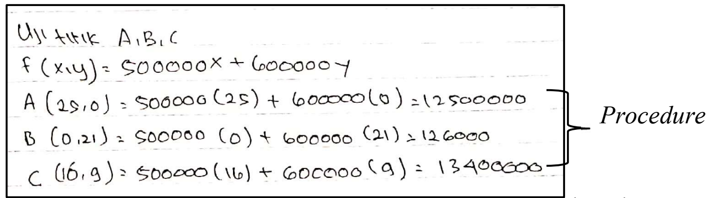

Figure 13. Subject tests points A, B, and C against $f(x, y)$

P : After getting points A, B, and C, what is the next step?
S1 : Test points A, B, and C to find the maximum profit obtained by Mr. John.
P : How do you do it?
S1 : For A(25,0) = 500,000(25)+600,000(0)=12,500,000, then for B(0,21)=500,000(0)+600,000(21)=12,600,000, and for C(16,9)=500,000(16)+600,000(9)=13,400,000
P : What do you get from solving the problem?
S1 : So the maximum profit obtained by Mr. John is IDR. 13,400,000, by selling 16 mountain bikes and 9 racing bikes.

Figure 14. Transcript of the researcher's interview with the subject

Based on Figures 13 and 14, the subject meets the situation stages, procedure stages, and language stages. The situation stages are proven by the subject writing f (x, y) = 500,000x + 600,000y which will be used as a function to solve the problem. The procedure stages are shown by the subject writing down sequentially the test of points A, B, C, against f (x, y). The language stage is proven by the subject stating that the conclusion obtained from completing the problem is that the maximum profit obtained by the trader is IDR 13,400,000 and selling 16 mountain bikes and 9 racing bikes.

Discussion

Based on the results of the study above; in solving mathematical problems on linear programming material, students have taken the right steps. The stages of students' onto-semiotic approaches in solving mathematical problems in this study are based on mathematical objects in the onto-semiotic approach. From the results of the answers and interview results, there are 6 mathematical objects that appear, namely language, concepts, propositions, procedures, situations, and arguments.

In solving mathematical problems, students go through language stage 1. Students can be said to have passed language stage 1 if students can use mathematical symbols. This is evidenced by students changing contextual objects into variable forms. Yadav et al. (2019) said that variables are symbols used to assume anything and are usually written in letters. This agrees with the results of research by Afifah (2016) which concluded that the language stage includes mathematical symbols used by students to solve mathematical problems.

After passing language stage 1, students pass the argument stage. Students can be said to have passed the argument stage if students can explain the reasons for the answers that have been written. This is proven by students saying the reason for writing variables is that students are used to doing it and following the teacher's teachings. This is in line with the results of Hasanah & Budiarto (2019) study which concluded that students in solving mathematical problems at the argument stage contain reasons related to the answers that have been written in solving mathematical problems.

After passing the argument stage, students pass the proposition stage. Students can be said to have passed the proposition stage if students provide statements about the answers that have been written where the statement contains the relationship between concepts from the steps used. This is evidenced by students stating the reason why variables cannot be replaced with other variables because there is a relationship with the steps that students will take, namely the relationship between the concept of using variables x and y with the concept of Cartesian diagrams which generally use the x and y axes. This is

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in line with the results of research by Afifah (2016) which concluded that students' proposition stages in solving mathematical problems contain concepts that are linked to other concepts in mathematics.

After passing the proposition stage, students' mathematics passes the concept stage. Students can be said to have passed the concept stage if students can group the objects in the problem and use a plan to make it easier to solve the problem. This is proven by students creating and filling in tables by grouping problem objects based on type of goods, units, prices, profits, and capacities. The results of Astuti & Saputra (2021) study concluded that the concept stage contains ideas that students use to group an object. This is supported by the results of Hasanah & Budiarto (2019) study which concluded that students in solving mathematical problems at the concept stage use designs used to solve mathematical problems.

After passing the concept stage, students pass the language stage 2. Students can be said to have passed the language stage 2 if they can use mathematical symbols. This is evidenced by students writing mathematical symbols in the table cells resulting from translating into mathematical language the desires of the questions, namely "want to buy" and "don't want to spend more money than" are written in the symbol ≤ and then students write down the mathematical model that has been obtained from making a table. This is in line with the results of Afifah (2016) study which concluded that the language stage includes mathematical symbols used by students to solve mathematical problems.

After passing the language stage 2, students' mathematics passes the situation stage 1. Students can be said to have passed the situation stage 1 if the students write down the mathematical model used to solve a problem. This is proven by the students writing down all the mathematical models used to solve the problem. This is in line with the results of research by Afifah (2016) which concluded that the situation stage of students in solving mathematical problems contains the accuracy in writing the formula or mathematical model that will be used to solve the mathematical problem.

