Cakrawala Pendidikan
Jurnal Ilmiah Pendidikan
Vol. 41 No. 3, October 2022, pp.830-847
https://journal.uny.ac.id/index.php/cp/issue/view/2348
DOI: https://doi.org/10.21831/cp.v41i3.39457

Investigating teachers' mathematics pedagogical content knowledge on ratio and proportion: Does it exist in teaching?

Rooselyna Ekawati[1*], Fou-Lai Lin[2], Ahmad Wachidul Kohar[1]
[1]Universitas Negeri Surabaya, Indonesia
[2]National Taiwan Normal University, Taiwan
*Corresponding Author: rooselynaekawati@unesa.ac.id

ABSTRACT

There is a widespread agreement that Mathematics Pedagogical Content Knowledge (MPCK) has become one of the key resources for teaching mathematics effectively. This qualitative research investigates the existence of primary teachers' MPCK in mathematics teaching practice on ratio and proportion. Data were gathered from videotaped teaching observations of three primary teachers with varying levels of Mathematics Content Knowledge (MCK) and MPCK as determined by a paper and pencil test. A video observation instrument considering the MPCK factors' framework for teaching ratio and proportion was used to explore the existence of MPCK in the teachers' teaching practices. The data were analyzed using a whole-to-part approach of video-based data on three components of the teachers' ratio and proportion teaching practices, namely, task level feature, problem-solving strategy teaching, and knowledge of students' conceptual understanding. Results indicate that all the components of teachers' MPCK can be observed in teaching practice appropriately or inappropriately due to teachers' different levels of MPCK (Good, Medium, and Low). All MPCK factors were activated by the good teacher in her teaching, which appropriately differs from medium and low teachers. The medium teacher needs more opportunities to learn about ratio and proportion task level features. The evidence leads to the opportunity to design a learning trajectory for in-service primary teachers that considers the integration of MPCK and MCK in balance.

Keywords: Mathematics Pedagogical Content Knowledge, Mathematics Teaching, Primary Teacher, Ratio and Proportion

Article history

Received:	*Revised:*	*Accepted:*	*Published:*
14 April 2022	*25 May 2022*	*22 June 2022*	*1 October 2022*

Citation (APA Style): Ekawati, R., Lin, F., & Kohar, A. (2022). Investigating teachers' mathematics pedagogical content knowledge on ratio and proportion: Does it exist in teaching?. *Cakrawala Pendidikan: Jurnal Ilmiah Pendidikan,* 41(3). doi:https://doi.org/10.21831/cp.v41i3.39457

INTRODUCTION

The attention of mathematics education research for more than fifty years has been paid to teachers' knowledge as a key resource for mathematics teaching behavior. Effective mathematics teaching depends on the depth of teachers' knowledge (Fawns & Nance, 1993; Fenstermacher, 1989; Mewborn, 2003; Shulman, 1986). Numerous studies revealed that teachers who had a thorough understanding of mathematics could use a variety of mathematical examples in their lessons. For instance, a teacher's understanding of the relationships between the ideas in a given topic has the ability to predict how well they will teach (Moliner & Alegre, 2022; Tchoshanov, 2011; Walshaw, 2012). In addition, teacher knowledge affects how they work with students in mathematics learning (Steele & Rogers, 2012). Research on the assessment of Mathematics Teacher Knowledge for Teaching (MKT) has already been conducted, for example by Hill et al. (2008) and was later expanded to examine how it relates to teaching practice (Munter & Wilhelm, 2021; Melhuish et al., 2021). The MKT concept takes into account the need for teachers to have a great level of knowledge and skill in order to carry out the teaching.

In terms of the mathematics used in the instrument for measuring teachers' proficiency in teaching mathematics, the content was typically not particular to one curriculum area. For instance, earlier studies (e.g. Porter, 2002) used a variety of mathematics topics in examining teachers' knowledge and teaching practice. In those studies, the content understanding of the teachers as well as the content for instruction lacked linearity. Therefore, it was difficult to assess precisely how well teachers were applying their expertise in their instruction. Although numerous studies have examined teachers' instructional practices from the perspective of learning methodologies like problem-solving (e.g. (Anderson et al., 2005; Andrews & Xenofontos, 2015)), inquiry-based learning (Engeln et al., 2013; Xie et al., 2014), and realistic or contextual approach (Abrahamson & Zolkower, 2020; Özdemir & Soylu, 2017; Zulkardi et al., 2020), there is a lack of research that explores teachers' knowledge, especially MPCK in teaching practices that focus on a particular area of mathematics. MPCK involves complex interactions between knowledge of generic pedagogy, a strong understanding of the discipline of mathematics, and a sound grasp of the principles of mathematics-specific pedagogy (Cheang et al., 2007). Some findings investigating the teaching practices covered by the framework of Teachers' MPCK were indeed investigated, such as the MPCK on data handling (Leavy, 2015), rational number (Depaepe et al., 2018), geometry (Brodie & Sanni, 2014), and ratio and proportion (Ekawati et al., 2018). However, there hasn't been any specific research on this topic with precise indicators that looks at how much teachers indicate their MPCK on ratio and proportion. More specifically, Glassmeyer, Brakoniecki, and Amador (2021) stated researchers still emphasize the importance of students' understanding of ratio and proportion, the difficulty of teaching ratio and proportion, and the need for further research to help teachers teach skills related to proportional reasoning to students. However, there has been a lot of recent study on this topic that focuses on students' abilities rather than cognitive processes, such as those linked to strategies and misconceptions (Adak & Aliustaoğlu, 2020; Im & Jitendra, 2020; Riehl & Steinthorsdottir, 2019;Jitendra et al., 2015). For teachers, research was also carried out related to their content knowledge (Glassmeyer et al., 2021; Izsák & Jacobson, 2017; Weiland et al., 2021) arriving at teacher training professional development (e.g., (Anat et al., 2019)) related to understanding ratio and proportion topic content. However, research on how mathematical pedagogical abilities relate to this has not been studied in detail, especially in terms of the effectiveness of teaching practices connected to ratio and proportion teaching. This study extends the studies of Ball et al. (2008) and Ekawati et al. (2014) about the construct of teachers' knowledge, especially MPCK and skills unique to teaching, on the topic of ratio and proportion with the scheme of vignettes on video analysis. Ekawati et al. (2014) described the component MPCK on ratio and proportion in their study, consisting of factor component Knowing students' conceptual understanding, Ratio and Proportion Task Level Feature, and Teaching Problem Solving. Regarding these phenomena, the existence of MPCK needs to be investigated within in-service primary teachers' teaching practices with more detailed indicators using video vignettes compared to relevant studies. To support the investigation, video-based research was done to capture behavior and enable a wide range of analysis (Jacobs et al., 2007). The scheme of vignette activity is used in the video analysis of this research. This scheme is recognized as a valuable way to encourage critical examination and personal reflection of their teachers in developing their professional knowledge (Forsythe et al., 2022). In addition, the vignette method in video-based research is used as a central instrument for data collection because of the methodological consistency that can be achieved, as it can help fulfill internal validity and support findings (Skilling & Stylianides, 2020). Through the video-based research, the mathematics' teachers' teaching behavior and its analysis could be done. Therefore, this research aims to investigate the MPCK of three primary teachers who have varying levels of MCK and MPCK in their teaching practices.

