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Investigating Student Difficulties on Integral Calculus Based on Critical Thinking Aspects

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Abstract

Students of Mathematics education often struggle with integration problem, but yet the root of the problem related to critical thinking is rarely investigated. This article reports research where the first-year students of Mathematics Education of PGRI University Semarang were given an integral problem, then individually they were interviewed related to the answer they have made. The findings of students' difficulties in working on integration problem were confirmed through several questions in the interview which aimed to uncover their critical thinking process related to concepts, procedures, and problem solving. This study shows that student difficulties in Integration by disc method such as failure in identifying radius of a rotary object, specify partition, and integration bounds are closely related to their failure to think critically related to concept, skills, and problem solving aspects of critical thinking.

Keywords: Critical thinking, Integration, difficulties.

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INTRODUCTION

Integration method is one of the important procedural abilities possessed by the students of mathematics education, but yet they often struggle with integral problem. It is seen from students in performance in this competence is always low. The importance of integration method is that it can be used in calculating the curve-bounded area or volume. The problem that students often encounter is that they are unable to determine the upper and lower limits of integration. In addition, Integration by Substitution method, as well as Integration by Part method are also not fully mastered by students. The problems that exist in the students need to find out the basic causes so that the best solution can be selected.

Some of the difficulties faced by students on calculus and integration have been demonstrated by the researchers. The difficulty of understanding the language of mathematics, the limit is not just a simple arithmetic or algebraic operation but many concepts emerge in it, the concept of a very small variable arbitrarily (Tall, 1993). In addition others concluded that students difficulties includes misusing of concepts, failed to justify, and less precise use of algorithms (Lestari, 2014; Zetriuslita, Ariawan, & Nufus, 2016). The error is seen from not being able to choose the right integration method, and the wrong selection of theorems.

In fact, the students difficulty is closely related to critical thinking which is one of the skills of the 21st century (Permendikbud, 2016). Critical thinking plays an important role in the problem-solving process (Savery & Duffy, 1995; Shaheen, 2016). In addition, critical thinking also affects other thinking abilities such as creative, reasoning and others. Therefore, it is necessary to further study the difficulty of solving the problem of integral to the critical thinking process. Do students fail to think critically so they can not solve the integral problem? What are their difficulties in working on integration problem?

This article focus on the question: what are the difficulties facing by students in solving

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the integration problem in related to their critical thinking process? We assume that by giving them test of Integration problem, student difficulties related to critical thinking process emerge.

The ability to think critically is not only an important ability, but also the main goal of Higher education In Indonesia (Permendikbud, 2016). The ability to think critically is important because critical thinking has a role to analyze problems by basing on matching data so that various best ways of problem solving and decision making can be obtained (Zamroni and Mahfudz, 2009). The role allows one with critical thinking to be able to solve problems more precisely, to make quick and precise decisions, to find facts between opinions, and to enable them to behave more calmly even under stressful conditions. Furthermore, Kelley (2008) suggests that critical thinking is also a benchmark for the ability to achieve maximum learning outcomes. It is the underlying importance of having the ability to think critically, as well as critical mathematical thinking.

The ability to think critically in mathematics is the same as thinking critically on educational goals. In mathematics, Karim (2011) argues that critical mathematical thinking involves identifying concepts, generalizing skills, analyzing algorithms and solving problems. In addition, Innabi (2003) has developed a critical thinking scoring framework based on these indicators. Referring to the fact that critical thinking has such a character it is essential that critical thinking skills are shared by every student of mathematics.

Unfortunately, critical thinking skills are rarely fully mastered by Indonesian students. Studies on ciritical thinking of students from Asian countries shows that the competence in comparing, evaluating, arguing and conveying perspectives is not fully developed, and students have difficulty in memorizing, as well as passive learning (Durkin, 2008; Egege & Kutiele, 2004). In compared to Western students, Asian students show a much worse performance in mathematical critical thinking tests. They suggest that the educational background of students in the previous education level has a great impact in critical thinking patterns. Whereas in higher education, the ability to think critically is absolutely necessary.

In Integration competence in higher education, especially in mathematics, critical thinking skills are indispensable. There are so many concepts and formulas that students can use in integration method. If one chooses the wrong algorithm, the final calculation becomes false. Evaluation of the problem demands proper identification. The most common mistake most students make is to copy the procedures used in the exercise sample without considering the other competencies that are also used (Tall, 1993).

