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An analysis of critical thinking and communication skills in geometric transformation problem solving using TransGeo application

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ABSTRACT

The research aimed to narratively analyze critical thinking and communication skills using the *Android*-based TransGeo application on geometry transformation in junior high school. This study used a qualitative type with a descriptive method obtained from test and interview data. The test contained contextual problems in the *android* application, while the interviews were based on student test answers. Then, test results were adjusted descriptively in percentages that show the student's achievement to be categorized further based on critical thinking and communication scoring. Next, one student was randomly selected from each category as the interview subject. The indicator of critical thinking skills was difficult to achieve in the fourth indicator as students wrote inaccurate conclusions related to the initial problem. However, most students achieved the first indicator, where they wrote what was identified and asked in the problems. The indicator of communication skills was the most difficult one to achieve in the second indicator, where students did not describe or wrote mathematical sentences to present an explanation of the transformation type or final position area in the problems. The indicator that many students achieve was the first indicator, where students wrote what was identified and asked from the problems.

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INTRODUCTION

There is a possibility that many students are still making mistakes in solving math problems. Some frequent mistakes are involving concepts understanding, performing calculations, and writing down facts, particularly in solving math problems in the form of story problems ('Ashri & Aini, 2021) one of which is geometry transformation material, where students continue to struggle with concepts. Albab et al. (2014) stated that students struggle to understand the concept of transformation, which includes translation, reflection, rotation, and the combination of transformations. Furthermore, students need to comprehend a form when it is reflected (Gammath et al., 2020). Students' imaginations are used to determine the final position of shadows at coordinates via dilatation, translation, reflection, and rotation when learning geometry transformation (Ristu Atmawati, Sutopo, 2019). Students have been unable to draw shadows of transformed objects due to the problem of learning geometry transformation based on the results of document studies in journal articles through interviews (Elvi et al., 2021). Also, students need help in understanding the shadow cast by the object of reflection. When some steps are used to solve geometric transformation problems, the difficulty of problem-solving is evident (Gammath et al., 2020).

The problem-solving requires special skills, specifically the skills that encourage the student to focus on awareness of prior knowledge and systematic thinking processes. To solve problems, students must have conceptual mastery (Fitria et al., 2018). In addition, problem-solving and reasoning skills are required because they focus on the student's thinking process to write down logical reasons based on the initial discovered information (Ardhiyanti et al., 2019). Thinking is essential in mathematics learning because learning mathematics requires every student to understand formulas, calculate, analyze, classify objects, create manipulative media, make mathematical models, and so on. These activities require not only ordinary thinking skills but also higher-order thinking skills (Marliani, 2015).

Critical thinking skills are examples of high-level thinking abilities (Noor, Fahriza and Ranti, 2019). Critical thinking is the ability to observe information, and problems encountered by making logical decisions (Satwika et al., 2018). Critical thinking is students' ability to think by comparing two or more pieces of information, such as information obtained from new knowledge and previous knowledge process (Siswono, 2016). Students' critical thinking ability is a specific and systematic cognitive process in which they analyze the problem at hand, distinguish problems carefully and meticulously, and plan problem-solving strategies by reviewing information in advance (Azizah et al., 2018).

Communication skills, in addition to critical thinking abilities, are important abilities that must be developed to assist students in the development of the times (Noor, Fahriza and Ranti, 2019). According to Hodiyanto (2017), mathematical communication includes both oral and written communication. Oral communication takes the form of explanations or discussions. In contrast, written communication takes the form of mathematical ideas depicted or written in graphs/tables, equations, or in the student's native language. Furthermore, through communication skills, students can express their opinions on the ideas they receive and dare to ask questions when they do not understand the learning material (Wigati & Wiyani, 2020).

There were several issues with transformation learning in previous research, including the use of printed LKS (students' worksheet) (Wulandari & Raditya, 2017), GeoGebra software (Rodiawati, 2016), and Macromedia Flash (Rizqi & Nurjali, 2020). LKS is used as teaching material to help students explore the concept of geometric transformation, particularly rotation, using a variety of approaches (Wulandari & Raditya, 2017). However, printed LKS still requires money and time to produce, making it less appealing to students in online learning (Wulandari & Raditya, 2017). GeoGebra Software can also assist in the visualization of geometry transformation spaces construction, where students can observe the elements and properties of geometric transformations (Rodiawati, 2016). Geogebra has the disadvantage of not being compatible with all student devices. Furthermore, Macromedia Flash is used to reduce learning difficulties and develop students' mathematical creative thinking skills through geometric transformation (Rizqi & Nurjali, 2020). Macromedia Flash also has a similar obstacle to Geogebra since it also might not be compatible with all student devices. As a result, because it has yet to be widely used to overcome the difficulty of learning geometry transformations, the Android application was elected. Students can use Android applications because of their practicality, convenience, and attractiveness through animation.

Previous researchers have conducted several studies on critical thinking and communication skills in solving mathematical problems. According to Zakaria et al., (2021), students' critical thinking ability in solving math problems obtained from test results is still relatively low, at 68.42%. It is supported by the research of Rosmalinda et al. (2021), who stated that through test results with the PISA question type, students' critical thinking skills remained low at 58.1%. According to Ahmad & Nasution (2018), the Realistic Mathematics Learning approach revealed that students' mathematical communication skills were 40% and classified as low. It differs from the findings of Nopiyani et al. (2018), who found that the Realistic Mathematics Learning approach using Geogebra software improved students' mathematical communication skills.