METHOD

Participants

This study used descriptive qualitative research in which it investigated the different teaching behaviors of in-service primary teachers with different MPCK categories on the content of ratio and proportion. We chose three in-service primary teachers that had been categorized based on paper and pencil tests of MPCK results from our previous study (Ekawati et al., 2014). The three teachers were chosen because they are all senior teachers with more than ten years of experience, have distinct knowledge profiles (Good, Medium, and Low), and are all of the same gender. The MPCK data from the paper and pencil tests was analyzed with cluster analysis as well as Exploratory Factor Analysis (EFA), giving the MPCK categories of Good, Medium, and Low. The participants of this study were three primary teachers selected from Ekawati's (2014) teacher participants categorized in the groups of Good (GG), Medium (MM), and Low (LL) for MCK and MPCK, respectively (Ekawati et al., 2014). Without any intervention, we recorded and gathered each teacher's natural teaching of ratio and proportion. Two video cameras are used by each teacher participant to record their teaching and the students' learning during the two meetings. Meanwhile, three observers with observation tools watched the teaching practice.

A Framework for analyzing MPCK on teaching video observation

The three MPCK factors resulting from Exploratory Factor Analysis on paper and pencil tests (Ekawati et al., 2014, 2015) were the main components for observation, such as Knowing students' conceptual understanding, Ratio, and Proportion Task Level Feature, and Teaching Problem Solving Strategy. Thus, these three factors become aspects of the analysis of teachers' teaching practices during teaching observation as well as the Mathematics Quality of Instruction (MQI) domain. Each aspect can be described as three possible codings, i.e., presented and appropriate, presented and inappropriate, and not presented. Table 1 shows the framework for analyzing the existence of MPCK in mathematics teaching together with the criteria for MPCK presentation in teaching ratio and proportion. We looked into a scenario that demonstrated MPCK components such as (1) Ratio and proportion task Level Feature, (2) Teaching problem solving strategy and (3) Knowing students' conceptual understanding.

When looking for the disclosed MPCK in teaching, there were two transformational paths available. First, the MPCK framework's criteria were applied to the teaching video in order to analyze the MPCK capture, and second, teaching-related observations were made. The MPCK factor of ratio and proportion task level feature is considered as the Mathematics Quality Instruction/MQI (coding related to task level feature in section III, such as task launch or can be interpreted as providing target task. In the Bell's principle of the design of teaching, it accounts for the differentiation by individualization/flexible task and the changes of task were also considered as the sub-components of this first MPCK factor. Furthermore, the MPCK factor of teaching problem solving strategy includes teaching strategies of ratio and proportion concept, teachers' problem-solving strategy, and their own pedagogical problems. We considered MQI components in section II (classroom work that is connected to mathematical idea or procedure) and intervention principle were considered and contextualized related to the teaching of ratio and proportion. It was referred to as a "directing proportional problem-solving strategy relating to a mathematical procedure or idea." In addition, a sub-factor of teaching problem-solving strategy that is connected to students was added: eliciting students' descriptions and explanations for the MQI coding component. Another sub-factor pointed to the 'feedback' principle to be included since feedback is an integral part of the process of discussion in pairs, in groups, and in the class as a whole (Bell, 1993). This is closely related to the interaction within the teaching process. Table 1 shows the framework of MPCK in teaching ratio and proportion, which is and is not presented. If the indicator of MPCK in teaching is presented, there is a possibility of it appearing appropriate or not appropriate.

Regarding the first MPCK factor (knowing the student's conceptual understanding), that pertained to students' misconceptions and thinking on ratio and proportion. Regarding this, the design principle of Bell (1993) on revealing misconceptions and using or addressing students' errors and misconceptions was elaborated as the sub-factor for video observation. Since the analysis was in the form of video, we applied the type of whole-to-part of Erickson's (2006) approaches to analyzing video-based data. With regards to the current references, the research flow is shown in Figure 1 below.

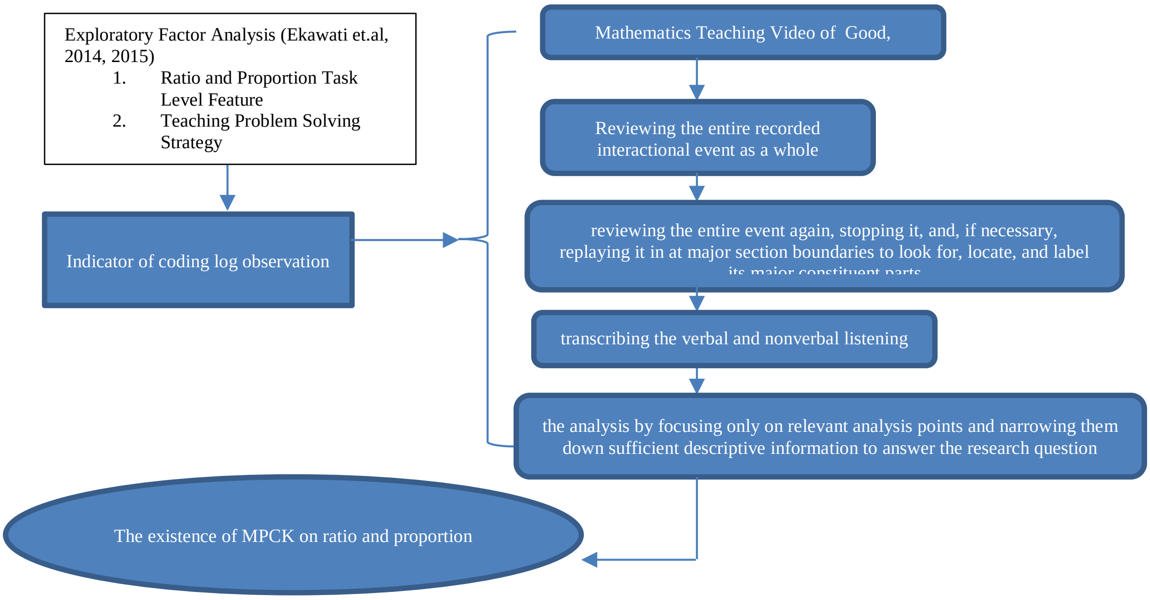


Figure 1. Data collection and analysis

Figure 1 takes into account Erickson's (2006) method of analyzing video. In addition, the indicator of data analysis was developed from the coding log observation of three MPCK factors. It was necessary to carefully analyze the classification criteria for the appearance of the MPCK factors to analyze the teaching video described above.

One of those three codings was given to each of the aspects of analysis based on raters' observations using the aspects provided in Table 1. To obtain valid and reliable data, we employed data source triangulation techniques by adding, modifying, and merging such codings with the observation data, which was not recorded by the video we obtained from some field notes (Patton, 2002). The minimum match rate of three observer coders on the coding log observation of MPCK in teaching ratio and proportion was 85%, which showed a reliable observation.