After passing through the situation stage 1, students pass through the procedure stage 1. Students can be said to have passed the procedure stage 1 if they describe the steps in solving the problem. This is evidenced by students simplifying the coefficients and constants in mathematical models whose numbers are considered large by dividing them based on the value of the largest factor. This is in line with the results of research by Hasanah & Budiarto (2019) which concluded that students in solving mathematical problems at the procedure stage contain a description of the steps used to solve mathematical problems.

After passing the procedure stage 1, in solving mathematical problems students pass the language stage 3. Students can be said to have passed the language stage 3 if students can use terms in mathematics. This is evidenced by the fact that in solving problems, students write that variables are members of whole numbers. Wahyuningtyas (2016) said that whole numbers are terms in mathematics that are defined as positive integers starting from 0 to infinity. This is in line with the research results of Afifah (2016) which concluded that the language stage includes mathematical terms used by students to solve mathematical problems.

After passing the language stage 3, students pass the situation stage 2. Students can be said to have passed the situation stage 2 if students write down the formula or mathematical model used to solve a problem. This is proven by students writing down the mathematical model used to find the value of each variable. This is in line with the results of research by Afifah (2016) which concluded that the situation stage of students in solving mathematical problems contains the accuracy in writing the formula or mathematical model that will be used to solve mathematical problems.

After passing the situation stage 2, students pass the procedure stage 2. Students can be said to have passed the procedure stage 2 if students describe the steps in solving the problem. This is evidenced by students explaining how to get the value of each variable, namely by assuming one of the variables with 0 to get the value of the other variable. This is in line with the results of Hasanah & Budiarto (2019) which concluded that students in solving mathematical problems at the procedure stage contain a description of the steps used to solve mathematical problems.

After passing the procedure stage 2, students pass the language stage 4. Students can be said to have passed the language stage 4 if students can use mathematical symbols. This is proven by students writing down the answers resulting from illustrating one variable with 0 to get the value of another variable. This is in line with the results of research by Afifah (2016) which concluded that the language stage includes mathematical symbols used by students to solve mathematical problems. And in line with

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the results of research by Suyitno (2008) which concluded that the results of the answers from the mathematical model are presented with mathematical symbols.

After passing the language stage 4, students' mathematics passes the situation stage 3. Students can be said to have passed the situation stage 3 if students write down the formula or mathematical model used to solve a problem. This is proven by students writing down the mathematical model that will be used to find the intersection of the lines. This is in line with the results of research by Afifah (2016) which concluded that the situation stage of students in solving mathematical problems contains the accuracy in writing the formula or mathematical model that will be used to solve mathematical problems.

After passing the situation stage 3, students pass the procedure stage 3. Students can be said to have passed the procedure stage 3 if students describe the steps in solving the problem. This is proven by students writing and telling the steps to get the intersection point of the two lines to get the value of one of the variables by multiplying the two mathematical models by a certain number to equalize one of the coefficients of the variable so that one of the variables can be eliminated by subtracting. This is in line with the results of research by Amin et al. (2018) which concluded that at the procedure stage, students provide accuracy and completeness in writing and explaining strategies in solving mathematical problems.

After passing the procedure stage 3, students pass the language stage 5. Students can be said to have passed the language stage 5 if students can use mathematical terms, namely elimination, in solving mathematical problems. This is evidenced by students saying "elimination" to get the value of one of the variables. This is in line with the results of research by Afifah (2016) which concluded that the language stage includes mathematical terms used by students to solve mathematical problems.

After passing the language stage 5, students pass the procedure stage 4. Students can be said to have passed the procedure stage 4 if students describe the steps in solving the problem. This is evidenced by students writing and telling the steps to get the intersection point of the two lines to get the value of the variable that has not been found by entering the value of the variable that has been previously found. This is in line with the results of research by Amin et al. (2018) which concluded that at the procedure stage students provide accuracy and completeness in writing and explaining strategies in solving mathematical problems.

After passing the procedure stage 4, students pass the language stage 6. Students can be said to have passed the language stage 6 if students can use the mathematical term substitution in solving mathematical problems. This is proven by students saying "substitution" to find the value of one of the variables. This is in line with the results of research by Afifah (2016) which concluded that at the language stage it includes mathematical terms used by students to solve mathematical problems.

After passing language stage 6 with students saying the word "substitution", students return to language stage 7. Students can be said to have passed language stage 7 if they can use mathematical symbols. This is evidenced by students writing down the answers to the results of finding the intersection of the two lines obtained from the mathematical model. This is in line with the results of research by Afifah (2016) which concluded that the language stage includes mathematical symbols used by students to solve mathematical problems. And in line with the results of research by Suyitno (2008) which concluded that the results of the answers from the mathematical model are presented with mathematical symbols.

After passing language stage 7 with students writing down the answer to the results of finding the intersection of the lines, students return to language stage 8. Students can be said to have passed language stage 8 if students can use mathematical images. This is evidenced by students drawing a graph in the form of a line on a Cartesian diagram which is a representation of the values of the two mathematical models and the intersection points that students have searched for. This is in line with the research results of Amin et al. (2018) which concluded that at the language stage, students use language from mathematical objects that include mathematical images to solve mathematical problems.