According to the description above, the solution to students' difficulties in determining the final object's position is to train students to solve problems gradually. The novelty of this study is the use of android application media that is different from the media used in previous studies, to help students imagine the position of objects after geometric transformations are performed. This study aims to conduct a narrative analysis of students' critical thinking and communication skills in junior high schools while using the android-based TransGeo application on geometry transformation material. Furthermore, unlike previous research, this study focuses on assessing students' critical thinking and communication

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skills using visual-contextual test questions contained in the android-based TransGeo application. The benefit of this research is that students' critical thinking and communication skills allow them to compile solutions correctly and adequately to geometric transformation problems using simulations on the Android-based TransGeo application.

METHOD

The analysis of critical thinking and communication skills in solving geometric transformation problems is qualitative research using a descriptive method as an analysis decomposer. The data came from various sources, including geometry transformation test questions, interview guidelines, and validation sheets (test questions with indicators of critical thinking and communication skills). Test questions are prepared as essay questions for researchers to find out students' critical thinking and communication skills. Mathematics education lecturers will validate the compiled test questions.

Time, location, and subject of research

The study was carried out at SMP Negeri 1 Turen, located at Jalan Panglima Sudirman No.1, Turen, Kec. Turen, Malang, East Java, from September to October 2021. The supervisor made the recommendation for the research location. Furthermore, the schools were chosen based on the results of interviews with teachers in areas where students continue to struggle with geometric transformation problems. This study focused on class IX B students with 22 students. The research subjects were chosen based on the lowest average year-end assessment score and students with strong writing and explaining skills. The interview subjects determined based on the legibility of writing and the ability to explain clearly and logically. The interview subjects included three students with various levels of ability, namely high, medium, and low, and who were in the range of their respective test scores, $79 < N \leq 100$, $59 < N \leq 79$, and $0 \leq N \leq 59$, respectively.

Stages of research

This investigation was carried out in several stages. The researcher's first step was preparation, which includes media development and the preparation of research instruments, as well as the processing of research permits to the destination school. Furthermore, media experts and practitioners were subjected to media validation and research instruments. Lecturers as material experts, media experts, and teachers as practitioners validate media and research instruments. According to the validation results, the test questions and media were very valid, and the interview guidelines were valid. The implementation of research in schools was the next stage of research. The study was conducted online using the Zoom application, with researchers and students providing tutorials on the application's operation and strengthening geometry transformation material through image visualization.

In this study, triangulation techniques used to collect data. Triangulation is a technique for validating data by comparing data outside the research data for research purposes (Guntur et al., 2020). The triangulation technique was used in this study by analyzing the results of students' answers to geometric transformation problems. To obtain accurate data, the data obtained from the test results were compared to the student's responses at the test time. Then, each category was analyzed as high, medium, or low. The identification of student interviews was based on the results of questions answer. At last, the findings would be described using the results of previous tests and interviews.

The research data

This study contains two types of data: test results and interview results. As shown in the table below, the test result data are classified based on the percentage of achievement of critical thinking and communication indicators in the emergence of student answers.

Table 1. Rubric for Assessing Critical Thinking

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The stages of problem solving	Description of critical thinking Indicators	Score
Understanding the Problem	**Understanding the problem indicated in the question** Students write down what they know and what they are asked in the form of Cartesius coordinates or coordinate points.	2. if the student precisely writes down the cartesius coordinate shape or coordinate point 1. if the student incorrectly writes down the cartesius coordinates or coordinate points 0 if the student does not write down the cartesius coordinates or coordinate points.
Planning a Problem-Solving Strategy	**Determine the connection between the statement and the concept presented in the questions.** 1. Record the type of transformation related to the problem information requested. 2. Create a solution plan by writing down the operator matrix based on the type of transformation.	2, if the student specifies the type of transformation and operator matrix 1. if the student incorrectly writes down the type of transformation and the operator matrix 0, if the student does not record the type of transformation and the matrix of operators
Implementation of Problem-Solving Strategies	**Using strategies with precise calculation operations in solving problems** 1. Create a column matrix from point coordinates. 2. Using a corresponding transformation type operator matrix to operate a column matrix from coordinates 3. Sketch the coordinate point changes. 4. Describes the changes occured as a result of transformation (shape, size, position)	2, if the student solves the problem using systematic and appropriate solving procedures 1, if the student solves the problem using an unsystematic or appropriate solving procedure 0, if the student does not write down the problem solving
Conclusion Drawing	**Giving the precise reason for the answer taken** Connecting the object's final position to the initial problem	2, if the student correctly writes down the conclusion 1, if the student incorrectly writes the conclusion 0, if the student does not write down the conclusion

Adapted from Karim & Normaya (2015)

Table 2. Rubrics for Assessing Communication Skill

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Stages of problem solving	Description of communication skill indicators	Score
Understanding the Problem	**Using precise symbols to illustrate mathematical ideas based on information interrelationships.** Students write what they know and are asked in the form of cartesius coordinates or coordinate points.	2, if the students write down what they know and are asked in the question using symbols precisely 1, if the students write down what they know and are asked in the question using symbols incorrectly 0, if the students do not write down the information contained in the question using symbols
Planning a Problem-Solving Strategy	**Use images or mathematical sentences to convey explanations** 1. Identify the type of information transformation required for the problem. 2. Describe a sketch in cartesian coordinates to explain the final position of the field region described in the question.	2, if the students write down the type of transformation correctly or draw the sketch correctly 1, if the students write down the type of transformation incorrectly or draw the sketch incorrectly 0, if the students do not write down the type of transformation or do not draw a sketch
Implementation of Problem-Solving Strategies	**Presenting solutions or concepts with calculation operations in solving problems in sequence** 1. Converting point coordinates to a column matrix 2. Operating a column matrix from coordinates with a corresponding transformation type operator matrix 3. Sketching coordinate point changes 4. Describing changes occured according to the type of transformation (shape, size, position)	2, if the student applies the settlement procedure systematically and appropriately 1, if the student applies unsystematic or improper completion procedures 0, if not applying the settlement procedure
Conclusion Drawing	**Giving the precise reason for the answer taken** Connecting the object's final position to the initial problem	2, if the student correctly writes down the conclusion 1, if the student incorrectly writes the conclusion 0, if the student does not write down the conclusion