Table 1. Content-related aspects of critical thinking according to Innabi

Content-related aspects	Concepts	Geberalization	Algorithm and skills	Problem solving
	1. Identifying the characteristics of the concept 2. Comparing concepts with each other 3. Identify examples of concepts by providing justification. 4. Identify concrete examples of such concepts by providing justification	1. Determine the concepts contained in the generalizations and relationships between them 2. Determine the conditions of application of generalizations 3. Determine the generalization formula (special situation) 4. Provide supporting evidence for generalization	1. Clarify the basic concepts of a skill 2. Compare student performance with exemplary performance	1. Establish a common form for the target solution 2. Determining the information provided 3. Determine the relevance and distrust of information 4. Choose and justify the strategy to solve the problem 5. Determine and conclude sub-goals that lead to the goal 6. Suggest alternative methods to solve the problem 7. Determine the similarities and differences between the given problem and other issues

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In looking at the many types of critical thinking skills, the classification of the type of critical thinking ability has been made by experts. Among which the most famous is the Innabi classification. Innabi (2003) classifies aspects of critical thinking in two types namely critical thinking of general aspects and critical thinking of content-related aspects. Because the interview was conducted to see student work related to integral conten, the discussion of critical thinking process in this article reported the second aspect of critical thinking. Table 1 classifies only critical thinking of content-related aspects it in detail.

METHOD

This article reports the investigation on difficulties of the first-year students of Mathematics Education of PGRI University Semarang who take Calculus courses in solving Integration problem in terms of content-related aspects. By using Individual Demonstration Interview (Goldberg & McDermott, 1986) on student work, obtained data about failure and thought process. Further, difficulties are identified by the type of critical thinking process. By checking the appropriateness of the student's work and the analysis of the interview result, the drawing of conclusions about student's mistakes and the identification of their relation to the critical thinking process can be emerged.

FINDINGS AND DISCUSSION

Identify Critical Thinking on Content-Related Aspects

Critical thinking process involves many aspects working to solve problem, and one of it is content-related aspects. Critical thinking on content-related aspects includes four sub-aspects: concepts, generalizations, algorithms and skills, and problem solving. In this article, We reported critical thinking on concepts, algorithms and skills, as well as problem solving. For that, an integration problem is used as a measure of students' thinking processes. The problem is presented below:

> An area is bounded by a curve and lines $y = 9 - x^2, x \geq 0, x = 0, \text{dan } y = 0$. When the area is rotated around $x = 3$ how much is the volume of the object formed?

Concept-Related Critical Thinking

There are four critical thinking indicators of content-related aspects of integration concept: identifying concept characteristics, comparing concepts with each other, identifying concepts by justifying them, and identifying concrete examples of those concepts by providing justification. These indicators serve as guidelines for the investigation of student performance demonstrations on an integration problem. Figure 1 shows the student work errors that indicated by the circle.

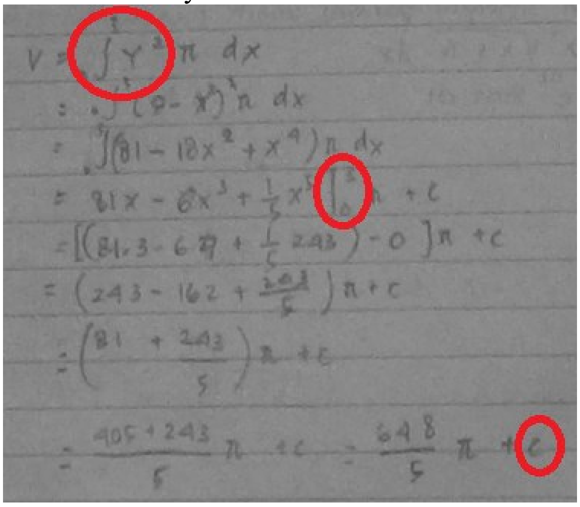

Figure 1. Errors made by subject 1

We thinks that Subject 1 answers the problem with the Disc Integration method where the volume is obtained using integral $\pi . r^2$ (Varberg, Purcell, & Susila, 2001). The error made by student is seen in the medium-circle sign where the variable y is squared multiplied by π where x should be squared. There is a inconsistency of the concept because the student incorrectly specifies the limit of integration that should be 0 to 9 instead of 0 to 3 as indicated by a large red circle. In addition, students also provide additional constants (C) on the answers as shown by small red circles. whereas the integral is the integral of course the result is a constant.