Table 1. A Framework for analyzing teaching videos concerning MPCK factor components

Aspects	Presented and Appropriate (PA)	Presented and Inappropriate (PI)	Not Presented (NP)
Ratio and Proportion Task Level Feature	The teacher launched a task that is mathematically appropriate and enables students to work productively. The teacher gave a flexible task to students in terms of number structure and situation presented (i.e., gave a different task to a different group of students). The teacher made the change to the task difficulty level regarding the number structures and situation presented (i.e., the task contains hierarchical number structures; Situation was changed from ratio as comparing quantity to situation with geometrical properties).	The teacher did not make any changes to the structures of the task (i.e., number structures and situation). The tasks presented used inappropriate number structures and situations. The students were disabled to work productively with the task given due to a lot of intervention from teachers. The tasks discussed were not a proportional problem. The task has a simple number structure, but there was no discussion or intention to include a multiplicative relation.	
Teaching Problem Solving strategy	The teacher guided and shared a unitary method with students for a proportional problem. The teacher guided students to understand several problem-solving strategies instead of directly sharing the strategy (i.e., discussing cross multiplication strategy, equivalence fractions, etc.). The teacher gave the students an opportunity to explore the solution to the proportional problem with their strategy. The teacher guided students to check their solutions and answers. The teacher shared guided questions when they had misconceptions about solving a proportional problem. The teacher guided students' misconceptions by reminding them of the previous lesson related to the problem. The teacher asked about the steps they used. The teacher asked "why" questions that contribute to building good explanations.	The teacher shared the formula to be memorized by the students. The teacher did not elicit the students' descriptions. There was no feedback given to students when they shared errors. If students shared errors, the teacher directly shared her/his evaluation by stating it was wrong.	
Knowing students' conceptual understanding	The teacher posed a task or question that was possible for students to have misconceptions about (i.e., number structure arranged for possible addition strategy to a proportional problem). The teacher asked a question that had more than one answer for the students. The teacher responded to, used, or otherwise addressed student errors. The teacher discussed students' errors with the whole class.	There was no specific support for students to be aware of misconceptions. The teacher did not respond to the students' questions. The teacher addressed the students' errors with pointed errors. The teacher gave hints to the students' errors by directly stating the formula without underlining the reasons. The teacher missed the point of the students' errors.	

FINDINGS AND DISCUSSION

Findings

Three observers examined the data from observation and video vignettes. Regarding the coding log observation, the numbers of observation cells were accumulated for all teaching stages and described in each factor. There are 132 cell, codes used for exploring MPCK in teaching. The observers made several codes in the 132 cells, and we searched for the inter-rater agreement of the three observers. The summary of inter-rater agreement of three observers in coding log observations was described in Table 2.

Table 2. Coding Observation of MPCK

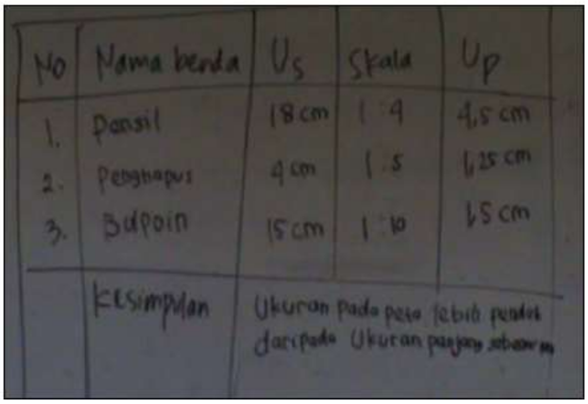
MPCK Factors	GG-O1	GG-O2	GG-O3	MM-O1	MM-O2	MM-O3	LL-O1	LL-O2	LL-O3
Ratio and Proportion Task Level Feature	88.64%	93.18%	100%	95.45%	95.45%	95.45%	90.91%	88.64%	95.45%
Teaching Problem Solving Strategy	85.42%	93.75%	91.67%	90.90%	91.67%	85.42%	85.42%	85.42%	91.67%
Knowing Students' conceptual Understanding	87.5%	95%	92.5%	95%	85%	85%	90%	85%	85%

The percentage of inter-rater agreement of video observation indicated a reliable observation result since the minimum acceptable reliability averaging across coders was 85%. However, there are several disagreements in the MPCK Knowing students' conceptual Understanding.The second pilot coder stated that it needed a clearer explanation of the problem-solving strategy such as the unitary method, cross multiplication, and giving formula. Therefore, it would be considered in the observations of three teachers. Furthermore, the observer discussed the disagreement components and finally reached and agreement.

The potential for the realization of MPCK in good teacher’s (GG) teaching

In the video vignettes of the first meeting, we found several findings about MPCK factor components. Three natural ratio and proportion meetings of GG were analyzed. To illustrate the use of ratio, GG provided a context of sharing watermelon and drawing a circle to be divided and shared into 6 people so that the first ratio number structure that students grabbed was 1: 6. She used ratio to compare quantities, showing three board markers and three rulers and asking a question about the ratio of the number of the ruler to all of the rulers and board markers. She pointed out the number of the ruler was three and the whole was six and formulated a ratio of 3: 6. In addition, there were some non-integer multiple number structures resulting from the students’ data on the number of boys and girls, such as the ratio of boys to all students in group AB and the ratio of girls to all students in group AB were 2/11 and 9/11, respectively. There was an error made by a group of students in determining the ratio of boys to girls in group AB. GG used this error for discussion and asked all students to explore why that group showed a different result. There was a verbal negotiation in the discussion between the teachers and students to get the correct answer. Afterward, GG shared another proportion problem involving boys and girls students and doubling number structure: "Given the total number of students is 30 students, and the ratio of boys to girls is 2: 4. How many boys and girls? GG guided students posited the ratio for boys was 2 in the numerator over the ratio of all students (here was 6) in the denominator multiplied by 30. Afterward, one of the students found the correct number of girls. We explored scenes that revealed MPCK factors such as ratio and proportion task level feature, teaching problem-solving strategy and knowing students' conceptual understanding. The connection to the number structures utilized in that task was underlined when referring to the MPCK factor of "ratio and percentage task level feature." In the second meeting, GG gave an illustration of scale on the map and explained that if students know the distance between two cities on the map, they might be curious about the real distance between those two cities. Then, she continued by providing a measurement context with a ruler. GG demonstrated measuring a board marker with a ruler and explained to students that if they wanted to draw it on the small paper, they needed a scale such as 1: 200. Furthermore, GG discussed the meaning of that scale by challenging students to express what it means by 1:200 or how to read that scale number. Through discussion with students, GG formulated a function of scale that could be used to find the real distance or size. Furthermore, she explained how to find the real distance if given the scale and distance on the map by giving the students an opportunity to learn the essence of the scale used. For example, GG stated *'We already discussed the meaning of the scale, and we see that since the real distance was longer than on the map, the way to find the real distance is by multiplying the scale and the size of the map. And you can find the size on the map by the real distance divided by the scale."* GG did not consider working with big integer multiple number structures but tended to challenge students with an easy integer multiple number structure.

A table was presented in the worksheet and students were asked to measure things nearby them, such as a book, pencil, and pen. The data would be submitted to the table and given some scale, such as 1: 4, 1: 5, and 1:10.



No	Nama benda	Us	Skala	Up
1.	Pensil	18 cm	1 : 4	4,5 cm
2.	Penghapus	4 cm	1 : 5	1,25 cm
3.	Bolpoin	15 cm	1 : 10	1,5 cm
	Kesimpulan	Ukuran pada peta lebih pendek daripada Ukuran panjang sebenarnya		

No	Things	The real size	Scale	Size on map/picture
1	Pencil	18 cm	1:4	4,5 cm
2	Eraser	4 cm	1:5	1,25 cm
3	Pen	15 cm	1:10	1,5 cm

Conclusion: The size on map is shorter than the real size

Figure 2. Students' worksheet

In the third meeting, GG focused on the proportional problem. She gave a clear illustration. GG showed a cake she brought and mentioned that it would be shared with three people. She asked how many parts each person got, and the students answered that it was 1/3. The next question is how many cakes were needed for six people? GG addressed this kind of problem and elaborated on it in that day's lesson. Students answered that it needed two cakes. GG continued by explaining the mathematics model to find that solution and wrote $\frac{1\ kue}{3\ anak} = \frac{?}{6\ anak}$ and pointed out the verbal mathematics of '*for every one cake is for 3 people*, then for 6 people, how many cakes are needed?' She challenged students to share their strategies. Two students shared different strategies, such as one student considered equivalence fractions and another shared a cross multiplication strategy. Instead of giving feedback directly to two different strategies, GG tends to share other proportional problems with non-integer multiple number structures within the context of time spent reading textbooks related to the number of pages. The problem was that "*Mrs. GG has a book with 30 pages that could be read within 2 days. If Mrs. GG read that book for 3 days, how many pages can Mrs. GG read?"* Another student shared her solution to this problem with the cross-multiplication strategy. Furthermore, GG led a discussion on the use of equivalence fraction to solve the problem that pointed to the non-integer multiplier.