Adapted from Danaryanti & Noviani (2015)

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The rubrics for assessing critical thinking and communication skills are shown in Tables 1 and 2. To obtain the assessment scores, data in the form of student answers and interview transcripts are combined in Tables 1 and 2. The scores are then added up, and the percentage of achievement for each indicator is calculated. After calculating the percentage, the percentage numbers that demonstrate the achievement of indicators of critical thinking and communication skills are classified. The interview subjects are then determined based on critical thinking and student communication scoring results.

Data analysis

This study employs both qualitative and quantitative data analysis. The forms of student writing on test answer sheets adapted to indicators of critical thinking and communication abilities yield qualitative data. Interview transcripts also generate qualitative data. Quantitative data is information presented as a percentage score of test results.

The descriptive method analyzes data from tests and interviews, and triangulation is performed. The test result data are given comments as evidence of the suitability of students' answers with indicators of critical thinking and communication skills in the first stage. The second stage is data reduction, in which test results are grouped according to the critical thinking and communication assessment rubric categories. Interview results are grouped according to the appearance of keywords in each indicator of critical thinking and communication skills. The third stage of data verification involves cross-referencing the test results with the interview results. The fourth stage is displaying scans of students' answers that have been given appropriate indicators of relational thinking skills, followed by the obtained interview results, whether they support the students' responses. The fifth stage is concluding test and interview results, as well as evidence and reasons, in the form of exposure to critical thinking indicators.

This analysis model contains two variables: independent and dependent variables. Independent variables are critical thinking and communication skills, and dependent variables are an android-based TransGeo geometry transformation application.

The validation results are calculated using a Likert Scale and a scoring rubric that includes critical thinking and communication skills indicators on the geometry transformation question test, followed by calculating the percentage of scores obtained. The following formula is used to calculate the percentage of achievement of students' critical thinking and mathematical communication skills.

$$P = \frac{score}{score\ total} \times 100\%$$

Tables 3 and 4 show the achievement levels of students' critical thinking and mathematical communication skills.

Table 3. Categories of Critical Thinking Ability Achievements

Percentages (P)	Category	Conclusion drawing
$0 \leq P \leq 59$	low	Achievement of low critical thinking indicators
$59 < P \leq 79$	moderate	Achievement of indicators of good critical thinking
$79 < P \leq 100$	high	Achievement of critical thinking indicators is excellent

Adapted from Muslimahayati (2020)

Table 4. Categories of Mathematical Communication Ability Achievements

Percentages (P)	Category	Conclusion drawing
$0 \leq P \leq 33$	low	Achievement of indicators of low communication ability
$33 < P \leq 66$	moderate	Achievement of indicators of good communication skills
$66 < P \leq 100$	high	Achievement of excellent communication skills indicators

Adapted from Wijayanto et al. (2018)

According to Table 3, if students' critical thinking ability reaches a score of more than 59%, it is concluded that there is success in terms of critical thinking ability when solving geometry transformation problems. Suppose the student's mathematical communication ability exceeds 33%. In that case, it can be concluded from Table 4 above that there is a success when solving geometric transformation problems in aspects of students' communication skills.

RESULT AND DISCUSSION

The research implementation at SMPN 1 Turen began on September 22, 2021 using Zoom, and included tutorials on how to use the TransGeo application, tests, and interviews. The research was carried out in two meetings. During the first meeting, a tutorial on using the TransGeo application was implemented online, and students downloaded the application using the http://bit.ly/Trans_Geo URL first. Furthermore, students were given a 30 minutes test downloaded on an Android application. Finally, the answer to the test result was uploaded to Google Form with the URL https://bit.ly/TransGeobyUMM. Based on the achievement of critical thinking and communication ability indicators, students' test answers were classified as high, medium, or low. Students in the high category have the highest critical thinking and communication test scores, students in the medium category have moderate critical thinking and communication test scores, and students in the low category have the lowest critical thinking and communication test scores. During the second meeting, each category representative was interviewed for 20 minutes. Interviews were conducted online through WhatsApp application.

The application is created with the Android Studio application. The transgeo application's components include simulations of reflection, translation, dilatation, and rotation materials based on the characteristics of contextual problems in the test question section. The simulation in the application created with geogebra software to help students understand the steps and complete the geometry transformation material. Then an example of a simulation of a reflective material with a point is provided. After that, students were asked to explain the changes occurred during the simulation. The study's findings were summarized in the following paragraph. Data analysis was based on the classification of the obtained scores, which are high, medium, and low. Students' answers were typed into MSWord to make it easier to read the text and not to reduce the elements in it.

Representation process carried out by high-scoring students

NK is the student with the highest ability score. Based on Polya's problem-solving stages, NK meets all critical thinking and communication skills indicators. It can be seen in the following figure when NK reads and describes the known issues.