After passing the language stage 8, students pass the procedure stage 5. Students can be said to have passed the procedure stage 5 if they describe the steps in solving the problem. This is proven after drawing a graph, students determine the solution set area by shading the solution set area by looking at the mathematical symbols used, if the symbol is $\leq$ then the solution set area is below the line and determining the corner points in the solution set area. This is in line with the research results of Hasanah

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& Budiarto (2019) which concluded that students in solving mathematical problems at the procedure stage contain a description of the steps used to solve mathematical problems.

After passing the procedure stage 5, students pass the situation stage 4. Students can be said to have passed the situation stage 4 if students use a mathematical formula or model that is used to solve a problem. This is proven by students writing the objective function which is generally written as f (x, y) in the form of a mathematical model that will be used to find the maximum value requested by the question. This is in line with the research results of Amin et al. (2018) which concluded that the situation stage includes writing a mathematical formula or model that will be used to solve mathematical problems.

After passing through situation stage 4, students pass through procedure stage 6. Students can be said to have passed procedure stage 6 if they describe the steps in solving the problem. This is evidenced by students writing down a description of the steps to find the maximum value requested by the question by multiplying the 3 corner points of the solution set area by the objective function. This is in line with the results of research by Hasanah & Budiarto (2019) which concluded that students in solving mathematical problems at the procedure stage contain a description of the steps used to solve mathematical problems.

After passing the procedure stage 6, students pass the language stage 9. Students can be said to have passed the language stage 9 if students can write an answer from a mathematical model into everyday language. This is evidenced by students writing down the maximum profit obtained by the seller from the procedure stage 6, namely testing using a mathematical model into everyday language. This is in line with the research results of Suyitno (2008) which concluded that at the language stage, students in solving mathematical problems include interpretation of the answers to mathematical models so that the answers to the problems are presented in everyday language.

Based on the data analysis and discussion results that the researcher has presented, it can be concluded that the stages of students' onto-semiotic approaches in solving mathematical problems are as follows:

1. Language stages include:
    a. Students change contextual objects into variable forms.
    b. Students write mathematical symbols in the table cells resulting from translating into mathematical language the desires of the contextual problem characters.
    c. Students write down the mathematical model that has been obtained from making a table.
    d. Students write that variables are members of whole numbers.
    e. Students write down the answer resulting from egulating one of the variables with 0 to get the value of another variable.
    f. Students use "elimination" to get the value of one of the variables. Students write down the answer resulting from finding the intersection point of the two inequalities.
    g. Students use "substitution" to find the value of one of the variables.
    h. Students write down the answer resulting from finding the intersection point of the two lines obtained from the mathematical model.
    i. Students draw a graph in the form of a line on a Cartesian diagram which is a representation of the values of the two mathematical models and the intersection points that students have searched for.
    j. Students write down the conclusions obtained from the problem-solving process that has been passed.
2. The concept stage includes students creating and filling in tables by grouping the objects in the problem to make it easier to solve mathematical problems.
3. The proposition stage includes students stating the reasons why variables cannot be replaced with other variables because of the relationship with the steps that students will take.
4. The procedure stage includes:
    a. Students simplify the coefficients and constants in mathematical models whose numbers are considered large by dividing them based on the value of the largest factor.
    b. Students explain how to get the value of each variable, namely by assuming one of the variables with 0 to get the value of the other variable.
    c. Students write and tell the steps to get the intersection point of the two lines to get the value of one of the variables by multiplying the two mathematical models by a certain number to equalize

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one of the coefficients of the variable so that one of the variables can be eliminated by subtracting.
   d. Students write and tell the steps to get the intersection point of the two lines to get the value of one of the variables by replacing the value of the variable that was previously found.
   e. After drawing the graph, students determine the area of the solution set, and determine the corner points in the solution set.
   f. Students write down the steps to find a value requested by the question by multiplying all the corner points of the area of the solution set by the objective function.
5. The stages of the situation include:
   a. Students write down all the mathematical models used to solve the problem.
   b. Students write down the mathematical models used to find the value of each variable.
   c. Students write down the mathematical models used to find the intersection points of the lines.
   d. Students write down the objective function in the form of a mathematical model that will be used to find the maximum value requested by the question.
6. The argument stage includes students stating the reasons for writing down the variables.

CONCLUSION

The data analysis and discussion reveal that students' onto-semiotic approaches in solving mathematical problems follow distinct stages. Initially, in the language stage, students transform contextual objects into variables, construct mathematical models, and use techniques like elimination and substitution to solve for variables. In the concept stage, students organize problem elements into tables to simplify the process. During the proposition stage, they articulate the rationale behind their variable choices. The procedure stage involves simplifying mathematical expressions and detailing steps to find solutions. In the situation stage, students systematically document the mathematical models employed. Finally, in the argument stage, students justify their choices and methods used throughout the problem-solving process. These stages highlight the structured approach students take in translating real-world problems into mathematical language and solutions.

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