GG: I have another problem. A thirty-page book could be read within 2 days. How many pages does Mrs. GG read if she has 3 days? What would you say if I used this strategy (refer to the equivalence fraction strategy)?

Students: It is more difficult.

GG: Yes, it is more difficult since the multiplier is non-integer or in the form of a fraction.

The way the GG teacher gave responses to two different students' strategies showed her appropriate feedback and evaluation to help students become aware of the differences. Besides, another event in the example stage of Meeting 3, the GG teacher intended to share the precise use of the language of the unit to solve the proportional problem. The use of "for every" phrase stated by the teacher could be interpreted that she was guiding the students to a unitary strategy. In the context of enlarging rectangular figures, GG continued to give another proportional problem. Students found the context of enlargement to be more challenging because they needed to apply multiplicative reasoning to figures of a similar size. Figure 3 illustrates how one student successfully responded to the first problem while another student made the same error when solving the second one.

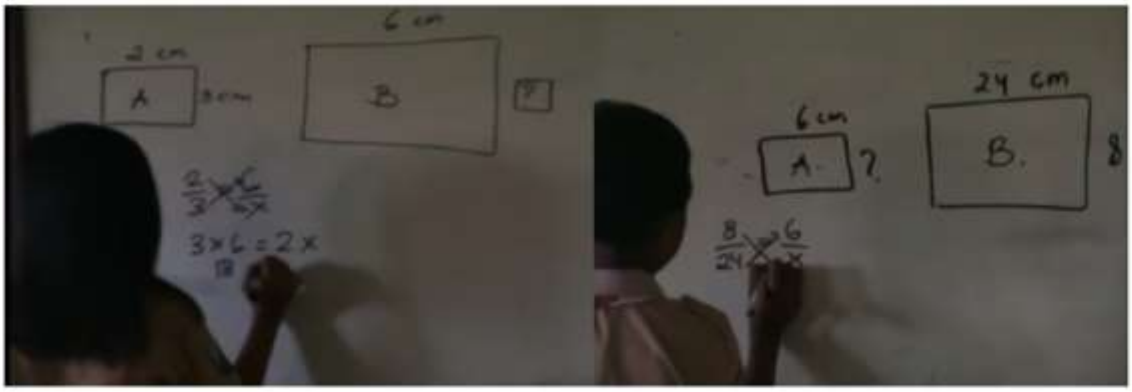


Figure 3. Students perform the problem solution on a whiteboard

From the exploration of MPCK in the GG teacher's teaching, it could be summarized that all MPCK factors were presented appropriately in her teaching as modeled in Figure 4.

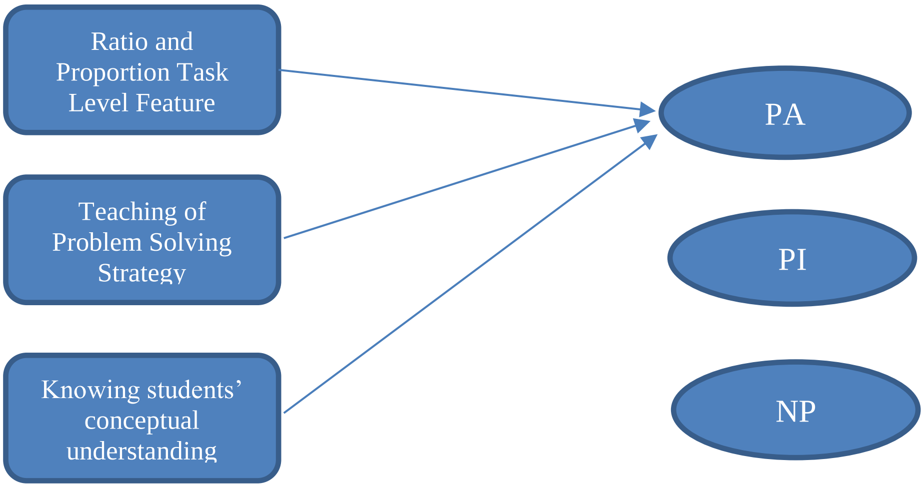


Figure 4. Model of realization of MPCK in GG teaching

The potential for realization of MPCK in medium teacher's (MM) teaching

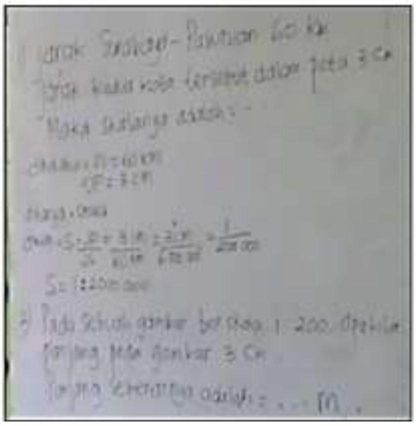
In the first meeting, MM opened the lesson by reminding students about the previous lesson learned. She tried to check students' understanding of the addition of fraction operations by giving a problem on the blackboard ½ + 2/4. In showing the solution, there was a semi monologue between MM and her students. Students need to make the common denominator by finding the least common multiple of the denominator. Afterward, MM explained the aim of the lesson on that day: (1) Students could read the ratio and proportion correctly. For example, the fraction of 12 could be read as 'one over two' or 'one to two'; (2) Students could write the ratio and proportion correctly. She mentioned that there were many applications of ratio and proportion in daily life. MM shared different contexts of *ratio as a comparison quantity*, such as the ratio of books and pens. Students could answer properly that the ratio of book to pen that MM showed was 5 to 4. She highlighted that students need to pay attention to what is mentioned first—It should be in the upper part or written first, and it should not be inverted.

There were four problems given to students. The three problems were about representing the ratio of boys and girls, the ratio of the triangle to rectangle given, and the ratio of students following school activities (i.e., volleyball and scouts) with a non-integer multiplier number structure such as 9: 6, 9:15, and 6:15. Another problem was the proportional problem with money: "The ratio of money between Ana and Yuli = 2:3. The amount of Yuli's money is IDR75,000,-. How much is Ana's money?". Some students raised a question about the solution's strategy for proportional problems, and MM came to them to guide them individually. From the teaching video scene, it could be observed that the teacher took most of the intervention in teaching, so that it's a lack of students productively working on the problem. The students' intention to work on the problems was to apply formulas to solve them.

In the second meeting, MM pointed out that the ratio is the simplest form of a fraction. She described the symbolization of ratio within the fractional form written with a division sign. She performed brief monologues that stated the simplest form of the fraction and constantly reminded students of the value of simplifying fractions.

Afterward, MM gave a scale problem, "*The distance from Sidowungu to Kedurus is 10 km. It was drawn on 2 cm paper. What scale that is used?* ". To answer that problem, MM shared the formula for finding the scale (See figure 4). She also intended to tell the manipulative of the scale. It was done by small number division such as assuming the scale was 3 and the distance on the map was 6. Therefore, the real distance should be 2. From the assumed number, MM guided students to find the 6 was from 2 times 3. This is implied in the second **formula**, finding the distance of map = scale x the real distance.

In the students' work time stage, MM started by posing two-scale problems on the board and asked students to share their findings. One problem was about finding the scale, and another one was about finding the real size of pictures given the scale and the size on the map.