Stage 1:
Let: The region at the intersection of five gumul is depicted rectangular with coordinates *A(2.2), B(7.2), C(7.5), D (2.5)*. Bounded by highways that form the coordinate axis and divide 4 Different subdistrict areas.

Asked:
If a contractor wants to buy new land with a dilatation of -2 with a center point *(0.0)*. How many field coordinates does the contractor get and determine the sub-district that is the area?

Figure 1. NK's Answer to Stage 1

Based on Figure 1, NK understands the problem and can write down what is known and asked in the question. This is supported by the interview results, which are as follows.

P: "Try to tell me what problems you had in the question!"
NK: "Four distinct subdistrict areas bound the coordinate axis," says NK.
P: "So, what are the facts and questions about this subject? And what should be written as preliminary information about this work?"
NK: "It is known that the coordinate point of the land is in Kediri district with the starting points A(2,2), B(2,7), C(7,5), D(2,5), and then we are told to determine the location of the district after being dilated by -2 to the center point (0.0)"

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Then, as shown in Figure 2, NK develops and implements a troubleshooting strategy and conclusion drawing.

Answer:
1. The coordinates of the field obtained were:

$B.\begin{pmatrix}7'\\2'\end{pmatrix}=\begin{pmatrix}-2&0\\0&-2\end{pmatrix}\begin{pmatrix}7\\2\end{pmatrix}=\begin{pmatrix}-14\\-4\end{pmatrix}$ $A.\begin{pmatrix}2'\\2'\end{pmatrix}=\begin{pmatrix}-2&0\\0&-2\end{pmatrix}\begin{pmatrix}2\\2\end{pmatrix}=\begin{pmatrix}-4\\-4\end{pmatrix}$

$C.\begin{pmatrix}7'\\5'\end{pmatrix}=\begin{pmatrix}-2&0\\0&-2\end{pmatrix}\begin{pmatrix}7\\5\end{pmatrix}=\begin{pmatrix}-14\\-10\end{pmatrix}$ $D.\begin{pmatrix}2'\\5'\end{pmatrix}=\begin{pmatrix}-2&0\\0&-2\end{pmatrix}\begin{pmatrix}2\\5\end{pmatrix}=\begin{pmatrix}-4\\-10\end{pmatrix}$

Coordinates $A\begin{pmatrix}-4\\-4\end{pmatrix}$ $B\begin{pmatrix}-14\\-4\end{pmatrix}$ $C\begin{pmatrix}-14\\-10\end{pmatrix}$ $D\begin{pmatrix}-4\\-10\end{pmatrix}$
located in Pesantren sub-district

Figure 2. NK's answers on Stages 2, 3, and 4.

Figure 2 shows that NK can develop and implement problem-solving strategies. NK can multiply its transformation until it finds the result of the value requested in the question. Because it can relate between statements and concepts requested in the problem, NK is also good at analyzing the use of calculation operation structures. Through the conclusions written, NK is also able to connect the final position with the initial problem. The results of the interview with NK are as follows.

P: "How do you intend to solve the problem?"
NK: "Based on the known results in the question, there is a statement that the land is dilated by -2, so I substituted -2 in the dilation transformation formula"
P: "How will you put your chosen strategy into action?"
NK: "Once I have the matrix value, I immediately operate on it by multiplying it."
P: "Did you write down a conclusion after solving the problem and determining the value of the result?"
NK: "Yes, I wrote a conclusion beginning with a known statement and ending with the result of my final answer."

According to the research findings in the interview results, students who demonstrated critical thinking skills in indicator 1 can write down what they knew and ask questions. In indicator 2, students can connect revelations and concepts in the problem and devise a solution strategy. In indicator 3, students can use strategies with precise calculation operations based on the use of structures in the dilatation formula to determine the obtained shadow points. In indicator 4, students also write the correct conclusions based on the initial problem. Furthermore, the interview results show that students achieve mathematical communication skills in indicator 1, which is being able to write down what is identified and asked by writing the appropriate symbols.
Furthermore, students can explain using the mathematical sentences outlined in indicator 2. In indicator 3, students can use strategies with precise calculation operations based on the use of structures in the dilatation formula by determining the obtained shadow points. Finally, according to indicator 4, students write rational reasons for the initial problem.

Representation process carried out by moderate-scoring students

SA students have moderate test scores. SA can solve problems that meet several critical thinking and communication skills indicators. Based on the results of his answer in Figure 3, SA can write down what was known in the question but did not write down what was asked in the question. This demonstrates that SA comprehends the issues raised by the questions.

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1. Let: A 2,2), B(2,7), C(1,5), D(2,5)
   Dilatation of -2 with a center point (0,0)

Figure 3. SA's Answer to Stage 1

According to figure 3 above and the interview results, SA conveyed several things about the problems described in the following.

P: "Try to tell me what problems you had with the question!"
SA: "When I read the question, I discovered a problem in which I was instructed to find the location of the new ground shadow point."
P: "So, what are the facts and questions about this subject? And what should be written as preliminary information on this matter's work?"
SA: "The land's starting point or coordinate with points A(2,2), B(2,7), C(7,5), and D (2,5). I believe the preliminary information is a matrix transformation with dilation of -2 to find a new ground shadow point ".

Then, as shown in Figure 4, SA develops a troubleshooting strategy.