To reassure whether Subject 1 uses the disc method but with incorrect bounds and the addition of C, the following commands and questions are asked

> Explain how to do the problem!
> How to get limit of 0 to 3?
> What does C mean in your answer?

Response from Subject 1

> Since (the region) is rotated against the $x = 3$ then y is squared, then it is integrated by

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> multiplying π first. The intersection of the curve with the x-axis is at the point (0.3) and the minimum limit is the x-axis intersection with the y-axis. Each integral result must include a constant of C

From the student response, it is seen that Subject 1: First, Unable to identify the concept's characteristics appropriately, where they misjudged the y variable that should be x, then continued the error in determining the integration bounds. Second, Unable to compare among concepts, where they add a constant C on the student work. The subject does not compare between the concept of the integral of course and the indefinite integral.

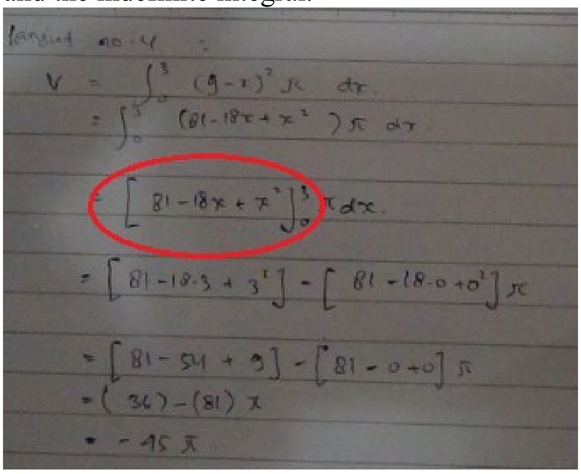

Figure 2. Error made by Subject 2

Besides making errors as did by subject 1, Subject 2 failed to integrate the equation $\int_0^3 81 - 18x + x^2\, dx$ where the problem is an addition integration problem skills (Varberg et al., 2001). Student errors are seen in a circle-shaped sign.

To reassure whether Subject 2 fails to integrate such a form, some questions are asked

> Explain how to do the problem!
> How to integral this form?

Response of Subject 2

> Form $9 - x^2$ is squared and then is integrated by multiplying π first. This process syntesizes form $81 - 18x + x^2$. I, then substitute the integration bounds to the form.

From the student response, it can be seen that Subject 2: (1) Unable to clarify the basic concept of a skill, where an integral operation should be performed before it is substituted with an integration bounds; (2) Unable to compare student performance with exemplary performance, wherein those integration by addition should be split in the form of $\int 81\, dx - \int 18x\, dx + \int x^2 dx$ to avoid oblivion, or integration errors.

Problem solving-related critical thinking

There are seven critical thinking indicators from the aspect of content related to problem solving: establishing common forms of target solutions, determining the information provided, determining the relevance and mistrust of information, selecting and justifying strategies for solving problems, defining and concluding sub-goals that lead to goals, suggest alternative methods to solve problems, and determine the similarities and differences between the given problem and other problems. These indicators serve as guidelines for the investigation of student performance demonstrations on an integral question. Figure 3 shows errors made by that indicated by the circle.

We think that Subject 3 responsed to the problem by generating information that is known and what is asked about the problem with a similar error with Subject 1 and 2. Subject 3 has attempted to describe the area bounded by curve an lines and also the area partition to be integrated. However, the error made by Subject 3 is that He make a wrong graph for quadratic curve and its partition. The quadratic curve should have been opened down and the partition for the area around the $x = 3$ axis should be layed horizontally, not vertically (Varberg et al., 2001). In addition, errors in calculation in the operations also occur.

To reassure whether Subject 3 really understands what is known and what is the importance of the information, so that it is sufficient or not to solve the problem. Then, We also want to make sure what goal Subject 3 wants to achieve by drawwing the curve and determine the partition, and the calculation, the following questions are asked

> What is known in the problem?
> Are all the known information important and enough to answer the question?
> What is asked in the question?
> What to do before answering questions?
> If the previous step is not done, is there any hope that can be continued? How to draw curves and partitions?
> Have you tried any other integral technique?