1. The distance of Surabaya to Pasuruan is 60 km, The distance of those two cities on map is 3 cm. What is the scale?

Given : Js (real distance) = 60 km
Jp (distance on map) = 3 cm
Asked: scale
Answer : S= Jp/Js = 3 cm/60km= 1/2000000= 1: 2000000

2. On the picture with scale 1 : 200, if the length on picture is 3 cm. The real length is .....m

Figure 5. Student solution to a scale problem

From the scene, MM tried to interrupt the student when he wrote his result. Directly, MM asked to change the scale into the fractional form and always reminded students to change the measurement unit from centimeter to meter. Figure 6 demonstrates these findings.

In the third meeting, MM shared a proportion problem: *"the ratio of red marbles to yellow marbles is 2 : 3. If the number of red marbles is 20 pieces, how many do the yellow marbles have?"* Students were asked to read aloud the problem. MM reminds students about the rule of solving proportional problems, such as "the number of ratio of what is being asked should be posited in the upper part (numerator). As problem number 1, it asked for the red one, so the ratio of the red should be in the upper part of the green one."

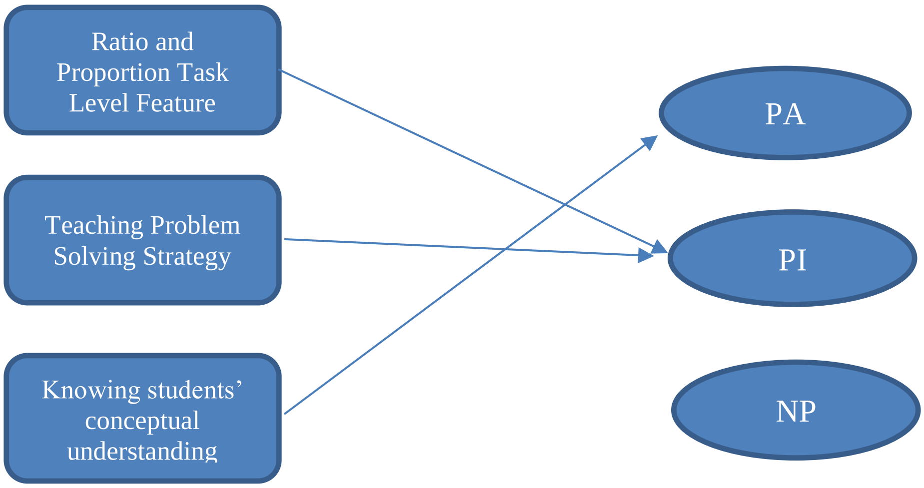


Figure 6. Model of realization of MPCK in MM teaching

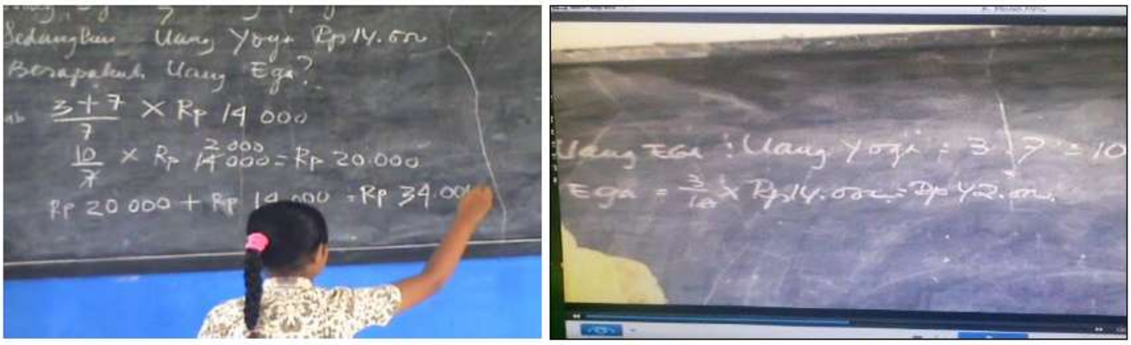
The potential for the realization of MPCK in low teacher's (LL) teaching

As to other teachers in this study, three MPCK factors were also explored in LL's classroom. In the first meeting, LL started the lesson by motivating students to learn mathematics that is applied in daily life. LL shared a question: "What is a ratio?" A student stated that a ratio is the simple form of a fraction. To illustrate, she wrote a fraction 4/7 and explained that a fraction could be regarded as a ratio of 4: 7. Another fraction she shared was 5/750, which could be interpreted in the ratio form of 5: 750.

In the example stage of LL's teaching, she considered giving some examples of the proportional problem. She had a problem with buying in a grocery store as when Toni was asked by his mother to buy rice. The price for 5 kg of rice is Rp. 30.000,- and the mother asked to buy 8 kg of rice. How much money does he need to bring to the store? She explained the solution and asked students about the calculation. LL also explained the equivalence fraction strategy to solve the proportional problem. She wrote 5/ 8 = b/40. To get b, LL shared the need to find the multiplicative factor for the numerator and denominator. For instance, the denominator is multiplied by 5 to range from 8 to 40, while the numerator is multiplied by 5 to equal 25.

In the students' work time stage, LL gave three proportional problems. She divided students into four groups to discuss the solution. The first problem she shared for students to work on was "Ira's age is 40 years old. The ratio of Ira and Ira's father is 2 :3. What is Ira's father's age?".
"Student 1: sixty"
"LL: We could take half of Ira's age and add it to Ira's age because Ayah is older than Ira."

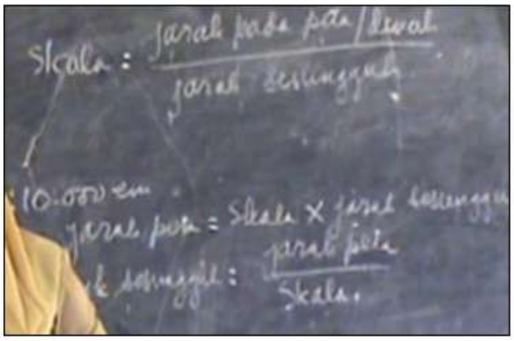
Furthermore, there is a video vignette in which LL gives a problem to the student: " Ega's money = 3/7 Yoga's money. Yoga's money is Rp. 14000,-. How much is Ega's money?".
One student raised her hand and was willing to share her solution. She made an error by adding the ratio, and putting it as the numerator, as shown in the figure below.



Ega's money: Yoga's money = 3:7 =10
Ega's money = $\frac{3}{10}\ x\ Rp.14.000 = Rp.42.000,-$

Figure 7. LL and her students show the wrong solution

LL did not realize that the problem should not be solved by adding the ratio as she mentioned before. In the next meeting, LL explained about scale. She always mentioned that students needed to memorize the formula and it would be easy for them to solve scale problems.



Scale = $\frac{distance\ on\ map/sketch}{real\ distance}$

Distance on map = scale x real distance

Real distance =$\frac{distance\ on\ map}{scale}$

Figure 8. Formula related to scale

LL shared more examples of the scale, such as on the map, the size would be in centimeters, for example, 1 cm, and the real size could be 100 meters or maybe in kilometers. During the students' work time, LL shared a scale problem for students to work out. "The length of the railway from Surabaya to Kediri on the map is 18 cm. The scale on the map is 1: 450000. What is the real length of the railway? When students worked on that problem, LL reminded them of the formula she shared. She pointed out that it would be easier for students if they could remember all the formulas to apply. The illustration above showed the MPCK factor of ratio and proportion task level feature which was presented inappropriately since the target task was not enabling the student to work productively. The phenomena of exploring the MPCK in teaching is represented in Figure 9.