Stage 1:

1. Let: $A(2,2); B(2,7); C(7,5), D(2,5)$
   Dilation of -2 with a center point of (0,0)

$$= \begin{pmatrix} a' \\ b' \end{pmatrix} = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} am \\ bm \end{pmatrix}$$

$$\frac{(2'\ 2')}{(7'\ 2')} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 7 & 2 \end{pmatrix} = \frac{(-4\ -4)}{(-14\ -4)} \quad \begin{matrix} Point\ A \\ Point\ B \end{matrix}$$

$$\begin{pmatrix} d' \\ c' \end{pmatrix} = \begin{pmatrix} m & o \\ o & m \end{pmatrix} \begin{pmatrix} d \\ c \end{pmatrix} = \begin{pmatrix} dm \\ cm \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 7 & 5 \end{pmatrix} = \begin{pmatrix} -2.2 & -2,5 \\ -2.7 & -2.5 \end{pmatrix}$$

$$= (-4, -10) \quad Point\ D$$

$$= (-14, -10) \quad Point\ C$$

Figure 4. SA's answers on stages 2

Figure 4 depicts SA selecting problem-solving strategies during the stage of developing a problem-solving plan. First, SA can create and analyze a settlement plan and strategy, as well as connect the concept to the statements on the question. Then, as shown in Figure 5, SA solves the problem using the plans and strategies that have been developed and conclusion drawing.

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$$\frac{(2'\ 2')}{(7'\ 2')} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 7 & 2 \end{pmatrix} = \frac{(-4\ \ -4)}{(-14\ \ -4)} \quad \begin{matrix} Point\ A \\ Point\ B \end{matrix}$$

$$\begin{pmatrix} d' \\ c' \end{pmatrix} = \begin{pmatrix} m & o \\ o & m \end{pmatrix} \begin{pmatrix} d \\ c \end{pmatrix} = \begin{pmatrix} dm \\ cm \end{pmatrix}$$

$$= \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 7 & 5 \end{pmatrix} = \begin{pmatrix} -2.2 & -2,5 \\ -2.7 & -2.5 \end{pmatrix}$$

$$= (-4, -10)\ \ Point\ D$$

$$= (-44, -10)\ \ Point\ C$$

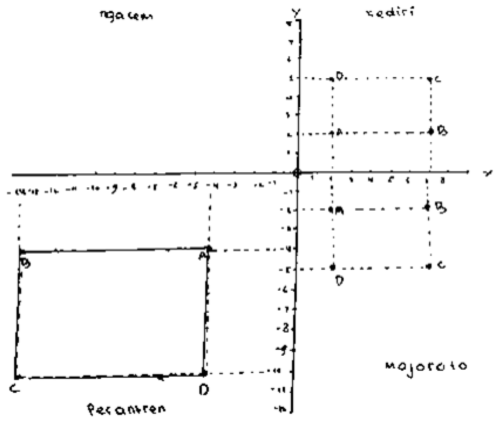

Figure 5. SA's answers on stages 3 and 4.

Figure 5 depicts SA's steps in analyzing the operating structure when executing the chosen plan. SA follows a predetermined plan to determine the final grade outcome. SA also creates a sketch for the final position of a known problem's object. Referring to the student's response in figure 5, SA revealed a few details during the interview.

P: "Given the most important information available right now, why do you employ such an approach to answering the question?" Then tell me how you implement the strategy in your work!"

SA: "Because the form of matrix transformation with a dilatation of -2 is known, I just need to substitute the value of -2 into the formula." As a result, the known starting point multiplied by the dilatation scale equals -2. And in the outcomes of A(-4,-4), B(-14,-4), C(-4,-10) and D(-14, -10)."

P: "Do you write a conclusion after solving the problem of the problem and discovering the result?"

SA: " No, after I solved the problem, I wrote down my conclusions and immediately began to sketch a solution."

According to the above description, after discovering the results of the requested grades, SA forgot to write down the conclusions, instead sketching out the new position of the land requested by the question. Because SA needs to write down conclusions, his critical thinking and communication skills are less apparent when re-reading the questions to evaluate the results of the answer decision.

According to the findings of the supportive interviews, students who demonstrated critical thinking skills in indicator 1 can write down what they knew but did not write down what was asked in the questions. As shown in indicator 2, students can make connections between revealations and concepts in the question. The achievement of indicator 3 demonstrates that students can use strategies with precise calculation operations based on the use of structures in the dilatation formula by determining the obtained shadow points. Nonetheless, as stated in indicator 4, students do not write down the conclusions. Furthermore, supportive interviews reveal that students who demonstrate

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mathematical communication skills in indicator 1 can write down what was known but cannot write what is asked about the question by using the correct symbols. Students can explain through sketch drawing when determining the object's final position, as stated in indicator 2. The student's explanation demonstrates the achievement of indicator 3; it demonstrates that students can use strategies with precise calculation operations based on the use of structures in the dilatation formula by determining the obtained shadow points. The results, as with students' critical thinking skills, show that students do not write down conclusions as the scoring rubric in indicator 4.

Representation process carried out by low-scoring students

RN students obtain the lowest scores. RN cannot carry out several indicators of critical thinking and mathematical communication, as well as explain the problems in the work results. According to the type of transformation, the RN writes down a strategy with calculation operations through a column matrix and an operator matrix based on the RN answer sheet in figure 6.

Stage 1.

Point A. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$; $x' = -2\,.\,2 = -4; y' = -2.2 = -4$

Point B. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$; $x' = -2\,.\,7 = -14; y' = -2.2 = -4$

Point C. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$; $x' = -2\,.\,7 = -4; y' = -2.5 = -10$

Point D. $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$; $x' = -2\,.\,2 = -4; y' = -2.5 = -10$

Therefore, the area of *kecamatan* is pesantren.