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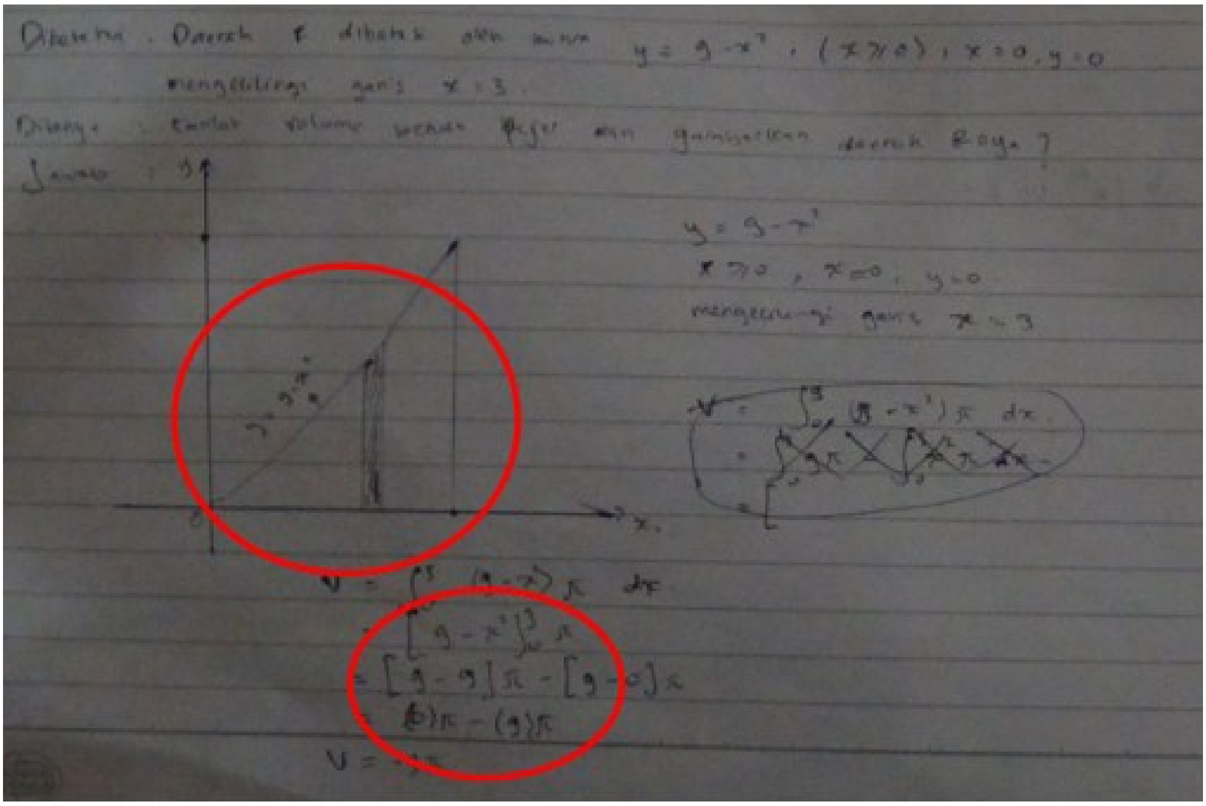

Figure 3. Errors made by Subject 3

Response of Subject 3

What is known is the quadratic function curve, and the borders $y = 0, x = 0$and rotated the $x = 3$s 3. I think what is important and all the same in the book (exercise example), is enough to answer the question. The question asks the volume of the known region and rotated against $x = 3$. To answer it, as I recall, I need to draw the area and its partition, set the bounds, and calculate it. Quadratic curve, how to make it, yes just make it. Put the partition in the area. That's the same thing that is in the practice questions. Yes, all the steps must be done, as an example in the main exercise. Then at first I split up when calculating the integration operation, then I fix it because it seems wrong. But I just remember, if this problem is like a sample problem, then I change it.

From the student respond, it ca be seen that Subject 3:

1. Unable to establish a general form for the target solution, where they follow only the procedures in similar exercise questions, so the general solution fails.
2. Have been able to compile the information that is known but unable to identify whether the known information is all important or not.
3. Unable to choose the right strategy in solving the problem even though it could fix the wrong responds as well.
4. Has been able to mention sub-goals even though not fully understand the usefulness of sub-goals. Subject 3 felt sub-goals should be elaborated and accomplished according to examples in practice questions.
5. Unable to determine the similarities and differences between the problems with one another.