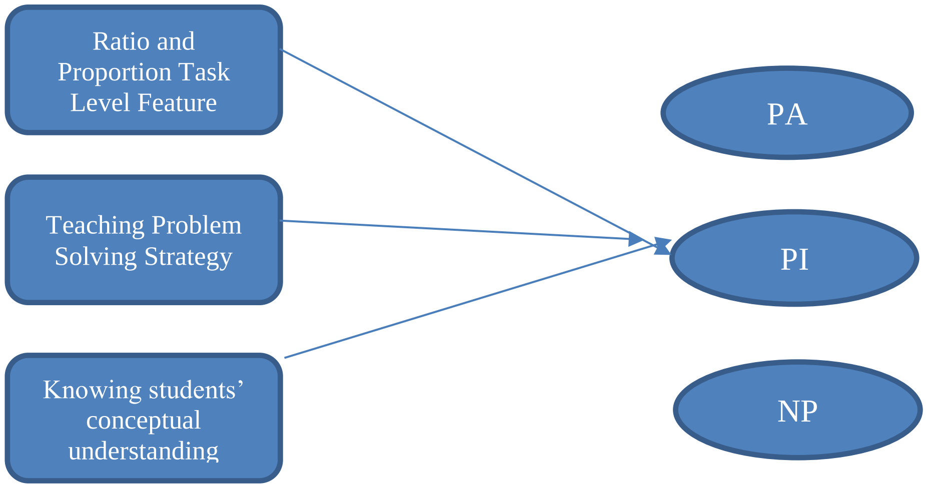


Figure 9. Model of realization of MPCK in LL teaching

DISCUSSION

The activation of MPCK in teaching ratio and proportion

This part discusses the different teachers with different levels of knowledge and performance in teaching practice. In practice, MCK, MPCK, and their relationships were dynamically explored and captured. Through teaching, the three participating teachers' MCK and MPCK may be captured. The interrelation of MCK and MPCK was observed in some teaching stages in good teacher (GG). Teachers with better MCK and MPCK knowledge show their knowledge's existence and appropriateness. This finding is in line with Kahan, Cooper, and Bethea (2003) and Yang et al. (2020), who share that strong Mathematics Knowledge becomes a factor in recognizing and seizing teachable moments. A good teacher might be able to read the teaching material comprehensively, and she/he might not just deliver the teaching material directly to students. This study's results valued and confirmed the translation of the dynamical relation of knowledge. It reflects that good teachers create a more active environment for students' learning by elaborating MCK and MPCK within their teaching, which then affects students' and teachers' interaction in the teaching environment. This finding can contribute to the study of Jordan et al. (2010) that found a positive relationship between teachers' pedagogical content knowledge and the quality of teacher-student interaction around the Mathematics task. To be more specific, the intertwinement relation finding in this study regards four components, such as providing task features regarding students' cognitive development, giving motivation to learning with a demanding systematic skills strategy, encouraging students to be aware of misconceptions by identifying different alternative solutions, and matching feedback to students' thinking with regards to mathematics connection. These elements stand in for student sensitivity and mathematical challenges in Jawroski's teaching triad (2002). Based on qualitative video analysis on the medium teacher, the teacher presented potential MPCK factors in her teaching, though the tendency of this teacher's teaching was to present a formula to be applied algorithmically to the student. The medium teacher directed students to find the solution to ratio and proportion problems with the rules as presented in the textbook, and less interaction between teachers and students happened. A similar situation appears in the study of Ross et al. (2003) which they used the term 'low reform teachers' who shared the activities of the textbook frequently but transformed them in subtle ways so that traditional practices were maintained. This finding corresponds the claims that many teachers merely use the textbook as an activity book (Lepik et al., 2015; Van Den Ham & Heinze, 2018), which frequently dominates their teaching practices (Pansell & Bjorklund Boistrup, 2018; Khalil, 2021). As a result, the students are unable to properly utilize the book as a versatile learning tool. These demonstrate that for many teachers, a textbook plays a crucial part in teaching mathematics.

In addition, in the case of the low teacher, she could perform the potential MPCK factor of teaching problem-solving strategy factor as a good teacher. To be more specific, she presented an appropriate primitive strategy for solving the proportional problem that is suitable for the primary level. This performance could not be found in teachers with a higher level of knowledge (i.e., medium teacher). It showed that even low-performance teachers based on paper and pencil tests could identify a good problem-solving strategy for the primary level in the classroom. From this phenomenon, it could be interpreted that teaching practice was not only dependent on the knowledge that a teacher had. It might also be affected by teachers' professional backgrounds (Graham et al, 2020; Podolsky et al, 2019) and sources that they used (Van Zanten & van den Heuvel-Panhuizen, 2018). Furthermore, compared to Medium and Good teachers, Low teachers had more inappropriate potential MPCK in practice.These findings supported Gamayao & Jr. (2021) which highlight the significant relationship between pedagogical content knowledge and teaching competence. Besides the teachers' knowledge that could be observed in the teaching practice as foregrounded in this study, it is also possible to explore knowledge that inhibited or could not be observed in teaching practice, such as those that are regarded as knowledge for practice and knowledge of the practice of Cohran & Lytle (1999).

The findings of this study also imply that professional development curricula should promote teachers' use of MPCK and MCK. In this regard, certain initiatives have been made to raise teachers' MPCK and MCK levels as well as to aid in the development of their teaching practices. According to several studies, teachers' participation in learning trajectory-based professional development has a considerable impact on their mathematical expertise for teaching (Sarama et al., 2016; Wilson et al., 2014). According to this study, when creating a professional development (PD) program, a designer should prioritize teaching problem-solving strategies, recognizing students' conceptual understanding, and task level features. Teachers in such a PD should also develop their skills in constructing learning trajectories that may be included in their teaching practices, in addition to having a solid understanding of MCK and MPCK. . In this case, Wilson et al. (2014) exemplify that in constructing a learning trajectory for specific content, a teacher may use their knowledge of instructional design to support learners' cognitive development along the trajectory they designed to aid students' voices in developing into mathematical viewpoints.

CONCLUSION

The study demonstrates the existence of MPCK within the teachers’ natural mathematics instruction. Regarding the potential of MPCK factors, there are three different direct transformations of MPCK into mathematics teaching practice (Good, Medium, and Low teachers). All MPCK factors were activated by the good teacher in her teaching, which appropriately differs from the medium and low teacher. The medium teacher needs more opportunities to learn about ratio and proportion task level features as well as teaching problem-solving strategies. Furthermore, the low teacher should connect and enhance her MCK and MPCK on the content of ratio and proportion. It showed that Mathematics Pedagogical Content Knowledge (MPCK) is very essential for mathematics teaching and students’ learning process. This has implications for future mathematics teacher education, which gives the teachers the opportunity to learn both mathematics content and mathematics pedagogy in balanced and integrated courses.