Figure 6. RN's Answer to Stage 1

Figure 6 depicts the RN conducting a strategy selection during the planning stage. The RN can construct and analyze a plan and link the concept to the problem's calculation operation. Given the question, the RN wrote down the reasons for the problem. According to the interview results, the RN communicated some of the problems discovered.

P: "What motivates you to write the matrix equation? What are the known and asked problems you are familiar with?"

RN: "To substitute the fixed point, write down the equation according to the steps in the application. There are four points, A(2,2), B(7,2), C(7,5), and D(2,5), and the point is dilated by -2 with a center point (0,0), then substituted, and the result is found."

P: "Did you write a conclusion after you solved the problem?"

RN: "I wrote a brief conclusion due to time constraints, so I immediately sent it to the link contained in the application."

According to research findings based on interview data, students with critical thinking skills cannot write down what is identified and asked, as indicated in indicator 1. Students are less able to relate statements and concepts in questions like those in the assessment rubric with indicator 2. The achievement of indicator 3 demonstrates that students can use strategies with precise calculation operations based on the use of structures in the dilatation formula by determining the shadow of the obtained object. In accordance with indicator 4, students write down their conclusions briefly. Furthermore, the interview results reveals that students who demonstrated mathematical communication skills cannot write down what they knew and asked questions by using the correct symbols, as stated in indicator 1. Students cannot convey explanations through sketch drawing or mathematical sentences in determining the object's final position in indicator 2 measurement. The measurement of indicator 3 demonstrates that students can use strategies with precise calculation operations based on the use of structures in the dilatation formula by determining the obtained shadow endpoints. Students jot down their conclusions in a brief manner (indicator 4).

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Table 5 shows the achievement of students' critical thinking skills based on the findings of the preceding research.

Table 5: Critical Thinking Ability Attained

Indicators	Percentage of Achievement	Category
Understanding the problem indicated in the question	81,06%	Very Good
Determine the connection between the statement and the concept presented in the questions	71,36%	Good
Using strategies with precise calculation operations in solving problems	80,09%	Good
Giving the precise reason for the answer taken	68,37%	Good

Adapted from Wijayanto et al. (2018)

Table 5 depicts the overall achievement of critical thinking ability indicators based on test and interview results. The analysis results in Table 5 show that overall achievement of the indicators of critical thinking ability is categorized as good.

Table 6: The Mathematical Communication Ability Attained

Indicators	Percentage of Achievement	Category
Using precise symbols to illustrate mathematical ideas based on information interrelationships.	81,06%	Very Good
Use images or mathematical sentences to convey explanations	60,67%	Good
Presenting solutions or concepts with calculation operations in solving problems in sequence	80,09%	Very Good
Giving the precise reason for the answer taken	68,37%	Very Good

Adapted from Wijayanto et al. (2018)

Table 6 shows the overall achievement of mathematical communication ability indicators based on test results and interviews. It can be concluded that the achievement of mathematical communication skills indicators is very well classified.

Based on the tests and interviews, it can be concluded that the achievement of indicators of critical thinking ability using android-based applications is well categorized. It is consistent with Rasyid et al. (2019)'s findings that using Android game media improves student learning outcomes in good categories compared to those who do not use Android-based learning media. According to Winata et al. (2019), the use of android improves students' critical thinking skills by 80% on average in cycle II.

The interview results show that RN can understand the problem, develop a solution strategy, and put the strategy into action. On the other hand, RN was less able to re-examine the outcomes of the answers obtained. The RN can form relationships based on information linkage in the problem or prior knowledge. When the RN re-examines the answers, however, RN is incorrect in evaluating the results of the answer decisions by writing down conclusions. It is consistent with previous research in which students cannot write down the problem's conclusions for it to be solved. (Pertiwi, 2018).

According to research findings on the subject of NK, NK is capable of reading and understanding problems, both in terms of preparing plans to be used and implementing plans, as well as writing rational reasons for a statement related to the initial problem. As a result, NK has been classified as meeting critical thinking and communication indicators. This is consistent with research by Albab et al. (2014), which states that if students give the correct reasons by reading the same number from the shadow, a pattern will be discovered. In terms of SA, he can devise problem-solving strategies by identifying the relationship between statements and concepts. As a result, SA is classified as meeting some of the critical thinking and communication indicators. Furthermore, this research is consistent with previous research

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in which students can write types of transformations using the information on the characteristics of each based on what they understand (Albab et al., 2014).

According to the study's findings, the indicator of critical thinking ability and mathematical communication that many students achieve is indicator 1, where students are used to writing what they know and being asked questions. It is consistent with previous research, which found that students with critical thinking skills can solve problems by writing down what they know and asking the right questions (Asria et al., 2021). Thus, 75.21% of the four critical thinking indicators on geometric transformation material are met. Furthermore, students' overall achievement in the four mathematical communication indicators on geometric transformation material was 72.5%.