From the students responds above, it it can be seen that Subject 3 response to the question based on an example of practice questions he has ever been able to. This is similar to alleged Tall (1993) where students tend to copy the procedures used in the sample exercise questions without considering other competencies that are also used.

From the description above, We found some common mistakes encountered in working on the Integration problem, (1) Incorrectly drawing graphs; (2) Falsely select the radius of a rotary object; (3) Incorrectly specified partition; (4) Incorrectly determining the integration bounds; (5) Adding C on the result of integration calculation; (6) Wrong perform integration calculations; (7) Wrong formula or concept.

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Table 2. The linkage between failure of critical thinking and form of error

Content-related Aspect	Critical Thinking Process	Errors
Concept	1. Unable to identify the concept's characteristics appropriately 2. Unable to compare the concept with each other	1. Incorrectly drawing graphs 2. Falsely select the radius of a rotary object 3. Incorrectly specify partition 4. Incorrectly determining the integration bounds 5. Adding C on the result of integral calculation
Algorithm and Skill	1. Unable to clarify the basic concept of a skill / procedure 2. Unable to compare student work with ideal job	Incorrectly doing the integration calculation, adding C to the final result of the the result of integral calculation
Problem Solving	1. Unable to set a general form for the target solution. 2. Unable to identify whether the known information is all important or not. 3. Unable to choose the right strategy in problem solving. 4. Unable to understand the sub-goals well. 5. Unable to determine the similarities and differences amongst problems.	Incorrectly select the right formula or concept in problem solving, and incorrectly draw the graph

Meanwhile, from the aspect of critical thinking with regard to content, such errors are classified as follows: (1) Unable to identify the concept's characteristics appropriately; (2) Unable to compare the concept with each other; (3) Unable to clarify the basic concepts of a skill; (4) Unable to compare student performance with exemplary performance; (5) Unable to set a common form for the target solution; (6) Unable to identify whether the known information is all important or not; (7) Unable to choose the right strategy in problem solving; (8) Not fully understand the sub-goals well; (9) Unable to determine the similarities and differences between the problems with one another.

The variety of errors and critical thinking relates each other as causal-effect relationship. For example, when student are unable to identify the concept of function, it leads to incorrect plotting curves. In the integration by disc method, when students are unable identify the concept of disc method, they would missed the radius of disc and the bounds. Another difficulties also arises such as incorrectly specify partition. This is because they are lack of mental image as suggested by Tall (1993). All critical thinking ability connect to performances in doing integration calculus. We linked the lack of critical thinking and the errors as see in Table 2.Critical thinking is still less mastered by student in our country. This is because critical thinking is poorly nurtured in schools in Indonesian schools (Syahbana, 2012). The same is also shown by critical thinking research in Asian countries. In comparing, evaluating, arguing and conveying views, it is not fully developed, and students are very difficult to memorize, as well as passive learning (Durkin, 2008; Egege & Kutiele, 2004).

Therefore, by knowing the error, the educator (lecturer or teacher) can adjust his teaching behavior (Tall, 1993). One of them is to try to make the teaching does not make learners just become imitators but able to think critically. In addition, materials and learning methods can also be designed in such a way that the learning to make learners can think critically and reasoning. Some RME study design studies such as transformation geometry learning design (Albab, Hartono, & Darmawijoyo, 2014). Moreover Collaborative learning methods by comparing other students' answers have also been exemplified to begin this improvement (Albab, Saputro, & Nursyahidah, 2017).

CONCLUSSION

From the above discussion it can be concluded that some errors such as mis-in drawing graphs, determining radius as partition, determining integration boundaries are strongly associated with the inability of students to think critically in identifying conceptual characteristics appropriately and comparing concepts with each other. In addition, the failure to think

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critically in clarifying the basic concepts of a skill / procedure and comparing student work with ideal work generates errors such as mis-calculations and adds C on the integration problem. While errors such as incorrectly choosing the right formula or concept in problem solving, and misinterpreting the graph are evidence of the student having difficulty thinking critically in setting the general form for the target solution, identifying whether the known information is all important or not, choosing the right strategy in problem solving, Understand the sub-goals well and not able to determine the similarities and differences between the problems with one another.

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