REFERENCES

Abrahamson, D., & Zolkower, B. (2020). Reinventing Realistic Mathematics Education at Berkeley—Emergence and Development of a Course for Pre-service Teachers. In M. Van Den Heuvel-Panhuizen (Ed.), *International reflections on the Netherlands Didactics of Mathematics* (pp. 255–277). Springer Open. https://library.oapen.org/bitstream/handle/20.500.12657/23013/1007148.pdf?sequence=1#page=260

Adak, B., & Aliustaoğlu, F. (2020). An Investigation of 7th Grade Students' Misconceptions about Proportion Ratio. *Online Journal of Mathematics, Science and Technology Education*, *1*(1), 55-74. https://uzak.kasefuk.org/ojomste/index.php/1/article/view/4

Anat, K., Einav, K., & Shirley, R. (2019). Development of mathematics trainee teachers' knowledge while creating a MOOC. *International Journal of Mathematical Education in Science and Technology*, *51*(6), 939–953. https://doi.org/10.1080/0020739X.2019.1688402

Anderson, J., White, P., Research, P. S.-M. E., & 2005, undefined. (2005). Using a schematic model to represent influences on, and relationships between, teachers' problem-solving beliefs and practices. *Springer*, *17*(2), 9–38. https://idp.springer.com/authorize/casa?redirect_uri=https://link.springer.com/article/10.1007/BF03217414&casa_token=yZRrUsIiNb0AAAAA:bbH-DHroDHMjuqCcpHZpjtBRgIek7GQNXWABB6xHGAeDesoV-Rge5j3vU7fqsVi2whn5FRKOe96d9X8

Andrews, P., & Xenofontos, C. (2015). Analysing the relationship between the problem-solvingrelated beliefs, competence and teaching of three cypriot primary teachers. *Journal of Mathematics Teacher Education*, *18*(4), 299–325. https://doi.org/10.1007/S10857-014-9287-2

Ball, D. L., & Thames, M. H. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher*, *59*(5), 389–407. https://doi.org/10.1177/0022487108324554

Brodie, K., & Sanni, R. (2014). "We won't know it since we don't teach it": interactions between teachers' knowledge and practice. *African Journal of Research in Mathematics, Science, and Technology*, *18*(2), 188–197. https://doi.org/10.1080/10288457.2014.930980

Cheang, W. K., Yeo, J. K. K., Chan, E. C. M., & Lim-Teo, S. K. (2007). Development of mathematics pedagogical content knowledge in student teachers. *The Mathematics Educator*, *10*(2), 27–54. https://repository.nie.edu.sg/handle/10497/102

Cochran-Smith, M., & Lytle, S. L. (1999). Relationships of knowledge and practice: Teacher learning in communities. *Review of Research in Education*, *24*, 249–305. https://doi.org/10.3102/0091732X024001249

Depaepe, F., Van Roy, P., Torbeyns, J., Kleickmann, T., Van Dooren, W., & Verschaffel, L. (2018). Stimulating pre-service teachers' content and pedagogical content knowledge on rational numbers. *Educational Studies in Mathematics*, *99*(2), 197–216. https://doi.org/10.1007/S10649-018-9822-7

Ekawati, R., Fiangga, S., & Siswono, T. Y. E. (2018). Historical aspect of mathematics on Indonesian mathematics textbook. *IOP Conference Series: Materials Science and Engineering*, *434*(1). https://doi.org/10.1088/1757-899X/434/1/012001

Ekawati, R., Lin, F. L., & Yang, K. L. (2014). DEVELOPING AN INSTRUMENT FOR MEASURING TEACHERS' MATHEMATICS CONTENT KNOWLEDGE

ON RATIO AND PROPORTION: A CASE OF INDONESIAN PRIMARY TEACHERS. *International Journal of Science and Mathematics Education*, *13*(1), 1–24. https://doi.org/10.1007/S10763-014-9532-2

Ekawati, R., Lin, F., & Mathematics, K. Y.-. (2015). Primary teachers' knowledge for teaching ratio and proportion in mathematics: The case of Indonesia. *Eurasia Journal of Mathematics, Science and Technology Education*, *11*(3), 513–533. https://www.ejmste.com/article/primary-teachers-knowledge-for-teaching-ratio-and-proportion-in-mathematics-the-case-of-indonesia-4394

Engeln, K., Euler, M., & Maass, K. (2013). Inquiry-based learning in mathematics and science: a comparative baseline study of teachers" beliefs and practices across 12 European countries. *Researchgate.Net*, *45*(6), 823–836. https://doi.org/10.1007/s11858-013-0507-5

Erickson, F. (2006). Definition and Analysis of Data from Videotape: Some Research Procedur. In J. Green, J. Camilli, & P. Elmore (Eds.), *Handbook of Complementary Methods in Education Research* (3rd ed., pp. 177–191). Routledhe. https://www.taylorfrancis.com/chapters/edit/10.4324/9780203874769-12/definition-analysis-data-videotape-research-procedures-rationales-frederick-erickson

Fawns, R., & Nance, D. (1993). Teacher Knowledge, Education Studies and Advanced Skills Credentials. *Australian Journal of Education*, *37*(3), 248–258. https://doi.org/10.1177/000494419303700303

Fenstermacher, G. D. (1989). Philosophy of research on teaching: Three aspects. *Handbook of Research on Teaching*, *3*(4), 37–49.

Forsythe, M. E., Criswell, B. A., Arias, A. M., Ellis, J. A., Escalada, L., Johnson, H. J., Palmeri, A. B., Riccio, J., & Parker, M. E. (2022). The Framework for Analyzing Video in Science Teacher Education (FAVSTE). *Taylor & Francis*, *33*(6), 621–640. https://doi.org/10.1080/1046560X.2021.1970698

Gamayao, M., & Binas, J. (2021). Teaching competence and pedagogical content knowledge of science teachers in the First District of Capiz, Philippines: Basis for a sustainable instructional. *European Journal of Humanities and Educational Advancement*, *2*(1). https://www.researchgate.net/profile/Enrique-Binas/publication/348834304_Teaching_Competence_and_Pedagogical_Content_Knowledge_of_Science_Teachers_in_the_First_District_of_Capiz_Philippines_Basis_for_a_Sustainable_Instructional_Program/links/60125f4745851517ef1eb7dc/Teaching-Competence-and-Pedagogical-Content-Knowledge-of-Science-Teachers-in-the-First-District-of-Capiz-Philippines-Basis-for-a-Sustainable-Instructional-Program.pdf

Graham, L. J., White, S. L., Cologon, K., & Pianta, R. C. (2020). Do teachers' years of experience make a difference in the quality of teaching?. *Teaching and teacher education*, *96*, 103190.

Glassmeyer, D., Brakoniecki, A., & Amador, J. (2021). Identifying and supporting teachers' robust understanding of proportional reasoning. *The Journal of Mathematical Behaviour*, *62*, 100873. https://www.sciencedirect.com/science/article/pii/S0732312321000341

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Loewenberg Ball, D. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Taylor & Francis*, *26*(4), 430–511. https://doi.org/10.1080/07370000802177235

Im, S., & Jitendra, A. (2020). Analysis of proportional reasoning and misconceptions among students with mathematical learning disabilities. *The Journal of Mathematical Behaviour*, *57*, 100753. https://www.sciencedirect.com/science/article/pii/S0732312319300550

Izsák, A., & Jacobson, E. (2017). Preservice teachers' reasoning about relationships that are and are not proportional: A knowledge-in-pieces account. *Journal for Research in Mathematics Education*, *48*(3), 300–339. https://pubs.nctm.org/view/journals/jrme/48/3/article-p300.xml

Khalil, I. (2021). Mathematics textbooks content development at the primary stage: a proposed vision. *Journal for the Mathematics Education and Teaching Practices*, *2*(2), 47-53.