CONCLUSION

Students can gain a better understanding of the concept of prerequisite materials such as matrices and coordinate systems by demonstrating their achievement towards certain indicators of critical thinking and communication skills in good categories. According to Polya, students can achieve the highest levels of critical thinking by using problem-solving stages in the first indicator, which measures their ability to understand or read problems. The level of achievement is lowest for the fourth indicator, which requires giving appropriate reasons for responses. Most students either do not write anything down or write conclusions that are not relevant to the initial problem. Students are able to comprehend concepts and precisely apply the operator matrix if the overall achievement of critical thinking indicators is within the good range. According to Polya, students achieve the highest level of achievement in the first indicator of mathematical communication, which is to appropriately illustrate mathematical ideas using symbols. Students also achieve the highest level of achievement in the second indicator of mathematical communication, which is to solve mathematical problems. The least amount of success was found with the fourth indicator, which offers a logical justification for a statement. The vast majority of students do not write incorrect conclusions that improperly connect the object's ultimate position to the original issue. The overall achievement of communication indicators is in the very good range, which indicates that students are able to comprehend ideas and precisely apply operator matrices through the use of images or mathematical sentences. The level of critical thinking and communication demonstrated by this study's fourth indicator remained at a relatively low level. It is important that suggestions for further research be able to generate questions that motivate students to write down conclusions or reasons related to the initial problem.

REFERENCES

’Ashri, H. Z., & Aini, I. N. (2021). Analisis Kesalahan Peserta Didik dalam Menyelesaikan Soal Matematika Transformasi Geometri Kelas IX. *GAUSS: Jurnal Pendidikan Matematika*, *4*(1), 22–31. https://doi.org/10.30656/gauss.v4i1.3191

Ahmad, M., & Nasution, D. P. (2018). Analisis Kualitatif Kemampuan Komunikasi Matematis Siswa Yang Diberi Pembelajaran Matematika Realistik. *Jurnal Gantang*, *3*(2), 83–95. https://doi.org/10.31629/jg.v3i2.471

Albab, I. U., Hartono, Y., & Darmawijoyo, D. (2014). Kemajuan Belajar Siswa Pada Geometri Transformasi Menggunakan Aktivitas Refleksi Geometri. *Jurnal Cakrawala Pendidikan*, *3*(3), 338–348. https://doi.org/10.21831/cp.v3i3.2378

Ardhiyanti, E., Sutriyono, S., & Pratama, F. W. (2019). Deskripsi Kemampuan Penalaran Siswa Dalam Pemecahan Masalah Matematika Pada Materi Aritmatika Sosial. *Jurnal Cendekia : Jurnal Pendidikan Matematika*, *3*(1), 90–103. https://doi.org/10.31004/cendekia.v3i1.82

Asria, H., Ahmad, A., & Joko, S. (2021). Analisis Kemampuan Berpikir Kritis Matematis Siswa SMP Negeri 1 Kota Ternate Dalam Menyelesaikan Soal Lingkaran. *Angewandte Chemie International Edition, 6(11), 951–952.*, *1*(2), 2013–2015.

Azizah, M., Sulianto, J., & Cintang, N. (2018). Analisis Keterampilan Berpikir Kritis Siswa Sekolah Dasar pada Pembelajaran Matematika Kurikulum 2013. *Jurnal Penelitian Pendidikan*, *35*(1), 61–70. https://doi.org/10.15294/jpp.v35i1.13529

Danaryanti, A., & Noviani, H. (2015). Pengaruh Gaya Belajar Matematika Siswa Kelas VII terhadap Kemampuan Komunikasi Matematis di SMP. *EDU-MAT: Jurnal Pendidikan Matematika*, *3*(2),

ISSN 2356-2684 (print), ISSN 2477-1503 (online)

204–212. https://doi.org/10.20527/edumat.v3i2.648

Elvi, M., Siregar, N. A. R., & Susanti, S. (2021). Pengembangan Lembar Kerja Peserta Didik Menggunakan Software Geogebra Pada Materi Transformasi Geometri. *Alifmatika: Jurnal Pendidikan Dan Pembelajaran Matematika*, *3*(1), 80–91. https://doi.org/10.35316/alifmatika.2021.v3i1.80-91

Fitria, N. F. N., Hidayani, N., Hendrian, H., & Amelia, R. (2018). Analisis Kemampuan Pemecahan Masalah Matematik Siswa SMP dengan Materi Segitiga dan Segiempat. *Edumatica*, *08*(1), 49–57.

Gammath, J., Maulani, F. I., & Zanthy, L. S. (2020). *Analisis kesulitan siswa dalam menyelesaikan soal materi transformasi geometri*. *5*, 16–25.

Guntur, M., Aliyyatunnisa, A., & Kartono. (2020). Kemampuan Berpikir Kreatif, Kritis, dan Komunikasi Matematika Siswa dalam Academic-Contructive Controversy (AC). *PRISMA, Prosiding Seminar Nasional Matematika*, *3*, 385–392. https://journal.unnes.ac.id/sju/index.php/prisma/

Hodiyanto, H. (2017). Kemampuan Komunikasi Matematis Dalam Pembelajaran Matematika. *AdMathEdu : Jurnal Ilmiah Pendidikan Matematika, Ilmu Matematika Dan Matematika Terapan*, *7*(1), 9. https://doi.org/10.12928/admathedu.v7i1.7397

Karim, K., & Normaya, N. (2015). Kemampuan Berpikir Kritis Siswa dalam Pembelajaran dalam Pembelajaran Matematika dengan Menggunakan Model Jucama di Sekolah Menengah Pertama. *EDU-MAT: Jurnal Pendidikan Matematika*, *3*(1). https://doi.org/10.20527/edumat.v3i1.634

Marliani, N. (2015). Peningkatan Kemampuan Berpikir Kreatif Matematis Siswa melalui Model Pembelajaran Missouri Mathematics Project (MMP). *Formatif: Jurnal Ilmiah Pendidikan MIPA*, *5*(1), 14–25. https://doi.org/10.30998/formatif.v5i1.166

Muslimahayati, M. (2020). Pengembangan Soal Kemampuan Berpikir Kritis Berbasis Kearifan Lokal Sumatera Selatan Pada Materi Trigonometri. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *9*(1), 12. https://doi.org/10.24127/ajpm.v9i1.2459

Noor, Fahriza dan Ranti, M. G. (2019). Relationship Between Critical Thinking and Mathematical Communication Ability of Junior High Schools Students in Mathematics Learning. *Jurnal Pendidikan Matematika*, *5*(1), 75–82.

Nopiyani, D., Turmudi, T., & Prabawanto, S. (2018). Penerapan Pembelajaran Matematika Realistik Berbantuan GeoGebra untuk Meningkatkan Kemampuan Komunikasi Matematis Siswa SMP. *Mosharafa: Jurnal Pendidikan Matematika*, *5*(2), 45–52. https://doi.org/10.31980/mosharafa.v5i2.259

Pertiwi, W. (2018). Analisis kemampuan berpikir kritis matematika peserta didik SMK pada materi matriks. *Jurnal Pendidikan Tamnusai*, *2*(4), 793–801.

Rasyid, A., Arif, A., & Kurnia, M. (2019). Pengembangan Media Pembelajaran Berbantuan Game Android untuk Meningkatkan Kemampuan Berpikir kritis Siswa. *Prosiding Seminar Nasional …*, 16. https://proceeding.unnes.ac.id/index.php/snpasca/article/view/239

Ristu Atmawati, Sutopo, L. F. (2019). Eksperimentasi Model Discovery Learning dengan Alat Peraga Petak Transformasi pada Pokok Bahasan Transformasi Geometri Ditinjau dari Kecerdasan Spasial Siswa Kelas XI SMK Negeri 1 Surakarta. *Jurnal Pendidikan Matematika Dan Matematika (JPMM) Solusi*, *III*(1), 102–116.

Rizqi, M., & Nurjali. (2020). Desain Bahan Ajar Berbantu Macromedia Flash Berbasis Kemampuan Berpikir Kreatif Matematis Pada Materi Transformasi Geometri. *Seminar Nasional Pendidi kan, FKIP UNMA 2020 "Transformasi Pendidikan Sebagai Upaya Mewujudkan Sustainable Development Goals (SDCs) Di Era Society 5.0"*.

Rosmalinda, N., Syahbana, A., & Nopriyanti, T. D. (2021). Analisis Kemampuan Berpikir Kritis Siswa Smp Dalam Menyelesaikan Soal-Soal Tipe Pisa. *Transformasi : Jurnal Pendidikan Matematika Dan Matematika*, *5*(1), 483–496. https://doi.org/10.36526/tr.v5i1.1185

Rodiawati, L. (2016). Pengaruh Metode Pembelajaran Demonstrasi Berbantuan Software Geogebra Terhadap Pemahaman Matematika Siswa Pada Pokok Bahasan Transformasi Geometri. *JES-MAT (Jurnal Edukasi Dan Sains Matematika)*, *2*(2), 67–80. https://doi.org/10.25134/jes-mat.v2i2.347

Satwika, Y. W., Laksmiwati, H., & Khoirunnisa, R. N. (2018). Penerapan Model Problem Based Learning untuk Meningkatkan Kemampuan Berfikir Kritis Mahasiswa. *Jurnal Pendidikan (Teori Dan Praktik)*, *3*(1), 7. https://doi.org/10.26740/jp.v3n1.p7-12

Siswono, T. Y. E. (2016). Berpikir Kritis dan Berpikir Kreatif sebagai Fokus Pembelajaran Matematika.

ISSN 2356-2684 (print), ISSN 2477-1503 (online)

Seminar Nasional Matematika Dan Pendidikan Matematika (Senatik 1), 11–26.

Wigati, M., & Wiyani, N. A. (2020). Kreativitas Guru Dalam Membuat Alat Permainan Edukatif Dari Barang Bekas. *As-Sibyan: Jurnal Pendidikan Anak Usia Dini*, *5*(1), 43. https://doi.org/10.32678/as-sibyan.v5i1.2700

Wijayanto, A. D., Fajriah, S. N., & Anita, I. W. (2018). Analisis Kemampuan Komunikasi Matematis Siswa Smp Pada Materi Segitiga Dan Segiempat. *Jurnal Cendekia : Jurnal Pendidikan Matematika*, *2*(1), 97–104. https://doi.org/10.31004/cendekia.v2i1.36

Winata, A., Sulistyaningrum, H., & Cacik, S. (2019). Peningkatan Kemampuan Berpikir Kritis Mahasiswa Menggunakan Pembelajaran Berbasis Android Pada Matakuliah Konsep IPA. *EduStream: Jurnal Pendidikan Dasar*, *3*(2), 1–9. https://journal.unesa.ac.id/index.php/jpd/article/view/6509

Wulandari, K. N., & Raditya, A. (2017). Pengembangan Lembar Kerja Siswa Pada Materi Geometri Transformasi Menggunakan Geogebra. *Prima: Jurnal Pendidikan Matematika*, *1*(1), 83. https://doi.org/10.31000/prima.v1i1.257

Zakaria, P., Nurwan, N., & Silalahi, F. D. (2021). Deskripsi Kemampuan Berpikir Kritis Siswa Melalui Pembelajaran Daring Pada Materi Segi Empat. *Euler : Jurnal Ilmiah Matematika, Sains Dan Teknologi*, *9*(1), 32–39. https://doi.org/10.34312/euler.v9i1.10539

ISSN 2356-2684 (print), ISSN 2477-1503 (online)