Jacobs, J., Hollingsworth, H., Methods, K. G.-F., & 2007, undefined. (2007). Video-based research made "easy": Methodological lessons learned from the TIMSS video studies. *Journals.Sagepub.Com*, *19*(3), 284–299. https://doi.org/10.1177/1525822X07302106

Jaworski, B. (2002). *Investigating mathematics teaching: A constructivist enquiry*. Falmer Press. https://www.taylorfrancis.com/books/mono/10.4324/9780203454213/investigating-mathematics-teaching-barbara-jaworski

Jitendra, A. K., Harwell, M. R., Dupuis, D. N., Jitendra, A. K., Harwell, M. R., Dupuis, D. N., & Karl, S. R. (2015). A randomized trial of the effects of schema-based instruction on proportional problem-solving for students with mathematics problem-solving difficulties. *Journal of Learning Disabilities*, *50*(3), 322–336. https://doi.org/10.1177/0022219416629646

Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., Tsai, Y.-M., Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klussmann, U., Krauss, S., Neubrand, M., & Tsai, Y.-M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*(1), 180. https://doi.org/10.3102/0002831209345157

Kahan, J., Cooper, D., & Bethea, K. (2003). The role of mathematics teachers' content knowledge in their teaching: A framework for research applied to a study of student teachers. *Journal of Mathematics Teacher Education*, *6*(3), 223–252. https://idp.springer.com/authorize/casa?redirect_uri=https://link.springer.com/article/10.1023/A:1025175812582&casa_token=-dxXrbrr_oAAAAAA:fQevzglETs8u6CIJhaPqntodo-DSEJuVeRa_taljlTEv2wiV8gE1WBuOgoxU-3qIsE4c8iLybEerMHM

Leavy, A. (2015). Looking at practice: revealing the knowledge demands of teaching data handling in the primary classroom. *Mathematics Education Research Journal*, *27*(3), 283–309. https://doi.org/10.1007/S13394-014-0138-3

Lepik, M., Grevholm, B., & Viholainen, A. (2015). Using textbooks in the mathematics classroom–the teachers' view. *Nordic Studies in Mathematics Education*, *20*(3-4), 129-156.

Mewborn, D. (2003). Teaching, teachers' knowledge, and their professional... - Google Scholar. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 45–52). NCTM. https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Teaching%2C+teachers'+knowledge%2C+and+their+professional+development.+In+J.+Kilpatrick%2C+W.+G.+Martin%2C+%26+D.+Schifter+%28Eds.%29%2C+A+Research+Companion+to+Principles+and+Standards+for+School+Mathematics+&btnG=

Moliner, L., & Alegre, F. (2022). Attitudes, beliefs and knowledge of mathematics teachers regarding peer tutoring. *European Journal of Teacher*, *45*(1), 93–112. https://doi.org/10.1080/02619768.2020.1803271

Munter, C., & Wilhelm, A. (2021). Mathematics teachers' knowledge, networks, practice, and change in instructional visions. *Journal of Teacher Education*, *72*(3), 342–354. https://doi.org/10.1177/0022487120949836

Özdemir, B. G., & Soylu, Y. (2017). Examination of The Pedagogical Content Knowledge of Mathematics Teachers. *International Online Journal of Primary Education*, *6*(1), 26–39.

Pansell, A., & Bjorklund Boistrup, L. (2018). Mathematics teachers' teaching practices in relation to textbooks: Exploring praxeologies. *The Mathematics Enthusiast*, *15*(3), 541-562.

Patton, M. (2002). *Qualitative research and evaluation methods* (3rd ed.). Sage Publications. https://scholar.google.com/scholar?hl=en&as_sdt=0%2C5&q=Qualitative+research+and+evaluation+methods+%283rd+ed.%29.+Thousand+Oaks%2C+CA%3A+Sage+Publications&btnG=

Podolsky, A., Kini, T., & Darling-Hammond, L. (2019). Does teaching experience increase teacher effectiveness? A review of US research. *Journal of Professional Capital and Community*, *4*(4), 286-308 https://doi.org/10.1108/JPCC-12-2018-0032

Porter, A. C. (2002). Measuring the Content of Instruction: Uses in Research and Practice. *Educational Researcher*, *31*(7), 3–14. https://doi.org/10.3102/0013189X031007003

Riehl, S., & Steinthorsdottir, O. (2019). Missing-value proportion problems: The effects of number structure characteristics. *Investigations in Mathematics Learning*, *11*(1), 56–68. https://doi.org/10.1080/19477503.2017.1375361

Ross, J., McDougall, D., Hogaboam-Gray, A., & LeSage, A. (2003). A survey measuring elementary teachers' implementation of standards-based mathematics teaching. *Journal for Research in Mathematics Education*, *34*(4), 344–363. https://doi.org/10.2307/30034787

Sarama, J., Clements, D. H., Wolfe, C. B., & Spitler, M. E. (2016). Professional development in early mathematics: Effects of an intervention based on learning trajectories on teachers' practices. *Nordic Studies in Mathematics Education*, *21*(4), 29-55.

Shulman, L. S. (1986). Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*, *15*(2), 4–14. https://doi.org/10.3102/0013189X015002004

Skilling, K., & Stylianides, G.-. (2020). Using vignettes in educational research: a framework for vignette construction. *Journal of Research & Method in Education*, *43*(5), 541–556. https://doi.org/10.1080/1743727X.2019.1704243

Steele, M. D., & Rogers, K. C. (2012). Relationships Between Mathematical Knowledge for Teaching and Teaching Practice: The Case of Proof. *Journal of Mathematics Teacher Education*, *15*(2), 159–180. https://doi.org/10.1007/S10857-012-9204-5

Sztajn, P., & Wilson, P. H. (2019). Learning trajectories and professional development. *Learning trajectories for teachers: Designing effective professional development formath instruction*, 1-14.

Tchoshanov, M. A. (2011). Relationship between teacher knowledge of concepts and connections, teaching practice, and student achievement in middle grades mathematics. *Educational Studies in Mathematics*, *76*(2), 141–164. https://doi.org/10.1007/S10649-010-9269-Y

Van Den Ham, A. K., & Heinze, A. (2018). Does the textbook matter? Longitudinal effects of textbook choice on primary school students' achievement in mathematics. *Studies in Educational Evaluation*, *59*, 133-140

Van Zanten, M., & van den Heuvel-Panhuizen, M. (2018). Opportunity to learn problem solving in Dutch primary school mathematics textbooks. *ZDM*, *50*(5), 827-838.

Walshaw, M. (2012). Teacher knowledge as fundamental to effective teaching practice. *Journal of Mathematics Teacher Education*, *15*(3), 181–185. https://doi.org/10.1007/S10857-012-9217-0

Weiland, T., Orrill, C. H., Nagar, G. G., Brown, R. E., & Burke, J. (2021). Framing a robust understanding of proportional reasoning for teachers. *Journal of Mathematics Teacher Education*, *24*(2), 179–202. https://doi.org/10.1007/S10857-019-09453-0

Wilson, P. H., Sztajn, P., Edgington, C., & Confrey, J. (2014). Teachers' use of their mathematical knowledge for teaching in learning a mathematics learning trajectory. *Journal of Mathematics Teacher Education*, *17*(2), 149-175.

Xie, M., Talin, R., & Sharif, S. (2014). The Relationship between Teachers' Knowledge, Attitude and Belief with the Implementation of Inquiry-Based Learning in Zhengzhou, China. *International Journal of Learning, Teaching, and Educational Research*, *8*(1), 149–161. http://ijlter.net/index.php/ijlter/article/download/1163/1169

Yang, X., Kaiser, G., König, J., & Blömeke, S. (2020). Relationship between pre-service mathematics teachers' knowledge, beliefs and instructional practices in China. *Zdm*, *52*(2), 281-294.

Zulkardi, Z., Putri, R., & Wijaya, A. (2020). Two decades of realistic mathematics education in Indonesia. In M. van den Heuvel-Panhuizen (Ed.), *International reflections on on the Netherlands Didactics of Mathematics* (pp. 325–340). Springer Open. https://library.oapen.org/bitstream/handle/20.500.12657/23013/1007148.pdf?sequence=1#page=328